Recovery of Fundamental Spectrum from Color Signals

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Abstract

Spectral reconstruction from color scanning signals is discussed based on inverse projection from color signal subspace. A fundamental spectrum without metameric black carries the tristimulus value essential to human vision. Here, we present a method to extract the fundamental spectra from XYZ or RGB color scanning signals using pseudo-inverse projection and estimate the skin color reproducibility.

Introduction

A spectral reflectance of natural object is known to be well reconstructed from an expansion of few basis functions.[1-2] Principal component analysis or K-L transform give a good solution for the description of spectra. However, human vision perceives the same color sensation as the original when stimulated by the metamers with different spectra. Among the many metamers, the fundamental carries the essential spectrum to human vision. Here the fundamental spectrum is tried to be recovered from color scanning signals. We need not to recover the original spectra but need to reproduce the fundamental spectra. A smoothed spectrum like a skin color is well reconstructed from XYZ or RGB color signals and also shown to be roughly reconstructed from the reduced color signals such as XZ or RB.

Extraction of Fundamental Spectra

An input color spectrum $C(\lambda)$ is described by n-dimensional vector $\boldsymbol{C}$ as follows.

$$\boldsymbol{C} = [c_1 \ \ c_2 \ ..... \ c_n]^t;\ c_i = C(\lambda_i),\ \text{t = transpose} \tag{1}$$

In Human Visual Sub-space (HVS), $\boldsymbol{C}$ is decomposed into the fundamental $\boldsymbol{C}^*$ and the residue $\boldsymbol{B}$.

$$\boldsymbol{C} = \boldsymbol{C}^* + \boldsymbol{B},\ \boldsymbol{C}^* = \boldsymbol{P}_v\boldsymbol{C},\ \boldsymbol{B} = (\boldsymbol{I}\text{-}\boldsymbol{P}_v)\boldsymbol{C} \tag{2}$$

Here, $\boldsymbol{I}$ means unit matrix and $\boldsymbol{P}_v$ represents the projection operator[3-4] onto HVS.

$$\boldsymbol{P}_v = \boldsymbol{A}\ (\boldsymbol{A}^t\boldsymbol{A})^{-1}\boldsymbol{A}^t$$

where, $\boldsymbol{A}$ denotes the 3xn color matching matrix given by CIE $x(\lambda),y(\lambda),z(\lambda)$ color matching functions as

$$\boldsymbol{A} = \begin{bmatrix} x(\lambda_1), x(\lambda_2), & \ldots, & x(\lambda_n) \\ y(\lambda_1), y(\lambda_2), & \ldots, & y(\lambda_n) \\ z(\lambda_1), z(\lambda_2), & \ldots, & z(\lambda_n) \end{bmatrix}^t \tag{3}$$

The fundamental $\boldsymbol{C}^*$ carries a true tristimulus vector $\boldsymbol{T}$ and a residue $\boldsymbol{B}$ is called metameric black with zero stimulus not perceived to human vision.

$$\boldsymbol{T} = [X,Y,Z]^t = \boldsymbol{A}^t\,\boldsymbol{C} = \boldsymbol{A}^t\,\boldsymbol{C}^* \tag{4}$$

An input spectrum $\boldsymbol{C}$ is estimated by using pseudo-inverse transform[5] from eq. (4) as

$$\hat{\boldsymbol{C}} = \boldsymbol{P}_{inv}\boldsymbol{T} + (\boldsymbol{I} - \boldsymbol{P}_{inv}\boldsymbol{A}^t)\boldsymbol{V} \tag{5}$$

where, $\boldsymbol{V}$ means an arbitrary vector. Here, we should notice the 1st term of Eq. (5) is equal to the fundamental $\boldsymbol{C}^*$ and the 2nd term denotes a metameric black.

Thus the fundamental $\boldsymbol{C}^*$ is simply recovered from $\boldsymbol{T}$ using colorimetric pseudo-inverse projection operator $\boldsymbol{P}_{inv}$ by

$$\boldsymbol{C}^* = \boldsymbol{P}_{inv}\boldsymbol{T}, \qquad \boldsymbol{P}_{inv} = \boldsymbol{A}\ (\boldsymbol{A}^t\boldsymbol{A})^{-1} \tag{6}$$

Similarly, a RGB tri-color signal $\boldsymbol{X}$ from a color scanner with spectral sensitivity $\boldsymbol{F}$ is given as follows.

$$\boldsymbol{X} = [R,G,B]^t = \boldsymbol{F}^t\boldsymbol{C}, \boldsymbol{F} = \begin{bmatrix} r(\lambda_1), r(\lambda_2), & \ldots, & r(\lambda_n) \\ g(\lambda_1), g(\lambda_2), & \ldots, & g(\lambda_n) \\ b(\lambda_1), b(\lambda_2), & \ldots, & b(\lambda_n) \end{bmatrix}^t \tag{7}$$

$\boldsymbol{F}$ is defined by its red, green, and blue color separation filter functions $r(\lambda),g(\lambda),b(\lambda)$. Thus, the fundamental $\boldsymbol{C}_F^*$ from color scanning signal $\boldsymbol{X}$ is estimated by a scanner pseudo-inverse projection operator.

[Solution-Direct Pseudo-inverse]

$$\hat{\boldsymbol{C}}_F^* = \boldsymbol{P}_{Finv}\boldsymbol{X}, \quad \boldsymbol{P}_{Finv} = \boldsymbol{F}(\boldsymbol{F}^t\boldsymbol{F})^{-1} \tag{8}$$

Figure 1 shows how a spectrum $\boldsymbol{C}$ of skin color is decomposed into a fundamental $\boldsymbol{C}^*$ and a metameric black $\boldsymbol{B}$. The figure schematically summarizes the relations between the projection operators and the estimated spectra.

Filter Correction

The color scanner signal can be approximated to the true tristimulus value by electronic color correction. The most simple correction is done by operating a matrix Mc on tri-color signal X to get the corrected signal Xc by

$$\boldsymbol{X}_c = \boldsymbol{M}_c\boldsymbol{X} = \boldsymbol{M}_c\boldsymbol{F}^t\boldsymbol{C} \tag{9}$$

The matrix $\boldsymbol{M}_c$ is determined to minimize the mean square error between $\boldsymbol{X}_c$ and the true tristimulus vector $\boldsymbol{T} = \boldsymbol{A}^t\boldsymbol{C}$ for the selected color chips $\boldsymbol{C}_j$ (j = 1~N) as follows.

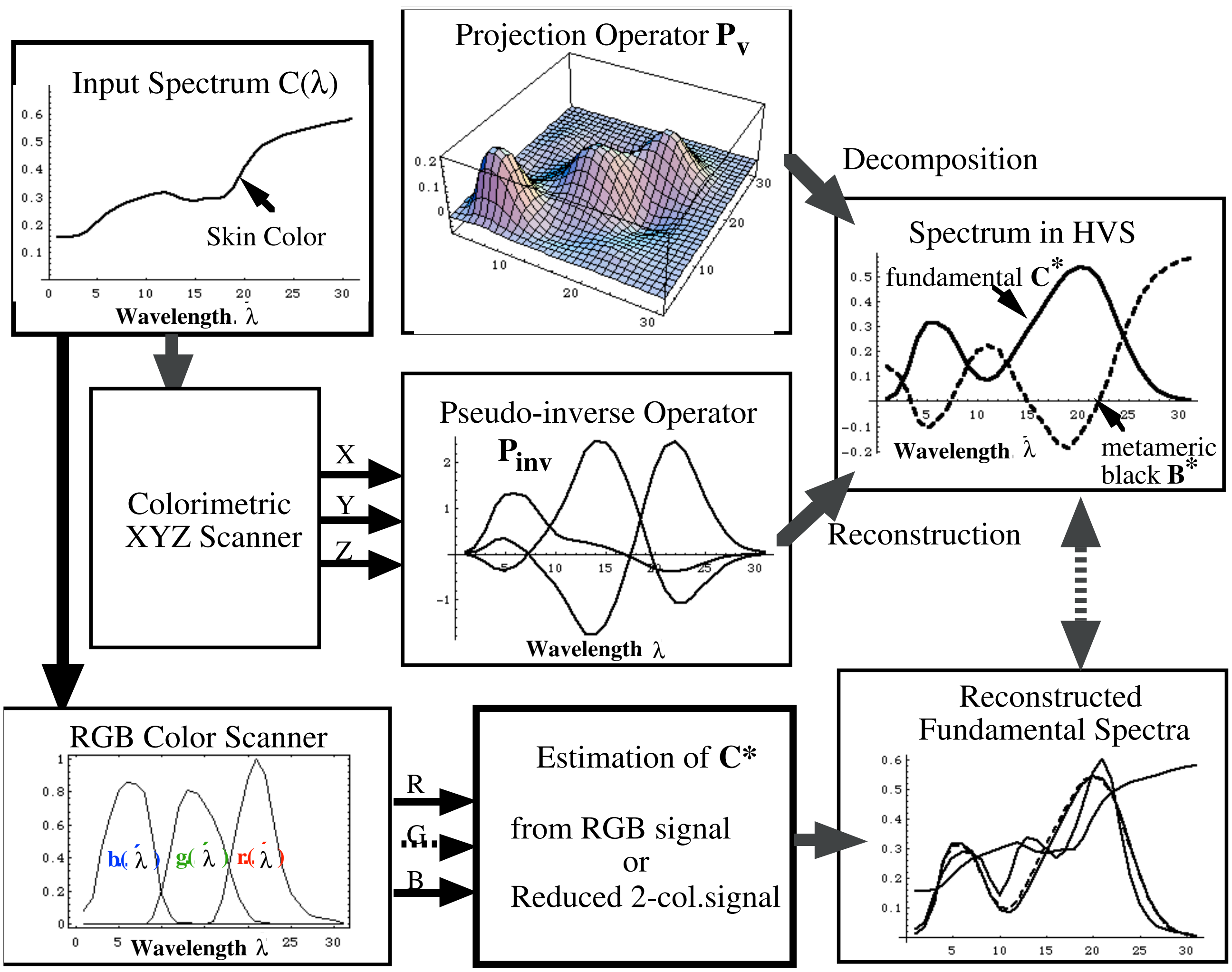

Figure 1. Basic Diagram for Estimating and Recovering Fundamental Spectra

$$M_c = (A^t R F)(F^t R F)^{-1};\ R = E[C_j C_j^t] \tag{10}$$

where, **R** denotes correlation matrix for C_j.

Applying the matrix M_c to Eq.(8), the corrected pseudo-inverse solution is given by

[Solution-B:Corrected Pseudo-inverse]

$$\hat{C}_{F_C} = P_{F_C inv} X\ ,\quad P_{F_C inv} = F_C (F_C^t F_C)^{-1},\quad F_C^t = M_C F \tag{11}$$

While, by substituting X_c for T in Eq. (6), the fundamental from a signal with colorimetric correction is given by

[Solution-C:Colorimetric Pseudo-inverse]

$$\hat{C}^*_{X_C} = P_{inv} X_C = P_{inv} M_C X \tag{12}$$

Smoothed Pseudo-Inverse Operator

Equation (8) gives a direct pseudo-inverse solution from a scanner RGB signal, but often results in irregular spectral shape.

A smoothed shape of spectrum may be recovered by operating a smoothing matrix S 5 with correlation coefficient r at adjacent wavelength of spectral components as follows.

[Solution-D:Smoothed Pseudo-inverse]

$$\hat{C}^*_{F_S} = P_{F_S inv} X,\quad P_{F_S inv} = SF(F^t SF)^{-1}$$

$$S = \begin{bmatrix} 1 & \rho & \rho^2 \ldots & \ldots & \rho^{n-1} \\ \rho & 1 & \rho & \rho^2 & \rho^{n-2} \\ \vdots & & & & \vdots \\ \vdots & & & & \vdots \\ \rho^{n-1}\ldots & \ldots & \rho^2 & \rho & 1 \end{bmatrix} \tag{13}$$

The smoothing matrix S can be also applied to a filter with spectral correction, giving the modified solution as **[Solution-E:Modified Pseudo-inverse]**

$$\hat{C}^*_{F_{CS}} = P_{F_{CS} inv} X\ ,\quad P_{F_{CS} inv} = SF_C (F_C^t SF_C)^{-1} \tag{14}$$

Recovery of Skin-color Spectra from Reduced Scanner Signals

A smoothed shape of spectrum such as skin color is known to be expanded by a small number of basis functions. We had reported a possibility of color image rendition from 2-color separation signals.[6] In this paper we examine a recovery of fundamental spectrum of skin color from the reduced color scanner signals, for example, cutting the Y in XYZ or G in RGB signals, the matrices A or F are reduced as follows.

$$A_2 = \begin{bmatrix} x(\lambda_1), x(\lambda_2), \ \ldots\ldots, x(\lambda_n) \\ z(\lambda_1), z(\lambda_2), \ \ldots\ldots, z(\lambda_n) \end{bmatrix}^t, \ F_2 = \begin{bmatrix} r(\lambda_1), r(\lambda_2), \ \ldots\ldots, r(\lambda_n) \\ b(\lambda_1), b(\lambda_2), \ \ldots\ldots, b(\lambda_n) \end{bmatrix}^t \tag{15}$$

A rough estimation of fundamental spectra is possible by applying the pseudo-inverse operators using reduced matrices $\boldsymbol{A}_2$ or $\boldsymbol{F}_2$ on the 2-color signals $\boldsymbol{T}_2 = [X,Z]^t$ or $X_2 = [R,B]^t$ as follows.

$$\boldsymbol{C}_2^* = \boldsymbol{P}_{inv2}\boldsymbol{T}_2 , \qquad \boldsymbol{P}_{inv2} = \boldsymbol{A}_2(\boldsymbol{A}_2^t\boldsymbol{A}_2)^{-1} \tag{16}$$

$$\hat{\boldsymbol{C}}_{F_2}^* = \boldsymbol{P}_{F_2 inv}\boldsymbol{X} , \qquad \boldsymbol{P}_{F_2 inv} = \boldsymbol{F}_2(\boldsymbol{F}_2^t\boldsymbol{F}_2)^{-1} \tag{17}$$

A smoothing matrix $\boldsymbol{S}$ can be also applied to these operators for reduced signals.

Results

A fundamental spectrum of skin color was estimated from CIE-XYZ value and also a color scanner signal RGB. Then we have examined how it can be recovered from the reduced 2-color signals, such as *XY* or *RB*.

Figure 2 shows a recovery from tri-color signals for a tested color scanner. It is shown that a fundamental spectrum is best recovered by operating the colorimetric pseudo-inverse projection $\boldsymbol{P}_{inv}$ on corrected color signal $\boldsymbol{X}_c$ [Solution:C]. The matrix $\boldsymbol{M}_c$ of the scanner was determined as shown in Eq. (18)..

$$\boldsymbol{M}_C = \begin{bmatrix} 0.72958, & 0.09636, & 0.17013 \\ 0.41505, & 0.60072, & -0.01550 \\ -0.01516, & 0.03999, & 0.97357 \end{bmatrix} \tag{18}$$

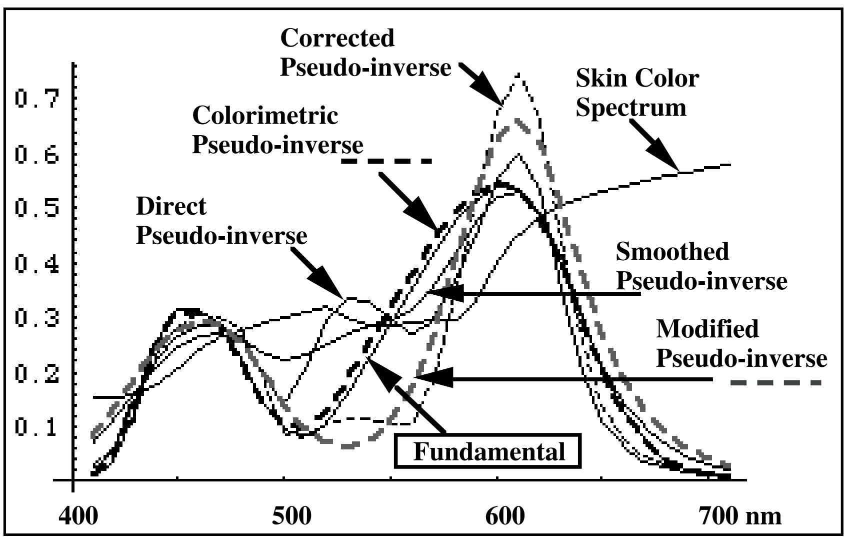

Figure 2. Recovery of Skin Color Spectrum from Tri-color Signal

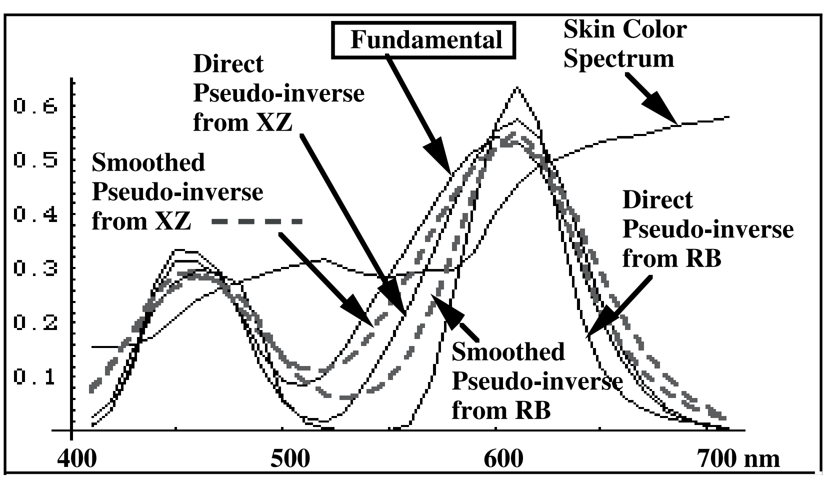

Figure 3. Recovery of Skin Color Spectrum from Two-color Signal

Figure 3 is a recovery from 2-color signals. Here, we cannot use a filter correction for 2-color signals but a skin color fundamental can be roughly recovered by applying a pseudo-inverse operator with smoothing effect from two components such as XY or RB in tri-color signals **[Solution:D]**. The larger the correlation coefficient is, the greater the smoothing effect becomes. In the above estimations, it was selected around $\rho \cong 0.6$.

Conclusions

A spectral reconstruction from color scanning signals is discussed based on the inverse projection. Here, not the real spectrum but the fundamental spectrum is shown to be recovered from the tri-color signals, because it is a principal component without metameric black and essential to human vision. A smoothed spectrum like as a skin color is possible to be approximately recovered from two color signals or two basis functions.

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