Dot Allocations in Dither Matrix with Wide Color Gamut

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Abstract

The color gamut in bi-level digital printers depends on the CMY primary ink dots allocations placed in dithering matrices. Typical color mixture model is based on Neugebauer theory, where CMY dots are placed in random, while the recent digital printers don't obey that theory. The most simple way used in digital printers is "**coaxial**" allocation, where the CMY ink dots are placed in the same positions in each color dither matrix. The coaxial model will produce sharp edges, but it may give the narrower color gamuts due to the area ratio occupied by secondary colors with unsaturated chromaticies. A mixture of C and M inks may produce more brilliant bluish color when C and M dots are placed side-by-side to avoid overlapping. In this paper, "**min-med**", "**min-max**" and "**min**" models with side-by-side dots allocations are discussed. These are designed to suppress the occurrence of secondary colors. The color gamuts are analyzed and compared each other.

Introduction

The color gamut in bi-level digital printers depends on the primary ink dots allocations placed in dithering matrices. Typical color mixture model in bi-level printer is based on Neugebauer equation1, where unit area is composed of at most 8 colored areas, that is, white W, primary colors, C, M, Y, secondary colors, R, G, B, and 3rd-order color, K caused by random mixture of C, M, Y colorants. These 8 basic colors make 12 outer edges of color solid independent of color mixing methods. On the other hand, the surfaces of color solid surrounded by these 12 edges are determined by the primary color dots allocations. We reported[2] the highly saturated colors come from the mixtures of 2 primary colors placed on paper white. The outer surfaces of color solid are determined by the following 6 planes.

(1) M and Y inks on W : W-M-Y-(R)
(2) C and Y inks on W : W-C-Y-(G)
(3) C and M inks on W : W-C-M-(B)
(4) M and Y inks on C : C-(B)-(G)-(K)
(5) C and Y inks on M : M-(B)-(R)-(K)
(6) C and M inks on Y : Y-(G)-(B)-(K)

The bracket denots a secondary and 3rd-order colors. Each plane is not flat but forms a tetrahedral surface based on the primary ink dots allocation.

Color Dot's Allocation Models

Classically, the theoretical model of color mixture in bi-level printer is described by well-known Neugebauer equation, which is applied for conventional printing using halftone screen. Fig.1(a) shows its random mixture model of 3 primary color inks given by Demichel. While the color dot's allocations in recent digital printer are different from Demichel's. For example, (b) illustrates a typical model, what we call "**coaxial**", where the CMY primary ink dots are placed in the same dot's position in each color matrix and unit area is composed of at most 4 colored areas, that is, one primary, one secondary, black K, and white W. The coaxial model is realized by the precious dot position control in digital technology. It will produce sharp edges, but may give the narrow color gamut due to the maximum occurrence of secondary colors with unsaturated chromaticies.

Fig.1(c), (d) and (e) show the three different models "**min-med**", "**min-max**" and "**min**" to suppress the occurrence of secondary colors. Here, the primary color inks P, Q, and S are placed in unit area with dot area ratios p, q, and s. In the **min-med** model, the dots of P and Q are placed in the opposite side to keep the minimum overlapping, while S is placed perpendicular to P and Q, which makes the medium overlapping. The **min-max** model is different from **min-med** only for S to be placed in the same position as P or Q. While in the **min** model, the starting points of P, Q, and S are equally separated by 1/3 of unit area to make the minimum overlapping one another.

Table 1 and 2 show the colored dot area ratios of **min-med** and **min-max** models in case of (P=C, Q=M, S=Y) as compared with (a) and (b). Apparently 8, 4, 6, and 4 colored areas appear in Neugebauer, **coaxial**, **min-med**, and **min-max** models individually. These mixtures of colored areas produce their own color gamuts.

Analysis of Color Gamut

A tristimulus vector **T** of unit area is given by the additive mixture of tristimulus vector $\mathbf{T}_i$ for each colored area **i** as follows.

[Neugebauer] As well known, 8 colored areas appear as a result of random mixture of C, M, and Y.

$$T = \sum_{i=1}^{8} a_i \mathbf{T}_i ; \quad i = W, C, M, Y, R, G, B, K \tag{1}$$

[coaxial]At most, 4 colored areas appear, where six different cases occur for the selections of P and Q from CMY.

$$T = \sum_{i=1}^{4} a_i \mathbf{T}_i ; \quad i = W, P, PQ, K \tag{2}$$

$$P = \max (C, M, Y)$$

$$PQ - \max (C, M, Y) \cap \mathrm{mid} (C, M, Y)$$

P and PQ denote one of the primary and secondary colors.

[min-med] Six different cases, three selections for S from CMY and two conditions for p + q < 1 or p+q + 1 , occur. Each case includes 6 colored areas.

$$T = \sum_{i=1}^{6} a_i \mathbf{T}_i \tag{3}$$

$$i \begin{cases} = W, C, M, Y, PS, QS; & \text{for } p + q < 1 \\ \quad P, Q, R, G, B, K; & \text{for } p + q \le 1 \end{cases}$$

[min-max] Six different cases occur as same as **min-med**. Each case includes at most 4 colored areas.

$$T = \sum_{i=1}^{4} a_i \mathbf{T}_i \tag{4}$$

[min] Totally 60 different cases including i = 4 ~ 6 colored areas appear according to the combinations of the ranges for

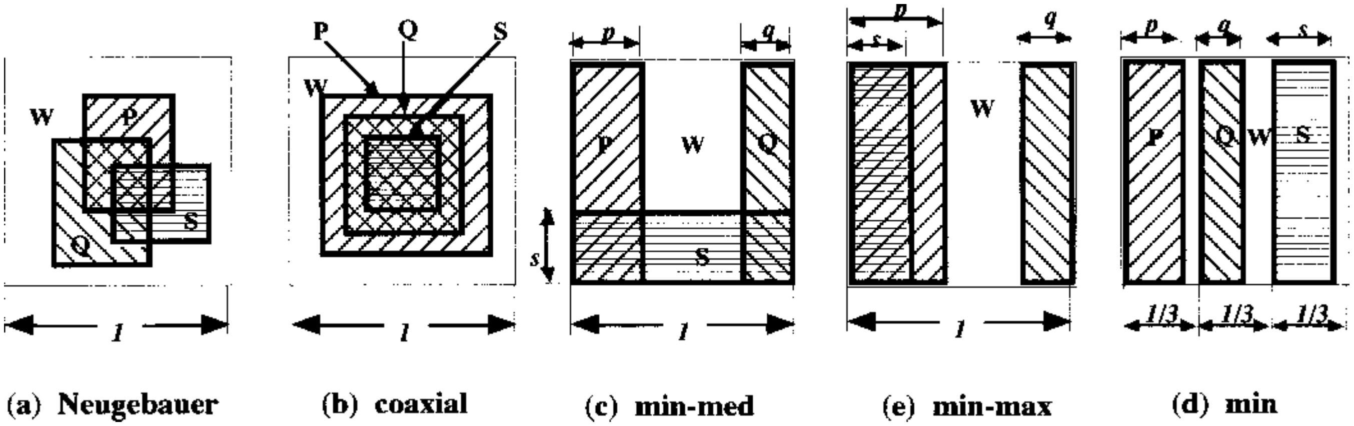
$$0 \le p, q, s < 1/3, \; 1/3 \le p, q, s < 2/3, \text{ or } 2/3 \le p, q, s \le 1.$$

Results

The outer surface of color solid is constructed from six planes, each comprising the tetrahedral color points W-M-Y-(R), W-C-Y-(G), W-C-M-(B), C-(B)-(G)-(K), M-(B)-(R)-(K), and Y-(G)-(B)-(K). Figure 2 shows an example of calculated W-C-M-(B) surfaces for wax-transfer thermal color printer. Generally W, C, M, and B are not on the same plane but make a specified tetrahedron in each dot allocation model. It is shown the **min-med**, **min-max**, and **min** models produce the wider outer surfaces than that of **coaxial.** Figure 3 shows the resultant outer surface of 3 - D color solid in each model. The **coaxial** model used in digital printers gives the smallest color gamut among them.

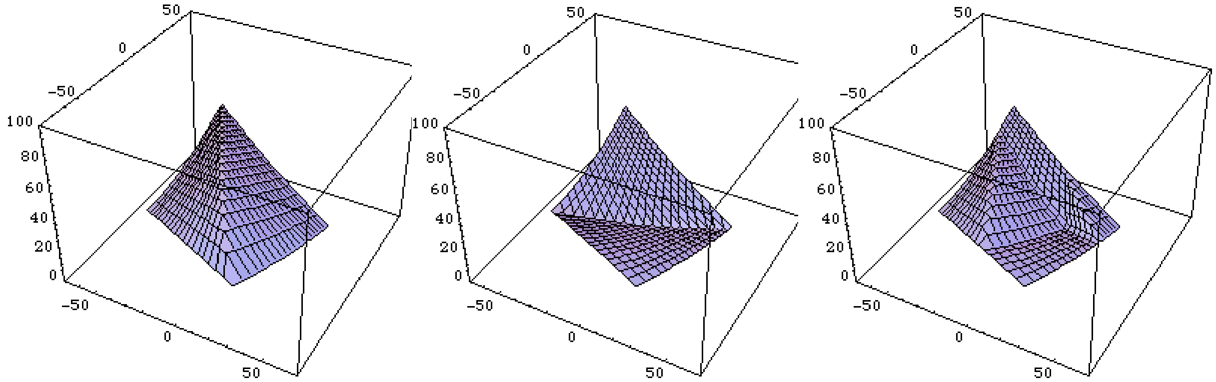
Figure 4 is a comparison of reproducible color plots in (a*-b*) plane sliced in 40 < L* < 60. Coaxial model is unable to fill the bluish color areas sufficiently due to the easy occurrence of secondary color B, while the proposed models give the wider color gamut especially in bluish colors. Figure 5 shows the comparison of 3-D plots inside the color solids when the (c, m, y) area ratios of primary color inks are changed from 0 to 1 by 0.1 step, generating 11^3 = 1331 colors. The min-med and min-max models show the similar distributions, while the min model acts as to produce the balanced mixtures of CMY primary colors.

An example of the dithering matrices for color imaging is shown in Figure 6, where the threshold values in C and M matrices are arranged in the forward and backward order each other to make the minimum overlapping in the **min-max** model (a). While these values are placed rotationally shifted by 1/3 in each color for the **min** model as shown in (b). Here the basic 4 × 4 ordered dither matrices are given and the larger matrices such as 8 × 8 or 16 × 16 are easily determined in same manner. The color images have been processed using these matrices, resulting in the more saturated color renditions than that of **coaxial** model.



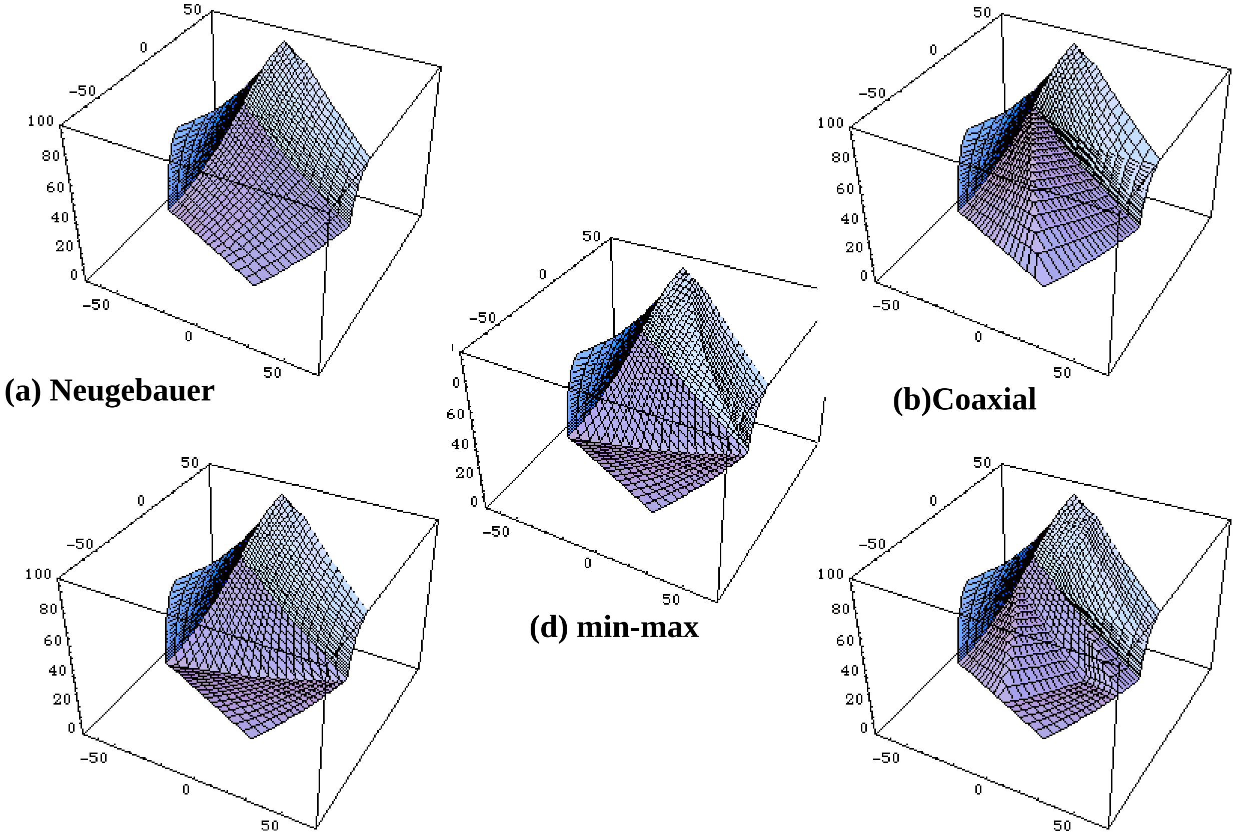
(a) Neugebauer **(b) coaxial** **(c) min-med** **(e) min-max** **(d) min**

Figure 1. Basic models of color dots allocations in bi-level printer



(b) coaxial **(c) min-med** **(e) min**

Figure 2 Comparison of tetrahedral W-C-M-(B) surface (wax-transfer thermal printer)



(a) Neugebauer

(b)Coaxial

(d) min-max

(c) min-med

(e) min

Figure 3 Outer surface of color gamut in each model (wax-transfer thermal printer)

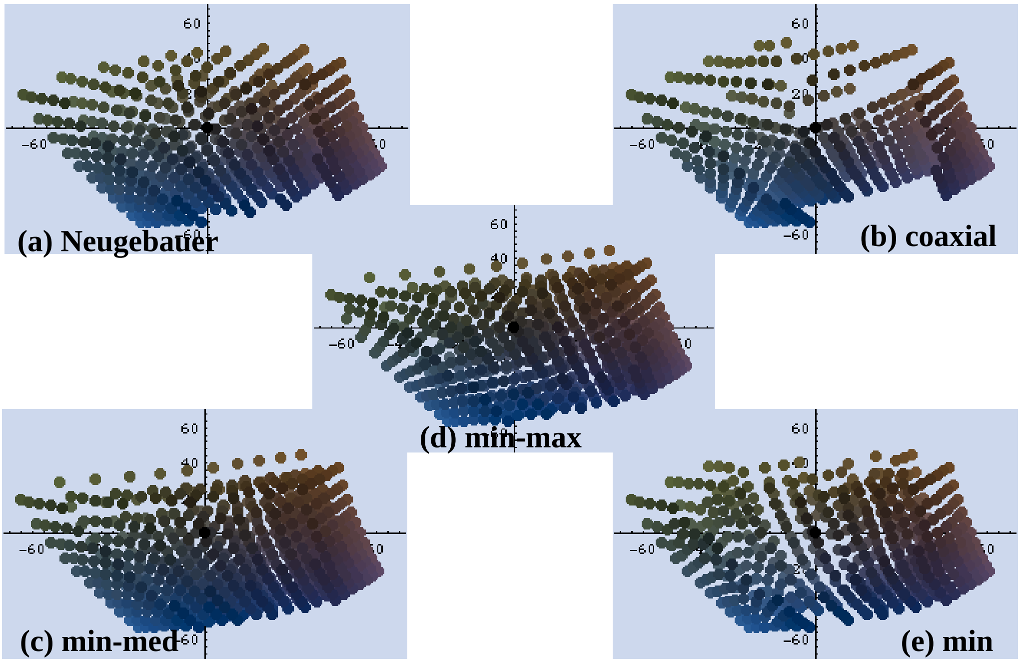


Figure 4 Comparison of color gamut in a-b* plane for 40 < L* <60*

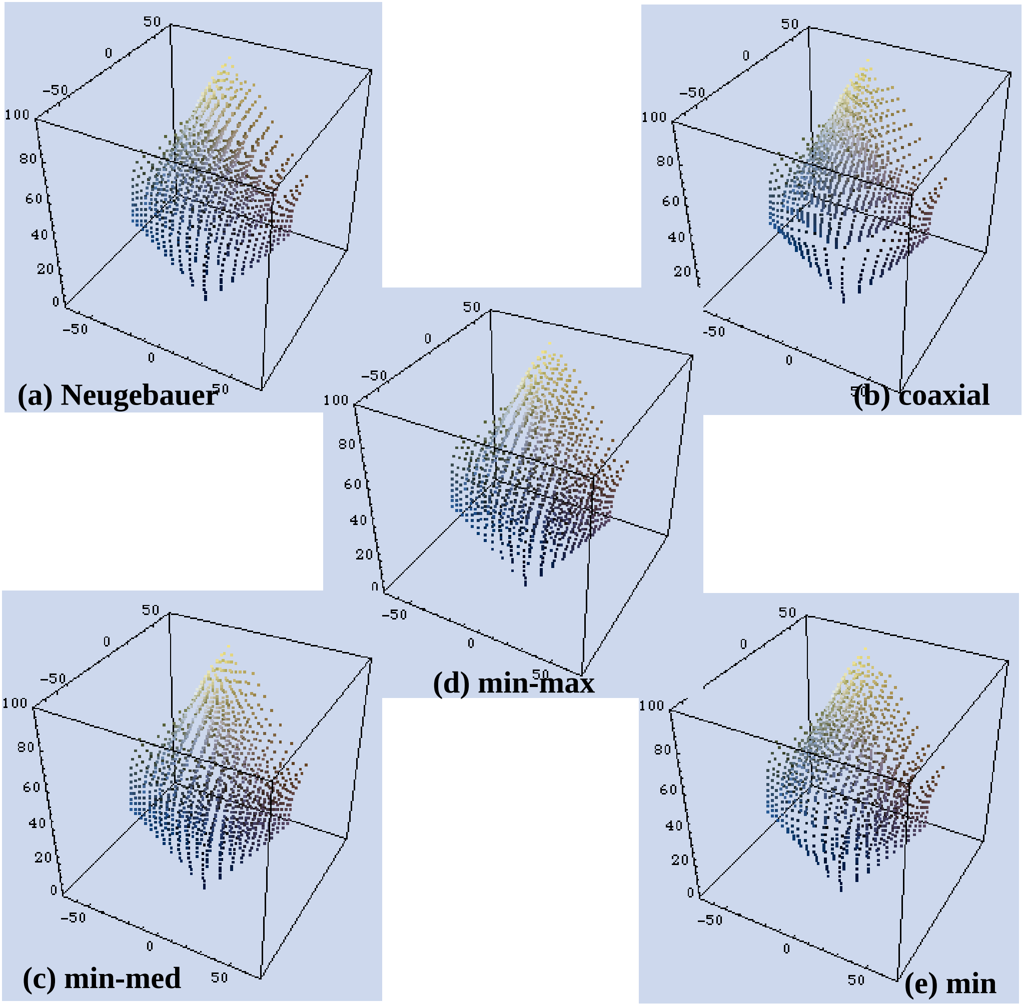


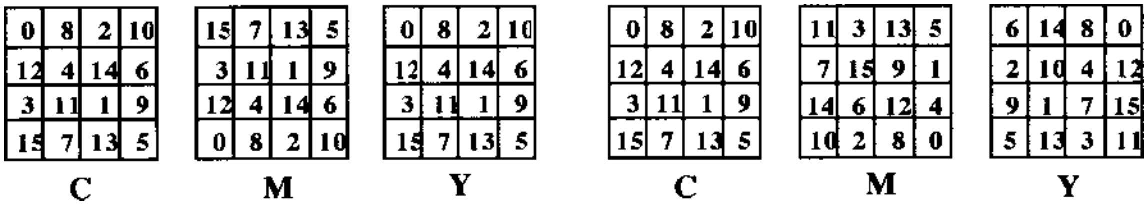
Figure 5 Three dimensional color plots reproduced by each model(wax transfer thermal printer)

Table 1 Colored dot area ratios in min-med model

Neugebauer		**Coaxial model**		**min-med model** (C||M⊥Y)		
color **i**	area ratio $\mathbf{a}_i$	color **i**	area ratio $\mathbf{a}_i$	color **i**	area ratio $\mathbf{a}_i$ (c+m<1)	area ratio $\mathbf{a}_i$ (c+m≥1)
W	(1-c)(1-m)(1-y)	W	1-Max(c,m,y)	W	(1-c-m)(1-y)	0
C	c(1-m)(1-y)	P	Max(c,m,y)-Mid(c,m,y)	C	c(1-y)	(1-m)(1-y)
M	(1-c)m(1-y)	PQ	Mid(c,m,y)-Min(c,m,y)	M	m(1-y)	(1-c)(1-y)
Y	(1-c)(1-m)y	K	min(c,m,y)	Y	y(1-c-m)	0
R	(1-c)my			B	0	(c+m-1)1-y)
G	c(1-m)y			G	cy	(1-m)y
B	cm(1-y)			R	my	(1-c)y
K	cmy			K	0	(c+m-1)

Table 2. Colored dot area ratios in min-max model

min-max model	W	C	M	Y	B	R	G	K
c+m<1, c>y	1-c-m	c-y	m	0	0	0	y	0
m+y<1, y>c	1-m-y	0	m	y-c	0	0	c	0
y+m>1, c+m>1, M+y<1	0	1-m-y	1-c	0	c+m-1	0	y	0
m+y>1, c+m<1	0	0	1-y	1-c-m	0	m+y-1	c	0
m+y>1, c>y	0	0	1-c	0	c-y	0	1-m	m+y-1
c+m>1, c<y	0	0	1-y	0	0	y-c	1-m	c+m-1



C

0	8	2	10
12	4	14	6
3	11	1	9
15	7	13	5

M

15	7	13	5
3	11	1	9
12	4	14	6
0	8	2	10

Y

0	8	2	10
12	4	14	6
3	11	1	9
15	7	13	5

(a) ***min-max*** *model*

C

0	8	2	10
12	4	14	6
3	11	1	9
15	7	13	5

M

11	3	13	5
7	15	9	1
14	6	12	4
10	2	8	0

Y

6	14	8	0
2	10	4	12
9	1	7	15
5	13	3	11

(b) ***min*** *model*

Figure 6. Examples of ordered dither matrix in side-by-side dot allocation model

Conclusion

The additive mixture of CMY inks can produce the wider color gamut than that of subtractive mixture in bi-level printers. Here the 3D color gamuts in three basic dot allocation models are analyzed and compared with that of typical coaxial model. The fundamental idea is hoped to be extended for the dot placements in error diffusion method and the further studies are to be continued.

Acknowledgement

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