Control Oriented Modeling of a Hybrid Two-Component Development Process for Xerography

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Abstract

This paper presents a methodology for developing a control oriented model for a hybrid development architecture. The resulting model is suited for multivariable controller design and analysis to stabilize and improve the development process. The proposed methodology extracts relevant characteristics from an experimentally validated complex model based on toner age distribution and development probability. Three lumped state variables are proposed to characterize the macroscopic properties of the process. Ordinary differential equations of the proposed states are then derived to characterize the system dynamics of the process. Simulation results show that, for the properties of interest, the control oriented model yields similar results as the complex model for a wide range of operating conditions. Although the results presented are for a development process, the methodology can also be extended to other xerographic processes.

Introduction

Xerography is the "dry-marking" process used in the majority of laser printers and copiers. A typical xerographic process includes 6 sequential steps around a photoreceptor (PR): charge, expose, develop, transfer, fuse and clean. Detailed description of the steps involved in a xerographic process can be found in [1]-[3]. In this study, we will focus on the development step.

The development step is one of the most important and challenging steps in xerography. It is a key factor in determining marking engine productivity and image quality.

A hybrid two-component development approach was introduced in 1975 by Liebman [4]. This approach is called hybrid since it uses both single-component and two-component development processes, where two-component development refers to a process that uses combination of toner and carrier particles. The hybrid two-component approach has since been used in various products for its high reliability and low operating cost [5,6].

In this paper, we will investigate an example of a hybrid two-component development system as given in Fig. 1. To work through the system, we begin with toner and carrier particles that are dispensed from the dispenser to the sump area, where the toner and carrier particles are mixed and transported through an auger. An important aspect of this mixing is that it facilitates tribocharging between the toner and carrier, which causes the toner to attach to the surface of the carrier. To further assist charging and particle flow, additives are typically blended with the toner prior to being dispensed. Eventually, the toner/carrier mixture is transported by the auger within proximity of the mag roll, where this combination of particles becomes electromagnetically attached to the mag roll. The toner particles are then detached from the carrier particles and are electrostatically transferred to the donor rolls. As the donor roll rotates and aligns with the development electrostatic field formed by the latent image on the PR and the development electrode, the field strength detaches the charged toner particles from the donor roll and moves them on to the PR surface.

Typically, models of the development process have been constructed from a physics perspective to provide phenomenological understanding, predict system behavior, and diagnose system defects. Although accurate, these first-principles based models are typically complex and computationally intensive. They tend to be difficult to use for process control synthesis and analysis.

Control oriented models for printer calibration have also appeared in the literature [7]-[9]. These models treat the development step as a static, nonlinear mapping between the development voltage V_{dev} and the developed mass per area (referred to as DMA) on the PR. One of the widely used models is the "solid area development curve" model [1], which takes the form

$$DMA = \alpha[1 - e^{-\frac{\gamma}{\alpha}(V_{dev} - V_{D0})}] \qquad (1)$$

where V_{D0} is the potential required for the onset of the development, γ is the slope of the development curve at V_{D0}, α represents the maximum achievable DMA. For process control purposes, under specific operating conditions, α, γ and V_{D0} are typically assumed to be constant. However, in practice, customer usage (e.g. area coverage of a print job) and the ambient conditions cause substantial change in α and γ, indicating that the constant-parameter model is limited. This limitation is more prominent for "low" area coverage (e.g., less than 10%).

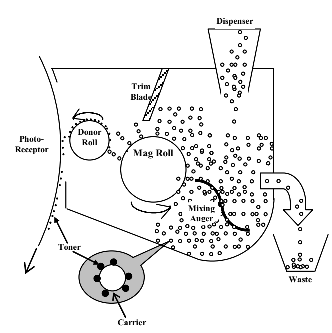

***Figure 1**. Schematic of a hybrid two-component development process*

With more stringent requirements on image quality, it is necessary to develop a methodology for deriving a model that captures the dynamic behavior more comprehensively, but is still manageable for controller analysis and synthesis. In this paper, we will discuss an aggregated approach for constructing such a control oriented model of the hybrid development process, which is based on a complex full-order model developed in [10]. Although we illustrate the methodology on a particular hybrid development process, the approach is generally applicable to other hybrid and two-component development processes.

The first-principle based, full-order model in [10] has been built to characterize the additive burial effect in two-component systems. It is derived based on 1) the toner age distribution at different locations within the development process and 2) the development probability that characterizes the developability of a toner particle. This approach has proven to be successful in predicting the developability of the two component development system. Both toner age distribution and development probability are discretized with respect to time and toner age, and thus, the number discretized states increases unbounded as the process evolves in time. In this paper we will discuss a model reduction approach to derive a control oriented state-space model, with only a few states, from the model in [10].

The remainder of this paper is organized as follows. The complex full-order model [10] for two component development will be summarized in the next section. A simple extension to a hybrid development system will discussed. The aggregated model reduction approach used to derive the control oriented model is discussed in section 3. Model verification results and observations are presented in section 4 followed by conclusions.

System Description and Full-order Model

The development system can be modeled as a cascade structure as Fig. 2, the block on the left is the development model, Eq. (1), and the block on the right is the toner aging dynamics investigated in [10]. In this section, we will briefly introduce the simplification of the development model and review the full-order model [10] based on toner age distribution and development probability, from which the control-oriented model is derived.

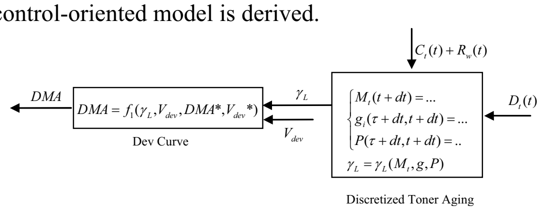

***Figure 2**. Structure of the hybrid two-component development system*

Solid Area Development Curve

As mentioned in the previous section, the solid area development curve model is typically parameterized by three process parameters α, γ and V_{D0}. However, the region of interest in a development curve is around the target mass, DMA*. Experiments suggest that in this region, a simpler local development model may be considered:

$$DMA = f_1(\gamma_L, V_{dev}, DMA^*, V_{dev}^*) \tag{2}$$

where V_{dev}^* is the development voltage needed to achieve DMA* and γ_L is a local process parameter. Experimental data also suggest that γ_L is affected by the *tribo* (charge status of the toners) and a function γ_d that is related to the toner adhesion state on the donor (as will be defined later), i.e., γ_L can be written as

$$\gamma_L = f_2(tribo, \gamma_d) = \frac{A_0}{tribo(M_t, RH)}\gamma_d \tag{3}$$

where A_0 is a scaling constant, *tribo* can be modeled as a function of toner mass M_t (the tribo is inversely proportional to the toner concentration which is the ratio of the toner mass to the carrier mass in the developer sump) and relative humidity (RH).

Evolution of Toner Mass, Development Probability, and Toner Age Distributions

For low throughput print jobs, it has been observed that newly dispensed toner particles are easier to develop compared with the toner particles with a longer residence time. As a result, the toner particles on the donor roll have a longer averaged (over all toner particle population) residence time than the toner particles in the sump. Based on these observations, Ramesh [10] proposed that the developability of the toner particle at different locations (e.g., in the sump or on the donor roll) is determined by the "averaged" burial status of the additives on the surface of toner particles. In addition, the development probability of a toner particle with residence time τ at time t, $P(\tau,t)$, is completely determined by the additive burial status and can be assumed independent of the location of the particle. It is also assumed that fresh toner particles from the dispenser can bring extra additives into the sump area, and part of the fresh additives are free to go to all the toners, instantly and uniformly. With these assumptions, the evolution of the toner mass M_t, the toner age distributions in the sump g_s, and the development probability of toner particles $P(\tau,t)$ have been derived in [10].

From the mass balance, the evolution of M_t can be written as

$$\dot{M}_t(t) = D_t(t) - C_t(t) - R_w(t) \tag{4}$$

where M_t is the toner mass in the sump, C_t is the throughput rate, D_t is the toner dispense rate and R_w is the rate of toner waste. Based on the definition, the evolution of development probability $P(\tau,t)$ can be written as

$$P(\tau+dt, t+dt) = \frac{p_0(t)}{1+p_0(t)} + e^{-\beta dt}\frac{1}{1+p_0(t)}P(\tau,t) \tag{5}$$

where $p_0(t) = p_a D_t(t)dt / M_t(t+dt)$, with p_a denoting the fraction of free additives.

Let $g_s(\tau,t)$ be the fraction of toners in the sump with residence time τ at time t. The evolution of $g_s(\tau,t)$ can be written as

$$g_s(\tau+dt, t+dt) = g_s(\tau,t)\left(\frac{M_t(t)}{M_t(t+dt)} - \frac{C_t(t)dt}{M_t(t+dt)}\frac{P(\tau,t)}{\gamma_s(t)} - \frac{R_w(t)dt}{M_t(t+dt)}\right)$$

$$g_s(0, t+dt) = \frac{D_t(t)dt}{M_t(t+dt)} \tag{6}$$

The development model described in [10] essentially keeps track of the infinite dimensional age distributions and development probability using Eqs. (5)-(6), and calculates the averaged toner developability in the sump (γ_s) using Eq. (7) at each time step, i.e.

$$\gamma_s = \sum_{\tau=0}^{t} P(\tau,t)g_s(\tau,t) \tag{7}$$

The above model for a two component development system can be extended to a hybrid development system by considering the toner age distribution on the donor roll. Let $g_d(\tau,t)$ be the fraction of toners on the donor rolls with residence time τ at time t, and M_d be the toner mass on the donor roll, which is assumed to be a constant. We will assume that the additive state of the toners given by the development probability $P(\tau,t)$ describes the toner age selectivity in the mag brush to donor loading as well as donor to PR development processes. Thus the evolution of $g_d(\tau,t)$ can be written as

$$g_d(\tau+dt,t+dt) = g_d(\tau,t)\left(1-\frac{C_t(t)dt}{M_d}\frac{P(\tau,t)}{\gamma_d(t)}\right)+\frac{C_t(t)dt}{M_d}g_s(\tau,t)\frac{P(\tau,t)}{\gamma_s(t)}$$
$$g_d(0,t+dt) = \frac{D_t(t)dt}{M_t(t+dt)}\frac{C_t(t)dt}{M_d} \tag{8}$$

where the averaged toner developability on the donor roll (γ_d) is given by

$$\gamma_d = \sum_{\tau=0}^{t} P(\tau,t)g_d(\tau,t) \tag{9}$$

Control Oriented Model of Development

From a controller design and analysis point of view, the dispense rate $D_t(t)$ is the "manipulated input" or "control input" to the *toner aging process* that is independently actuated to keep the process on target, and the throughput rate together with the waste rate, $C_t(t)+R_w(t)$, is a "disturbance input" to the process that is largely determined by customer usage (e.g. area coverage of a print job). The toner mass $M_t(t)$, toner age distributions, $g_s(\tau,t)$ and $g_d(\tau,t)$, and development probability $P(\tau,t)$ are the internal states.

The approach we adopted here to reduce the number of states associated with $g_s(\tau,t)$, $g_d(\tau,t)$, and $P(\tau,t)$ is to examine time evolution of a set of aggregated developabilities over all age distributions at specific spatial locations of the process. Specifically, we propose to keep track of the developabilities of the toners in the sump and at the donor roll, which we will refer to as the sump state $\gamma_s(t)$ and the donor state $\gamma_d(t)$ respectively, as defined by Eq. (8). The toner mass in the sump $M_t(t)$ will still be retained as an internal state that characterizes the change in the flow of material. Without loss of generality, the developability states are normalized to within 0 and 1 with an initial value of 1.

The time evolution of the toner mass in the sump $M_t(t)$ is given by Eq. (4). The time evolutions of the toner age distribution and development probability will be derived in the following subsections.

Evolution of the Sump State

Based on the definition of $\gamma_s(t)$ as in Eq. (6), the evolution of $\gamma_s(t)$ for a short duration dt can be written as

$$\gamma_s(t+dt) = \sum_{\tau=0}^{t+dt} P(\tau,t+dt)g_s(\tau,t+dt)$$
$$= \frac{D_t(t)dt}{M_t(t+dt)} + \sum_{\tau=0}^{t} P(\tau+dt,t+dt)g_s(\tau+dt,t+dt) \tag{10}$$

Using Eqs. (5)-(6) and approximating $P(\tau,t)/\gamma_s(t)$ with 1, we can rewrite Eq. (10) as

$$\gamma_s(t+dt) = \frac{D_t(t)dt}{M_t(t+dt)} + \frac{p_0(t)}{1+p_0(t)}$$
$$+\frac{e^{-\beta dt}}{1+p_0(t)}\frac{M_t(t)-R_w(t)dt}{M_t(t)-R_w(t)dt+D_t(t)dt}\gamma_s(t) \tag{11}$$

Note that in the equation above, the first term on the right hand side represents the effect of newly dispensed toner particles, which only has new additives with burial time zero. The second term represents the effect of new additives on old toners and the third term characterizes the decay of the effectiveness of "old additives" on old toners. The time derivative $\gamma_s(t)$ can then be derived to be

$$\dot{\gamma}_s(t) = \frac{D_t(t)}{M_t(t)}\left(1+p_a\right)\left(1-\gamma_s(t)\right) - \beta\cdot\gamma_s(t) \tag{12}$$

From Eq. (12), when $D_t(t) = 0$, i.e. no dispense, the sump state decays exponentially. When $D_t(t) > 0$, the rate of the decay can be reduced. The amount of reduction is proportional to $(1+p_a)$ and inversely proportional to the amount of toner in the sump $M_t(t)$. More free additives, i.e. larger p_a, results in larger reduction. On the other hand, the more toner in the sump, i.e. larger $M_t(t)$, the less effective will a constant dispense rate D_t have in reducing the decay rate. Another interesting observation from Eq. (12) is that the dispenser is more effective in reducing the decay rate for older toner particles, where $\gamma_s(t)$ is small, i.e. the term $(1-\gamma_s)$ is larger.

Evolution of the Donor State

Based on the definition of $\gamma_d(t)$ as in Eq. (8), following similar approach used for developing the sump state and approximating $P(\tau,t)/\gamma_d(t)$ with 1, the time derivative of the donor state can be written as:

$$\dot{\gamma}_d(t) = \frac{D_t p_a}{M_t(t)}(1-\gamma_d(t)) + \lambda\frac{C_t}{M_d}(\gamma_s(t)-\gamma_d(t)) - \beta\gamma_d(t) \tag{13}$$

where the value of λ needs to be determined based on a more complicated simulation models or experiment.

Equation (13) is very similar to Eq. (12). Without dispensing supply, the donor state also exhibit an exponential decay. The dispensing has a factor of p_a instead of the $(1+p_a)$ factor, which implies that compared with $\gamma_s(t)$, the dispenser is less effective to $\gamma_d(t)$. The second term on the right hand side of Eq. (13) is a proportional control that will drive the donor state to track the sump state. Note that the proportional gain is also proportional to C_t/M_d, which implies a higher gain for higher throughput prints. However, without any integral action, with different gains for different throughputs, the donor state will be quite different between high throughput prints and low throughput prints. For higher throughput prints, the donor state tracks the sump state closely, but for lower throughput prints, the donor state may be quite different from the sump state. Since the dispenser is not as effective to the donor state as it is with the sump state, if we assume $\gamma_s(0) = \gamma_d(0) = 1$, as time goes on, the donor state will be smaller than the sump state. Another note is that the above modeling of the donor state ignores the exchange of toners between the donor roll and the sump.

Combining the evolution of all the states, the hybrid two-component development system can be modeled as a system with three internal states (M_t, γ_s and γ_d), one output (DMA) and two control inputs (D_t and V_{dev}), as shown in Fig. 3. The state equations are

$$\dot{M}_t(t) = D_t(t) - C_t(t) - R_w(t)$$
$$\dot{\gamma}_s(t) = \frac{D_t(t)}{M_t(t)}\left(1+p_a\right)\left(1-\gamma_s(t)\right) - \beta\cdot\gamma_s(t) \tag{14}$$
$$\dot{\gamma}_d(t) = \frac{D_t(t)p_a}{M_t(t)}(1-\gamma_d(t)) + \lambda\frac{C_t}{M_d}(\gamma_s(t)-\gamma_d(t)) - \beta\gamma_d(t)$$

The corresponding output equation is given by combining Eqs. (2) and (3):

$$DMA = f_1(\gamma_L, V_{dev}) = f_1(f_2(tribo(M_t, RH), \gamma_d), V_{dev}) \quad (15)$$

If we further assume that the variation in toner mass M_t is small over time, then Eq. (14) can be further simplified to a structure of a bilinear dynamic system, as shown in Eq. (16), with a static nonlinear output equation of Eq. (15).

$$\dot{\gamma}(t) = \begin{bmatrix} \dot{\gamma}_s(t) \\ \dot{\gamma}_d(t) \end{bmatrix} = \underbrace{\begin{bmatrix} -\beta & 0 \\ \lambda C_t / M_d & -\beta - \lambda C_t / M_d \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \gamma_s(t) \\ \gamma_d(t) \end{bmatrix}}_{\gamma}$$
$$+ \underbrace{\begin{bmatrix} (1+p_a)(1-\gamma_s(t))/M_{t0} \\ p_a(1-\gamma_d(t))/M_{t0} \end{bmatrix}}_{(B_0 + B_1\gamma)} D_t(t) = A \cdot \gamma + (B_0 + B_1\gamma) \cdot D_t \quad (16)$$

In Eq. (16), the (2,1) element of the system matrix A characterize the effect of the toner transport between the sump and the donor roll.

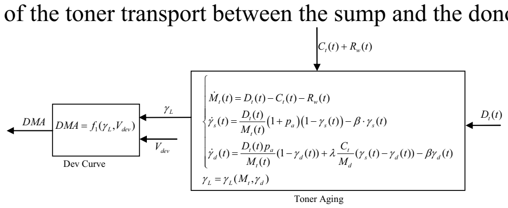

***Figure 3**. Block diagram of the control oriented model*

Model Verification and Observations

To verify that the control oriented model described by Eqs. (14) and (15) is able to capture the characteristics of the hybrid two-component development system, simulation results from the control-oriented model are compared with the simulation results from the complex full-order model. Numerical simulations were performed using the control oriented model and the complex full-order model under the following conditions: a throughput value C_t corresponding to roughly 2% area coverage, a constant 4% toner concentration, and a relative humidity of less than 20%. Figure 4 shows the comparison between the two models for the sump state $\gamma_s(t)$ and donor state $\gamma_d(t)$. In Fig. 4, the red line is the response from the full-order model and the blue line is the response from the control oriented model. It can be seen that the control oriented model is able to capture the main transient dynamics of the complex full-order model. Comparisons using different area coverages as well as different toner concentrations give similar results for area coverages less than 10%.

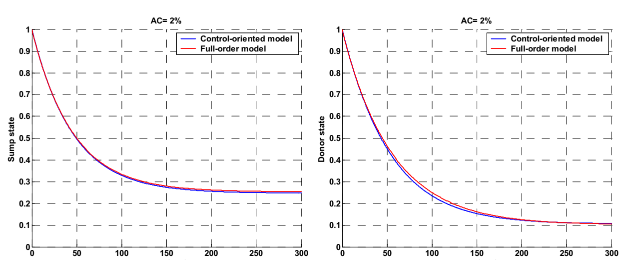

***Figure 4**. Comparison between control oriented model and the full order model*

Given the system level control oriented model, a couple of observations can be made: (1) The hybrid two-component development system is not operating around an equilibrium point. Notice that traditionally, the dispenser is used to maintain a constant toner concentration. For constant throughput printing, the dispense rate. $D_t(t)$ is also a constant (assuming constant waste rate). However, this constant $D_t(t)$ can not maintain other states at constant values, a direct result is that the toner developability will degrade with respect to time, which has been observed in practice. (2) As the developability of the system degrades to a certain level, the performance of the development step will become unacceptable. If changes of some states (such as toner mass M_t) within certain bounds are acceptable, can the toner dispense rate $D_t(t)$ be used to modify some of the states (such as sump state $\gamma_s(t)$) to improve developability? The solution to this issue is not trivial. With the physical limitations such as non-negative dispense rate and desired bounds on each states, there is no immediate criterion to judge if there exists a particular trajectory of $D_t(t)$ that can drive the development system from any initial state to a particular final state.

Conclusions

This paper presented a methodology to derive a control oriented development model for a hybrid two-component xerographic process based on a set of complex full order model with some reasonable assumptions. Numerical simulation shows that, for the properties of interest, the control oriented model yield similar results as the complex model for a wide range of operating conditions. The model can be used for analyzing and synthesizing the process response to different input strategies.

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