# Optimized Basis Functions for Coding Reflectance Spectra Minimizing the Visual Color Difference

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## Abstract

For exact color reproduction of objects under varying illuminants it is necessary to provide information on the complete reflectance spectrum. Since no analytical solution exists a stochastic optimization algorithm is applied to calculate basis vectors for representing a set of test spectra that lead to a minimal color deviation.

## Introduction

The colors of objects depend on their reflectance spectra and on the spectrum of their illumination. To allow color reproduction for different light sources it is necessary to record and store more information on the reflectance spectrum than only the three-stimulus values supplied by usual cameras. A common method to compress the overwhelming amount of data to describe reflectance spectra is transform coding. The spectra are represented as linear combinations of an orthonormal basis vector set.

Some work has already been carried out concerning the problem of choosing appropriate basis functions. Exact reproduction can be achieved for a limited number of $N$ different light sources using $3N$ basis functions.[1] If the number of light sources increases, small color errors must be admitted to maintain a practicable number of basis functions.

The use of Fourier basis functions is a strategy that exploits the band limited characteristic of reflectance spectra. A different approach aims for a minimization of the mean squared error (MSE) between the original and the reconstructed spectrum using as few basis vectors as possible. Here an analytical solution exists called the principal component analysis (PCA) which depends on the statistical properties of the spectra set to be represented[2,3]. It offers the best data compression efficiency because of optimal energy concentration in few components.

Nevertheless a correct solution would aim for a different error measure to minimize. The MSE between the spectra allows no statement concerning the visual color difference. It disregards the human visual system perceiving the colors as well as it does not take different illuminations into account. To analyze the visual color difference resulting from coding a reflectance spectrum it is necessary to calculate the tristimulus color values under all of the considered illuminants. These color triples must be transformed into a visually uniform color space before calculating the difference between the original and the coded reflectance spectrum. Transformation into standardized visually uniform color spaces implies nonlinear operations. The error measure might contain nonlinearities as well. Thus minimizing such an error measure one encounters a nonlinear quality criterion for which no analytical solution for the optimal basis function set can be given. Therefore an unconstrained stochastic optimization algorithm is applied to calculate sets of basis functions in order to minimize the visual color difference.

## Basic Elements

The optimization is performed with respect to the reflectance spectra published by Vhrel et al.[4] They contain the spectra of 64 Munsell chips, 120 Du Pont paint chips and 170 natural objects sampled in the range between 400 and 700 nm at intervals of 10 nm.

Orientation of the first three basis vectors is organized according to Keusen et al.[5] to span the human visual system for a spectrally equal energy white illumination (the choice of this illuminant E is arbitrary). This space equals the space spanned by the three color matching functions $X(\lambda)$, $Y(\lambda)$ and $Z(\lambda)$. Hence the first three vectors yield a perfect color reconstruction for illuminant E. Orthogonal to these three vectors remains the so called metameric black space. The following basis vectors (that span a subspace of the metameric black space) are optimized in a manner that for a set of further illuminations the visual color difference is minimized.

Usually the quality of color reproduction is given after transforming the three-stimulus value $XYZ$ into a visually uniform color space, such as the standardized CIELab space.

The Euclidean distance between two color coordinates in the CIELab color space is abbreviated $\Delta E_{ab}$. According to a set of original and reconstructed three-stimulus values the quality is stated by the mean $\Delta E_{ab}$, the maximum $\Delta E_{ab}$, and the number of visible errors, that is the number of $\Delta E_{ab}$ exceeding the value of 3.

Consequently the optimization criterion is chosen as one of these quality measures. In this paper results are given optimizing first of all the maximum $\Delta E_{ab}$ for a given set of test spectra combined with a set of illuminants. Thus, it is guaranteed to offer an overall good color reproduction quality without single outliers. Additionally, optimization was performed minimizing the mean $\Delta E_{ab}$.

In Vhrel et al.[6] the Euclidean distance in the CIELab color space was evaluated in order to optimize filter transmission spectra for recording of colors. An unconstrained numerical gradient optimization method was employed to approach the optimum filter set. Presuming that it is not ensured to hit the global optimum by that means a different optimization strategy was chosen here.

## Stochastic Optimization

The most famous stochastic optimization scheme was introduced using the term "simulated annealing". It is designed to find among a variety of "configurations" the one that minimizes a specific "quality criterion". In our problem the "configurations" are the different sets of orthogonal basis vectors, and the "quality criterion" is represented by the maximum $\Delta E_{ab}$ or the mean $\Delta E_{ab}$, respectively.

At every iteration a small stochastic perturbation is performed on the configuration and the corresponding new quality criterion is calculated. If the new quality is better, then the new configuration is accepted as the starting configuration for the next iteration. If the new quality is worse than the old one, then the new configuration is accepted only with a certain small probability. This probability is lowered gradually during the optimization process. Thus, towards the end of the optimization only the better configurations are accepted so that a convergence near the global optimum can be expected.

The scheme according to which the probability is lowered has a great influence on the performance of the strategy. Therefore a sophisticated schedule is needed how to adapt the probability to the time during the optimization and to the current quality.

## Improvements on Stochastic Optimization

Some important enhancements to simulated annealing were published by Dueck et al.[7,8] They found a significant improvement by slightly changing the scheme for accepting worse configurations. Two different strategies were presented, operating as follows.

**Threshold Accepting (TA)**

The difference between the previous and the new quality measure is compared to a threshold. The new configuration is accepted, if the threshold is not exceeded (if the new configuration is not *much* worse).

During the optimization process the threshold is lowered gradually towards zero, leading to the acceptance of only the better configurations in the end. At a threshold of zero, the optimization is finished.

**Great Deluge Algorithm (GDA)**

The quality measure is always forced to stay above a quality level which is slowly raised during the optimization process.

When this level has become too high to find configurations with a quality measure above, the optimization is finished. In practice this condition is met either when the quality level has risen above the current quality measure or when there is no acceptance of a new configuration for a long time.

After largescale experiments both strategies were reported to yield remarkably better results than simulated annealing, requiring less computational time. Therefore they are well suited for this optimization problem which implies rather time consuming quality calculation.

Furthermore they are distinguished by the dependency of only one single parameter, the threshold decline or the level rising speed, respectively. The quality of the optimization turns out to be very insensitive to slight changes of these parameters. In comparison with that, simulated annealing requires a complicated schedule, in order to lower the possibility to accept worse configurations.

Threshold accepting offers another advantage: its computational time can be predetermined. The optimization is finished when the threshold reaches zero, whereas it is not predictable, when in the GDA no solution better than the level can be found anymore.

## Implementation

The whole algorithm can be outlined as follows:

- Adjust the first three basis vectors to build an orthonormal set parallel to the human visual system for illuminant E (arbitrary), which is the *XYZ*-space.
- Choose a set of further illuminants to optimize for, which is D65, D50, C, A, F2, and F11 in our case.
- Initialize the basis vectors to be optimized with random coefficients keeping orthonormality.
- Iterate the following steps:
  - Perform a small perturbation on the basis vectors to be optimized: randomly choose a vector, choose a coefficient and choose an offset.
  - Orthonormalize the basis vectors again.
  - Calculate the error criterion, which is the maximum $\Delta E_{ab}$ or the mean $\Delta E_{ab}$ for the chosen illuminants over all test spectra.
  - Decide whether to accept the new basis vector set or not.
  - Lower threshold (TA) or raise quality level (GDA), respectively.
  - When threshold has declined to zero (TA) or when there is no acceptance of a new basis vector set for a long time (GDA): FINISH.

To generate a complete set of basis vectors two different optimization strategies were pursued.

**Successive refinement.** Each of the basis vectors is optimized on its own, so the basis vector set can be used hierarchically. A set of *N* vectors is constructed out of the set of *N-1* vectors plus one additional vector, leaving the previous set of *N-1* vectors unchanged. With each new coefficient the metameric black space is refined further.

**Independent calculation.** The optimization is applied on every whole basis vector set (except for the first three vectors). Hence, a set of *N* vectors may differ completely from the set of *N-1* vectors. Because of the missing constraint to be hierarchical and the greater number of degrees of freedom during the optimization process this strategy is expected to yield better results.

It must be emphasized that there was no sophisticated scheme employed, to lower the threshold (TA) or to raise

the quality level (GDA). Especially, there was no fine-tuning of parameters for the different sets of basis vectors to be optimized. The parameters only differ with the quality measure, since the values of the maximum $\Delta E_{ab}$ are always somewhat greater than of the mean $\Delta E_{ab}$.

For the TA the number of runs was set to 1,000,000 and the threshold was linearly declined towards zero starting at values of 0.02 (maximum $\Delta E_{ab}$) and 0.01 (mean $\Delta E_{ab}$). The GDA was arranged to raise the quality level (precisely: lower the error level) gradually after every 500 iterations, starting with the qualities resulting from random initialization of the basis vectors. The level raising speed was set to values of 0.002 (maximum $\Delta E_{ab}$) and 0.001 (mean $\Delta E_{ab}$). When there has been no acceptance of new configurations for a period of such 500 iterations, the optimization was finished.

Concerning the 'small perturbation' on the basis vectors, the randomly chosen coefficient was added by an offset that was equally distributed in the interval [-0.02, 0.02].

To demonstrate the still immense amount of calculations for one single optimization run, here is a little example:

For calculating the three color values *X*, *Y*, and *Z*, for every of the 354 spectra, which consist of 31 samples each, under all 6 different illuminants, about 200,000 operations are required. This has to be done at each iteration. Therefore one whole run of the TA consists of at least $2 \cdot 10^{11}$ multiplications and additions. Implemented on a SUN SPARC 20-502 using the programming language C this one run requires about 22 hours.

## Results

The TA turned out to yield very stable results for various random starting conditions. As far as the optimization of the mean $\Delta E_{ab}$ was concerned, similar values could be achieved using the GDA. The optimal values differed throughout by less than one percent. Actually the GDA required significantly less iteration steps for the optimization. For calculating all 9 basis vector sets, for which the results are given in the tables, the TA needed altogether $9 \cdot 10^6$ iteration steps, versus about $2.410^6$ iteration steps for the GDA.

Surprisingly the GDA failed in optimizing the maximum $\Delta E_{ab}$. Even with a very low level rising speed, leading to much more iteration steps than the TA takes, rather bad values are obtained. The "maximum" operation seems to produce very many local optima which serve as traps for the GDA. Once a configuration of basis vectors got stuck in such a local optimum in a late phase of the optimization, the way out was often obstructed by the continuously rising quality level. The TA algorithm is more flexible because it enables the configuration to make many steps each of them a little worsening, thus leaving local optima.

In the following tables a summary of the results obtained by a typical TA run is given. Table 1 shows the maximum $\Delta E_{ab}$ over all 354 reflectance spectra taking into account the 6 illuminants D65, D50, C, A, F2, and F11. The calculation was performed after representing the reflectance spectra with 3 to 8 basis vectors. The first three basis vectors were organized as stated above, spanning the human visual system for illuminant E. Basis vectors 4 to 8 result from the optimizations, regarding the criteria maximum $\Delta E_{ab}$ and mean $\Delta E_{ab}$, respectively. Both of the optimization criteria were combined with the strategies *independent calculation* and *successive refinement*. For comparison, the principal component analysis has been applied to the projection of the reflectance spectra set onto the metameric black space, to get a set of basis vectors.

Tables 2 and 3 show the corresponding mean $\Delta E_{ab}$ and the number of $\Delta E_{ab}$ exceeding the value of 3. It is important to mention, that illuminant E, which we always have perfect color reconstruction for, is not considered. Otherwise the values would be even smaller.

**Table 1. Maximum $\Delta E_{ab}$**

| basis vectors | PCA | **maximum** | | mean | |
|---|---|---|---|---|---|
| | | **indep.** | **succ.** | indep. | succ. |
| 3 | 26.053 | | | | |
| 4 | 24.625 | **12.246** | | 20.672 | |
| 5 | 13.695 | **3.748** | **8.814** | 7.539 | 8.143 |
| 6 | 19.031 | **2.967** | **7.018** | 6.744 | 7.741 |
| 7 | 10.870 | **1.328** | **6.840** | 5.957 | 5.002 |
| 8 | 10.587 | **0.837** | **6.397** | 1.729 | 5.051 |

**Table 2. Mean $\Delta E_{ab}$**

| basis vectors | PCA | maximum | | **mean** | |
|---|---|---|---|---|---|
| | | indep. | succ. | **indep.** | **succ.** |
| 3 | 3.783 | | | | |
| 4 | 1.984 | 2.798 | | **1.542** | |
| 5 | 1.015 | 0.910 | 2.636 | **0.667** | **0.846** |
| 6 | 0.706 | 0.867 | 1.946 | **0.433** | **0.683** |
| 7 | 0.478 | 0.615 | 1.822 | **0.245** | **0.508** |
| 8 | 0.428 | 0.336 | 1.765 | **0.117** | **0.315** |

**Table 3. Percentage of $\Delta E_{ab} > 3$**

| basis vectors | PCA | maximum | | mean | |
|---|---|---|---|---|---|
| | | indep. | succ. | indep. | succ. |
| 3 | 41.1 % | | | | |
| 4 | 19.1 % | 37.2 % | | 12.9 % | |
| 5 | 7.3 % | 2.2 % | 36.0 % | 1.9 % | 1.7 % |
| 6 | 4.1 % | 0 | 22.3 % | 1.4 % | 1.7 % |
| 7 | 2.1 % | 0 | 18.5 % | 1.1 % | 1.0 % |
| 8 | 1.5 % | 0 | 17.4 % | 0 | 0.6 % |

As a result of choosing the maximum $\Delta E_{ab}$ or the mean $\Delta E_{ab}$ as the optimization criteria the other error measures are sometimes worse than the ones produced using the PCA basis vectors. This is a well expected fact. With the described algorithm every combination of error measures can be optimized regardless of any other constraints.

Using the *independent calculation* strategy optimizing the maximum $\Delta E_{ab}$ it can be concluded that two further basis vectors added to the first three *XYZ*-spanning vectors yield an almost visually perfect reconstruction for all reflectance spectra under all used illuminations. Only 2.2% of all combinations of reflectance spectra and illuminations exceed the visibility threshold value of 3 and the maximum $\Delta E_{ab}$ is not much above (3.748).

Figure 1 gives an impression of the corresponding basis vectors.

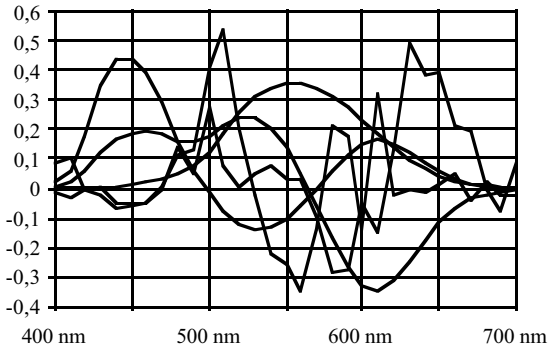


*Figure 1. 5 Basis vectors, minimizing the maximum $\Delta E_{ab}$, independent calculation. Dotted are the XYZ-spanning vectors.*

The values obtained with the *successive refinement* strategy were throughout significantly worse than following the *independent calculation* strategy. Therefore to get a hierarchical set of basis vectors, other optimization criteria should be used, based on a combination of qualities at all levels of the hierarchy.

## Conclusion

Two different stochastic optimization strategies were employed to achieve optimal sets of basis vectors for the transform coding of reflectance spectra. The aim of the optimization was to offer minimal visual color deviation for a given set of reflectance spectra under a given set of different illuminations.

The remaining color deviation is essentially smaller than that achieved using the principal component analysis (PCA).

The result from this work can be regarded from two different viewpoints.

At first it demonstrates the achievable color correctness using transform coding for reflectance spectra. If the first three basis vectors span the space of the human visual system the corresponding three components are sufficient to offer perfect color reconstruction for spectrally equal energy white illumination. Only two more basis vectors (or components, respectively) yield an almost perfect color reproduction for a large set of test reflectance spectra under a set of widely varying illuminants.

The second aim of this paper was the presentation of a powerful optimization algorithm which can be adapted to maximize any nonlinear quality criterion and which is therefore a good choice when problems regarding the nonlinearity of human color perception are concerned.

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