A Modified Error Diffusion Scheme Based on the Human Visual Model

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Abstract

Various techniques have been proposed to reduce the artifacts resulted from the Floyd-Steinberg's error diffusion algorithm. Recently, an adaptive error diffusion scheme which dynamically adjusts the weights for the error propagation was reported.[1] In order to further reduce the undesirable artifacts, this paper presents a modified adaptive error diffusion scheme. In the proposed scheme, the human visual characteristics are incorporated into the weighted error criterion as an additional low pass filter. Instead of the causal neighboring pixel set, a 3×3 set of the noncausal neighboring pixels is utilized during the minimization of the weighted error criterion. Experiments are performed to examine the effects of the proposed modifications. Experimental results of the proposed scheme are compared with those obtained by the Floyd-Steinberg's algorithm and the adaptive error diffusion scheme.[1]

Introduction

Many image output devices exhibit binary states, black and white. In order to generate the gray level images by such devices, it is necessary to convert the gray level image to binary image. The process of binarization is often called as digital halftoning. Various digital halftoning methods have been reported.[2,3,4] They can be divided into two categories; the point processing methods and area processing methods. The typical point and area processing methods are the dithering and error diffusion techniques, respectively.

In the error diffusion method, the quantization error of the pixel under binarization is propagated to the neighboring pixels. The propagated errors are then utilized to modify the gray level values of the neighboring pixels. The quality of binary image produced by the error diffusion method depends on the values of the weights for the error propagation and the locations of the neighboring pixels.* The values of weights and locations of neighboring pixels have been determined experimentally by the trial and error method. Most popular error diffusion algorithm proposed by Floyd and Steinberg[5] utilizes four neighboring pixels. The values of the weights for the error propagation are constants, i.e., independent of the contents of the gray level image. The undesirable correlated artifacts appear on the binary image generated by the Floyd-Steinberg's algorithm.

As an effort to reduce the artifacts, the adaptive error diffusion scheme which dynamically adjusts the weights for the error propagation was recently reported.[1] The values of weights were no longer constants in the adaptive error diffusion method and determined based on the contents of the gray level image. They were determined by minimizing the error criterion using the least mean square estimation method in the spatial domain. The error criterion utilized were the expected squared error weighted by the causal low pass filter. The coefficients of the low pass filter were determined by the radial power spectrum of the constant gray level image. Compared to the binary image obtained by the Floyd-Steinberg's algorithm, the binary image with the adaptive scheme showed less correlated artifacts.

In order to further reduce the undesirable artifacts on the halftoned image, this paper proposes modifications to the adaptive error diffusion scheme.[1] An additional low pass filter representing human visual characteristics is incorporated to the weighted error criterion for the least mean square minimization. Two low pass filters, one calculated from the radial power spectrum and one representing the photopic modulation transfer function, are calculated in the frequency domain. To implement the minimization in the spatial domain, the coefficients of two low pass filters were estimated to have values at the 3×3 noncausal windows of pixels, whereas the causal set of pixels located at the positions defined† by the Floyd-Steinberg's algorithm was utilized in the adaptive error diffusion scheme.[1] Experiments are performed to examine the effects of the proposed modifications. Experimental results indicate that the proposed modification further reduces the correlated artifacts on the halftoned image.

First, the Floyd-Steinberg's error diffusion algorithm will be briefly outlined. Also, the adaptive error diffusion scheme[1] will be described. The proposed scheme will be explained next. The aforementioned three error diffusion

**For clarity, the author has replaced this phrase as follows:*
the locations of the neighboring pixels, the quantization scheme, and the processing sequence

† this phrase should now be disregarded
‡ original publication inadvertently misprinted this authors' name as Wang

algorithms, the Floyd-Steinberg's algorithm, the adaptive error diffusion scheme,[1] and the proposed algorithm, are applied to the gray level ramp and Lena images. The effects of the proposed modifications will be examined. Finally, experimental results will be discussed.

Floyd-Steinberg's Error Diffusion Algorithms[5]

Let x(m,n) be the gray value of the pixel at the (m,n)th location on the input gray level image. The value of x(m,n) ranges from 0 to 255. Assume that b(m,n) be the halftoned binary pixel value, either 0 or 255 at the (m,n)th location on the output binary image. Floyd-Steinberg's error diffusion algorithm can be formulated by the following Equations (1)-(3).

$$e(m,n) = b(m,n) - u(m,n) \qquad (1)$$

$$u(m,n) = x(m,n) - \sum_{(k,I)\in R} w(k,I)e(m-k,n-l) \qquad (2)$$

$$u(m,n) = x(m,n) - \sum_{(k,l)\in R} w(k,l)e(m-k,n-l) \qquad (2)$$

$$b(m,n) = \begin{cases} 255 & \text{if } u(m,n) \geq 127 \\ 0 & \text{otherwise} \end{cases} \qquad (3)$$

where u(m,n) is the corrected gray value of the (m,n)th pixel and e(m,n) is the difference between binary value and corrected gray value. w(k,l) is the weight for the error propagation at the (k,l)th neighboring pixel. R represents the set of neighboring pixels considered for the error propagation. In the Floyd-Steinberg's error diffusion algorithm, four neighboring pixels are utilized and they are causal with respect to the halftoning* sequence. The location of neighboring pixels and the value of weight at each location shown in Equation (4) were determined experimentally by the trial and error method.

$$\begin{bmatrix} w(0,0) & w(0,1) & w(0,2) \\ w(1,0) & * & \end{bmatrix} = \begin{bmatrix} 1/16 & 5/16 & 3/16 \\ 7/16 & * & \end{bmatrix} \qquad (4)$$

As will be shown in the experiments, the correlated artifacts appear on the halftoned images obtained by the Floyd-Steinberg's error diffusion algorithm. Various efforts have been reported to improve the image quality by utilizing different threshold value, changing the number or location of the neighboring pixels, or modifying the values of the weights.[1-4,6,7] Among them, the adaptive error diffusion scheme[1] which dynamically adjusts the weights for the error propagation will be described next.

Adaptive Error Diffusion with Dynamically

* halftoning *is now replaced with the phrase:* raster scanning

Note: throughout the paper, equations (original in gray) are followed by revised equations.

Adjusted Weights[1]

In order to adjust the weights for the error propagation, the following weighted error criterion $Q_1(m,n)$ was first defined.

$$Q_1(m,n) = E[\{v(m,n) * (N(m,n) - x(m,n))\}^2] \qquad (5)$$

$$Q_1(m,n) = E[\{v(m,n) * (b(m,n) - x(m,n))\}^2] \qquad (5)$$

where * denotes the convolution operation. In Equation (5), the squared error is weighted by v(m,n). The weight filter v(m,n) serves as a low pass filter and is determined by taking inverse Fourier transform of V(u,v) defined in the frequency domain. Calculation of V(u,v) are carried out by the following procedure; First, 256 constant gray level images having only one gray value are constructed and halftoned by the Floyd-Steinberg's error diffusion algorithm. The radial power spectrum is then calculated from each of the halftoned constant gray level image. The cutoff frequency of the low pass filter V(u,v) for a given gray level is determined such that the energy within the passband contains 99% of the total energy. Thus, V(u,v) is a function of the gray level.

The weighted error criterion $Q_1(m,n)$ in Equation (5) is minimized to determine the adjusted weights by the iterative least mean square method. When taking the inverse Fourier transform of V(u,v) for the calculation of minimization, the values of v(m,n) are approximated at the causal locations shown in Equation (4), i.e., the set of neighboring pixels defined by the Floyd-Steinberg's error diffusion algorithm. It is due to the fact that the values needed for noncausal neighboring pixels are not available with respect to the halftoning* sequence. The details of the adaptive error diffusion scheme can be found in the reference.[1]

As will be shown in the experiments, the aforementioned adaptive error diffusion scheme reduces the correlated artifacts. In order to further reduce the artifacts, a modified adaptive error diffusion algorithm proposed in this paper will be described next.

Modified Adaptive Error Diffusion Scheme

In order to improve the quality of halftoned binary image, the weighted error criterion $Q_1(m,n)$ in Equation (5) is modified to $Q_2(m,n)$ in the following Equation (6).

$$Q_2(m,n) = E[\{v(m,n) * h(m,n) * b(m,n) - x(m,n)\}^2] \qquad (6)$$

where h(m,n) is an additional low pass filter representing the human visual characteristics. The coefficients of h(m,n) are independent of the gray value of the pixel whereas v(m,n) is a function of the gray value. The coefficients of h(m,n) are calculated by taking inverse Fourier transform of H(u,v), which is the photopic modulation transfer function.[8] The values of H(u,v) are obtained experimentally and expressed in terms of radial frequency f_r as follows

$$\mathbf{H}(f_r) = \begin{cases} 2.2(0.19 + 0.114 f_r)\exp(-0.114 f_r)^{1.1}), & \text{if } f_r > f_{\max} \\ 1.0 & \text{if } f_r > f_{\max} \end{cases} \quad (7)$$

$$\mathbf{H}(f_r) = \begin{cases} 2.2(0.19 + 0.114 f_r)\exp(-0.114 f_r)^{1.1}), & \text{if } f_r > f_{\max} \\ 1.0 & \text{if } f_r \le f_{\max} \end{cases} \quad (7)$$

where $f_{\max}$ is the frequency yielding the maximum value of H(f_r). By using Equations (1) and (2), Equation (6) can be rewritten as the following Equation (8)

$$Q_2(m,n) = E[\{a(m,n) * (\delta(m,n) - w(m,n))\}^2] \quad (8)$$

where

$$a(m,n) = h(m,n) * v(m,n) * e(m,n) \quad (9)$$

and $\delta(m,n)$ is two-dimensional delta function.

The coefficients of h(m,n) are approximated from H(u,v) in Equation (7) by the least squares estimation method to minimize the errors occurred during the inverse Fourier transform.[9] H(u,v) are sampled having 64×64 values and then inverse Fourier transformed to yield h(m,n) in the 3×3 window. The coefficients of v(m,n) are also approximated by the same method to have the 3×3 window in the spatial domain. Calculation of V(u,v) is described in previous section. As mentioned earlier, in the adaptive error diffusion scheme,[1] the coefficients of v(m,n) are calculated at the causal locations of neighborhood R shown in Equation (4). It is due to the fact that the values of e(m,n) in Equation (1) are not available for the noncausal neighboring pixels outside of R. In the proposed error diffusion algorithm, the values of e(m,n) in the neighborhood R are determined by Equation (1). The values of e(m,n) for the remaining noncausal pixels within the 3×3 window are determined by thresholding the value of x(m,n) with 127 and subtracting the resulting halftoned value from x(m,n).

The method to calculate a(m,n) in Equation (9) has been explained. The weights w(m,n) for the error propagation are to be determined by minimizing the weighed error criterion $Q_2(m,n)$ in Equation(8). The weight adjusting algorithm by the iterative least square estimation method will be described next.

Proposed Error Diffusion Algorithm

Step 1.

Let the initial values of the weights $w_1(m,n)$ be those values of the weights defined in Equation (4), i.e., the values of weights in the Floyd-Steinberg's error diffusion algorithm.

Step 2.

At *ith* iteration where $i = 1,2,3,\Lambda$, calculate a(m,n) in Equation (9). The coefficients of h(m,n), v(m,n), and e(m,n) are calculated to have values at the 3×3 windows. The coefficients of h(m,n) and v(m,n) are approximated by taking inverse Fourier transform of H(u,v) and V(u,v), respectively. The values of e(m,n) in the neighborhood R are determined by Equation (1). The values of e(m,n) for the remaining noncausal pixels are determined by thresholding the value of x(m,n) with 127 and subtracting the resulting halftoned value from x(m,n).

Step 3.

By the values of a(m,n) obtained in Step 2, calculate the following quantity $q(m, n)$

$$q(m,n) = [a(m,n) - a(m = \tilde{k}, n - \tilde{l}) - \sum_{(k,l)\in R_1 - (\tilde{k},\tilde{l})} w_i(k,l)(a(m-k,n-l) - a(m-\tilde{k})n - \tilde{l})] \quad (10)$$

$$q(m,n) = [a(m,n) - a(m = \tilde{k}, n - \tilde{l}) - \sum_{(k,l)|R_1 - (\tilde{k},\tilde{l})} w_i(k,l)(a(m-k,n-l) - a(m-\tilde{k})n - \tilde{l})] \quad (10)$$

where R_1 represents the set of 3×3 noncausal neighboring pixels and $(\tilde{k},\tilde{l})$ is the location of a neighboring pixel arbitrarily selected from R_1.

Step 4.

Adjust the values of weights by the following Equations (11) and (12)

$$w_{i+1}(k,l) = w_i(k,l) - \mu \frac{\partial q(m,n)}{\partial w_i(k,l)} \quad (11)$$

$$w_{i+1}(k,l) = w_i(k,l) - \mu \frac{fq(m,n)}{fw_i(k,l)} \quad (11)$$

$$w_{i+1}(\tilde{k},\tilde{l}) = 1 - \sum_{(k,l)\in R_1 - (\tilde{k},\tilde{l})} w_{i+1}(k,l) \quad (12)$$

$$w_{i+1}(\tilde{k},\tilde{l}) = 1 - \sum_{(k,l)|R_1 - (\tilde{k},\tilde{l})} w_{i+1}(k,l) \quad (12)$$

where, μ is a constant.

Step 5.

If the squared value of the absolute difference between $w_{i+1}(k, l)$ and $w_i(k, l)$ is larger than a specified constant threshold value, then increment i, $i = i + 1$ and go back to Step 2.

Otherwise, perform the binarization of the pixel of interest. Then, go to Step 1 for the next pixel to be halftoned.

The proposed adaptive error diffusion algorithm has been described. Experimental results will be presented next.

Experiments

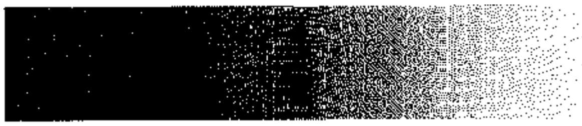
In order to examine the effects of the proposed modifications on the image quality, three aforementioned error diffusion algorithms, the Floyd-Steinberg's error diffusion algorithm,[5] the adaptive error diffusion algorithm,[1] and the proposed algorithm[1] are applied to the gray level ramp and Lena images.

Λ should now be read as ...

[1] *should now disregarded*

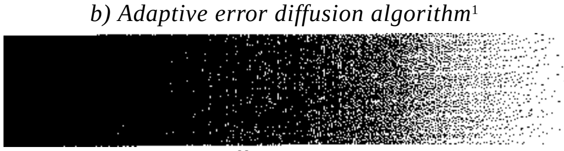
Experiment 1: Gray Level Ramp Image

A gray level ramp image of size 100×512 is constructed and utilized as an input image. Four experiments are performed on the gray level ramp image. First, the Floyd-Steinberg's error diffusion algorithm is applied to the gray level ramp image. The resulting binary image is shown in Figure 1 (a). The correlated artifacts appear on the halftoned image, specially near the gray level 65, 86, 128, 171, and 192. Second, the halftoned ramp image obtained by the adaptive error diffusion algorithm[1] is shown in Figure 1 (b). It is shown in Figure 1 (b) that the undesirable artifacts near the gray value of 128 are reduced. But, the artifacts on the other areas are still visible.

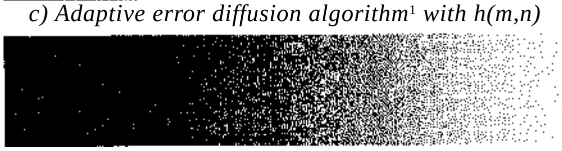


a) Floyd-Steinberg's error diffusion algorithm

b) Adaptive error diffusion algorithm[1]



c) Adaptive error diffusion algorithm[1] with h(m,n)



d) The proposed algorithm

Figure 1. Experimental results with gray level ramp image

In order to analyze the effect of the additional low pass filter h(m,n) on the halftoned image quality, the weighted error criterion $Q_2(m,n)$ in Equation (6) is minimized by the adaptive error diffusion algorithm.[1] The coefficients of h(m,n) and v(m,n) are approximated at the same causal locations utilized as in the original adaptive error diffusion algorithm.[1] It should be noted that the method for this experiment is different from the proposed algorithm in terms of the neighboring pixels utilized in the minimization. The resulting halftoned image is shown in Figure 1 (c). Comparing the images in Figure 1 (b) and (c), it can be said that improvements on image quality are achieved by incorporating h(m,n) into the weighted error criterion.

Finally, the proposed error diffusion algorithm summarized in previous section is applied to the gray level ramp image. In this experiment, H(u,v) and V(u,v) are sampled to have 64×64 values in the frequency domain. They are inverse Fourier transformed to yield the coefficients of h(m,n) and v(m,n) at the 3×3 windows whereas the four causal neighbors are utilized in the images of Figure 1 (a), (b) and (c). The values of e(m,n) are determined by the method described in the Step 2 of the proposed algorithm. The value of μ utilized in Equation (11) is 3×10^{-6}. The constant threshold value in Step 5 of the proposed algorithm is designed such that the number of iteration should be less than 15. The resulting binary image is shown in Figure 1(d). It is shown in Figure 1(d) that the correlated artifacts are considerably reduced.

The results of four different experiments performed on the gray level ramp image justify the modifications made in the proposed adaptive error diffusion algorithm. The effects of h(m,n) can be examined by comparing the images in Figure 1(b) and (c). The improvements made by utilizing 3×3 window set of neighboring pixels can be clarified by comparing the images in Figure 1(c) and (d).

(a) Floyd-Steinberg's error diffusion algorithm

(b) The proposed algorithm

Figure 2. Experimental results with the Lena image

Experiment 2: Lena Image

The Floyd-Steinberg's error diffusion algorithm is applied to the Lena image of size 256×256. The corresponding result of binarization is shown in Figure 2)Y). The correlated artifacts appear on the top of the hat, face, and shoulder of the image shown in Figure 2 (a). The proposed adaptive error diffusion algorithm is also applied to the Lena image. The resulting binary image is shown in Figure 2 (b). It can be noticed by comparing the images in Figure 2 (a) and (b) that the undesirable artifacts are reduced on the image in Figure 2 (b), specially on the top of the hat.

Conclusion

The halftoned image obtained by the Floyd-Steinberg's error diffusion algorithm exhibits the correlated artifacts. As an effort to improve the image quality, the adaptive error diffusion algorithm[1] which dynamically adjust the weights for the error propagation has been reported. This paper presented a modified adaptive error diffusion algorithm. The human visual characteristics are incorporated into the weighted error criterion. Instead of the causal neighboring pixel set, a 3×3 noncausal neighboring pixel set is utilized during the minimization procedure. Experiments are performed to examine the effects of the proposed modifications. The experimental results with the gray level ramp image indicated that the undesirable artifacts are considerably reduced with the modifications made in the proposed algorithm.

Acknowledgment

This work was supported by Inha University Research Fund.

References

1. P. W. Wong, "Error Diffusion With Dynamically Adjusted Kernel", *IEEE Int. Conf. on ASSP* pp.V.113-V.116, April 1994.
2. J. C. Stoffel and J. F. Moreland, "Survey of Electronic Techniques for Pictorial Image Reproduction", *IEEE Trans. on Communications*, Vol. **COM-29**, No.12, pp.898–1925, Dec. 1981.
3. J. F. Jarvis et.al., "A Survey of Techniques for the Display of Continuous Tone pictures on Bilevel Displays", *Computer graphics and image processing*, Vol. **5**, pp. 13–40, 1976.
4. R. A. Ulichiney, "Digital Halftoning". Cambridge MA: MIT Press, 1987.
5. R. Floyd and L. Steinberg, "An adaptive algorithm for spatial gray scale", *SID International Symposium*, Digest of Technical Papers, pp. 36–37, 1975.
6. Z. Fan, "Error diffusion with a more symmetric error distribution", *SPIE Conf. on Human Vision, Visual Processing, and Digital Display V*, Vol. **2179**, pp. 150–158, 1994.
7. Shiau and Z. Fan, "A set of easily implementable coefficients in error diffusion with reduced worm artifacts", *SPIE Conf. on Color Imaging: Device Indep. Color, Color hard copy, and Graphic Arts*, Vol. **2658** (to appear), 1996.
8. J. Sullivan, L. Ray and R. Miller, "Design of Minimum Visual Modulation Halftoning Pattern", *IEEE Trans. on systems, man, and cybernetics*. Vol. **21** No.l, pp. 34–39, January/February 1991.
9. R. C. Gonzalez and R. E. Woods, "Digital Image Processing", Addison Wesley Press, 1992.

❋ Previously published in *IS&T's 49th Annual Conference Proc.* pp. 341–345, 1996.