

# A Printer Model for Color Printing

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## Abstract

A new model to predict color for dot-on-dot color printing is presented. The Neugebauer narrow-band color mixing model was applied with modifications. The Yule-Nielsen factor  $n$  is optimized by minimizing  $\Delta E^*_{L^*a^*b^*}$  or  $\Delta E^*_{94}$ . Dot area at each wavelength was calculated by the Balasubramanian's cellular model with eighty-one primaries. Neugebauer colorimetric quality factor (CQF) was applied as a weighting function for the optimization of dot areas. The application of the CQF decreases average color difference significantly. We also analyzed the difference of optimizing the Yule-Nielsen  $n$ -value by minimizing  $\Delta E^*_{L^*a^*b^*}$  and by minimizing  $\Delta E^*_{94}$ . There is almost no further improvement in the optimization of the  $n$ -value by using  $\Delta E^*_{94}$  instead of  $\Delta E^*_{L^*a^*b^*}$  with the data set we used.

## 1. Introduction

In 1936, Murray and Davies<sup>1</sup> published a model to predict the binary tone reproduction printing process. This model was extended by Neugebauer<sup>2</sup> in 1937 for color halftoning. However, variation from this linear optical model is often significant. Yule and Nielsen<sup>3</sup> modified the tone reproduction model in which they incorporated an empirical factor  $n$  to fit the non-linear relationship between total reflectance and dot area. Viggiano<sup>4,5</sup> further improved the model by applying Yule-Nielsen factor  $n$  into the Neugebauer narrow-band model. This model and its variations have been widely used to model binary color printers.<sup>6-9</sup> Arney's model<sup>10,11</sup> reserves the linearity of the Murray-Davies model, and explains the reflectances of paper and inks as functions of ink area. Balasubramanian<sup>12</sup> extended the Neugebauer model into a cellular model in which the primary colors are not limited to sixteen. Due to the increase number of primaries in the cellular model, it yields more accurate result from a finer interpolation.

In this paper, the Neugebauer colorimetric quality factor<sup>13,14</sup> (CQF) is applied to optimize the dot areas of colorants. The Neugebauer 16-primary model and the Balasubramanian's cellular model are used to predict colorimetric values. Yule-Nielsen factor  $n$  is optimized by minimizing  $\Delta E^*_{L^*a^*b^*}$  or  $\Delta E^*_{94}$ ,<sup>15,16</sup> and the differences between using both color difference equations are analyzed.

This model was tested on a Novajet Pro wide-format ink jet color printer which was based on a four-color dot-on-dot printing process.

## 2. Neugebauer Equation and Its Yule-Nielsen Modification

The simplest tone reproduction model, Murray-Davies model, can be expressed by the following equation

$$R = (1 - a) R_0 + a R_i, \quad (2.1)$$

where  $R$  is the total reflectance,  $R_0$  is the reflectance of the base which is usually paper,  $R_i$  is the reflectance of ink, and  $a$  is the relative dot area of the ink.

The narrow-band equation based on this model can be written as

$$R(\lambda) = (1 - a) R_0(\lambda) + a R_i(\lambda), \quad (2.2)$$

where  $R(\lambda)$  is the total reflectance at wavelength  $\lambda$ ,  $R_0(\lambda)$  is the reflectance of paper at wavelength  $\lambda$ , and  $R_i(\lambda)$  is the reflectance of ink at wavelength  $\lambda$ .

In the case of a four-color printing process, a color is the combination of four colorants, cyan (C), magenta (M), yellow (Y), black (K), and white paper (W). The Neugebauer equation for four-color binary printing process is given by

$$R(\lambda)_{CMYK} = \sum_{i=1}^{16} a_i R_i(\lambda), \quad (2.3)$$

where  $R(\lambda)_{CMYK}$  is the total spectral reflectance in a given wavelength  $\lambda$ ,  $i$  is the  $i$ -th Neugebauer primary,  $R_i(\lambda)$  is the spectral reflectance of the  $i$ -th primary in the given wavelength  $\lambda$ , and  $a_i$  is the relative area of the  $i$ -th primary. The  $a_i$ s are given by

$$\begin{aligned} a_0 &= (1-c)(1-m)(1-y)(1-k) \\ a_1 &= c(1-m)(1-y)(1-k) \\ a_2 &= m(1-c)(1-y)(1-k) \\ a_3 &= y(1-c)(1-m)(1-k) \\ a_4 &= k(1-c)(1-m)(1-y) \\ a_5 &= cm(1-y)(1-k) \\ a_6 &= cy(1-m)(1-k) \\ a_7 &= ck(1-m)(1-y) \\ a_8 &= my(1-c)(1-k) \\ a_9 &= mk(1-c)(1-y) \\ a_{10} &= yk(1-c)(1-m) \\ a_{11} &= cmy(1-k) \\ a_{12} &= cmk(1-y) \\ a_{13} &= cyk(1-m) \\ a_{14} &= myk(1-c) \\ a_{15} &= cmyk \end{aligned} \quad (2.4)$$

where  $c$ ,  $m$ ,  $y$ , and  $k$  are dot areas of cyan, magenta, yellow, and black colorants, respectively.

The Murray-Davies equation is valid only for zero optical gain behavior. Yule and Nielsen improved the Murray-Davies equation by incorporating an  $n$  factor. Thus, equation (2.1) becomes

$$R^{1/n} = (1 - a) R_0^{1/n} + a R_i^{1/n} \quad (2.5)$$

This is the Yule-Nielsen equation. Viggiano extended the Yule-Nielsen effect to the Neugebauer equation to predict the color of halftone printing by equation

$$R(\lambda)_{CMYK}^{1/n} = \sum_{i=1}^{16} a_i R_i(\lambda)^{1/n} \quad (2.6)$$

The dot area  $a_i$  can be obtained by minimizing the color difference or by minimizing the difference between the predictive reflectance and the corresponding measured reflectance.

### 3. A New Model

The Neugebauer color mixing equation (2.6) will be used for printer color formation with some modifications, and the Neugebauer colorimetric quality factor (CQF) will be applied to optimize dot areas. In the Neugebauer model, sixteen primaries are always the same for any CMYK values. With the application of the cellular model, sixteen primaries are determined based on CMYK values. The processes are described in following sections.

#### 3.1 The Determination of Dot Areas

When equation (2.6) is used to calculate dot areas for each wavelength, dot area becomes a function of wavelength. Thus  $a_i$  can be treated as a function of  $\lambda$ ,

$$R(\lambda)_{CMYK}^{1/n} = \sum_{i=1}^{16} a_i(\lambda) R_i(\lambda)^{1/n} \quad (3.1)$$

The wavelength independent dot area coverage  $a_i$  is then obtained by different techniques. The simplest technique is by averaging  $a_i(\lambda)$  in the visual spectrum, or by using a wide-band color mixing model. This kind of algorithm does not give very good result. Since the human vision sensitivities in different wavelengths are different, errors at different wavelengths should not be treated evenly. Another technique to optimize  $a_i$  is by minimizing the color difference between the measured colorimetric value and the corresponding predictive colorimetric value in a uniform color space, such as CIE  $L^*a^*b^*$  color space. This algorithm gives good results but takes a lot of computation.

A new technique is to calculate  $a_i(\lambda)$  in the visual spectrum (we used wavelengths from 400nm to 700nm) by equation (3.1), and  $a_i$  is then optimized by equation

$$a_i = \int_{\lambda=400}^{700} w(\lambda) a_i(\lambda) d\lambda, \quad (3.2)$$

where  $w(\lambda)$  is a weighting function.  $w(\lambda)$  is determined based on the characteristics that the human visual sensitivity is a function of wavelength.

Neugebauer's CQF is used as the weighting function  $w(\lambda)$ . Figure 1 shows the CQF function. Since human eyes are almost non-sensitive to the wavelengths below 400nm and wavelengths above 700nm, the weights in these areas are set to zero. Three peaks approximately correspond to the most sensitive wavelengths of three cone types of the human eye.<sup>17</sup>

#### 3.2 The Determination of Primaries

Balasubramanian's cellular framework model was used to determine primaries. The eighty-one Neugebauer primaries were implemented. In the 16-primary model (the Neugebauer model), primaries are the combinations of C, M, Y, and K being 0 and 1. In the 81-primary model, the primaries are the combinations of C, M, Y, and K being 0, 0.5, and 1 (the medium value is not necessary to be exactly 0.5). Primaries to predict an unknown color are still sixteen in the 81-primary model. The difference between the 16-primary model and the 81-primary model is that the sixteen primaries in the 16-primary model are unchanged, while the sixteen primaries in the 81-primary model are changed for different colors. For example, if cyan dot area  $c \leq 0.5$ , cyan dot areas to compose primaries are 0 and 0.5, and  $c$  is re-scaled to  $c' = 2 \cdot c$  (see Figure 2-a); if cyan dot area  $c > 0.5$ , cyan dot areas to compose primaries are 0.5 and 1, and  $c$  is re-scaled to  $c' = 2 \cdot (c - 0.5)$  (see Figure 2-b).

#### 3.3 The Determination of Yule-Nielsen Factor $n$

The Yule-Nielsen factor  $n$  can be obtained by minimizing the average color difference, such as  $\Delta E^*_{L^*a^*b^*}$  or  $\Delta E^*_{94}$ , between the measured colorimetric values and the corresponding predictive values from a set of colors which are almost uniformly distributed in the whole color gamut.

## 4. Experimental Results

An X-Rite 938 spectrophotometer was used to measure spectral reflectances and CIE  $L^*a^*b^*$  values. The measuring geometry is 0/45. The validation of the predictive model was tested on an Encad Novojet Pro wide-format ink jet color printer.

The 4 by 12 cyan, magenta, yellow, and black primary ramps (CMYK digital values from 0 to 255) were printed and the spectral reflectances were measured. The dot areas were calculated by equations (3.1) and (3.2). Four one-dimensional look-up-tables for looking the CMYK dot areas from CMYK digital counts were obtained by linear interpolation.

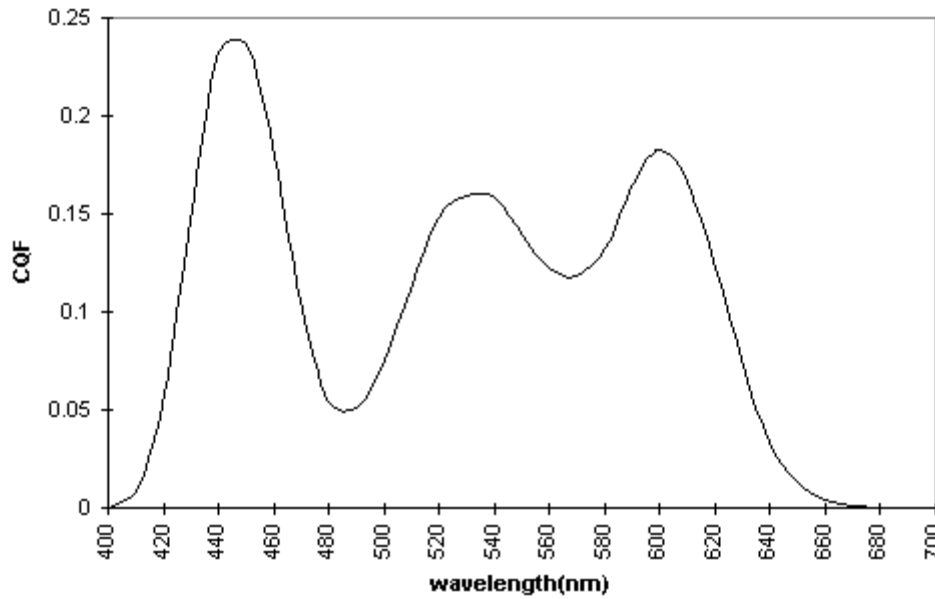


Figure 1. The Neugebauer's colorimetric quality factor (CQF).

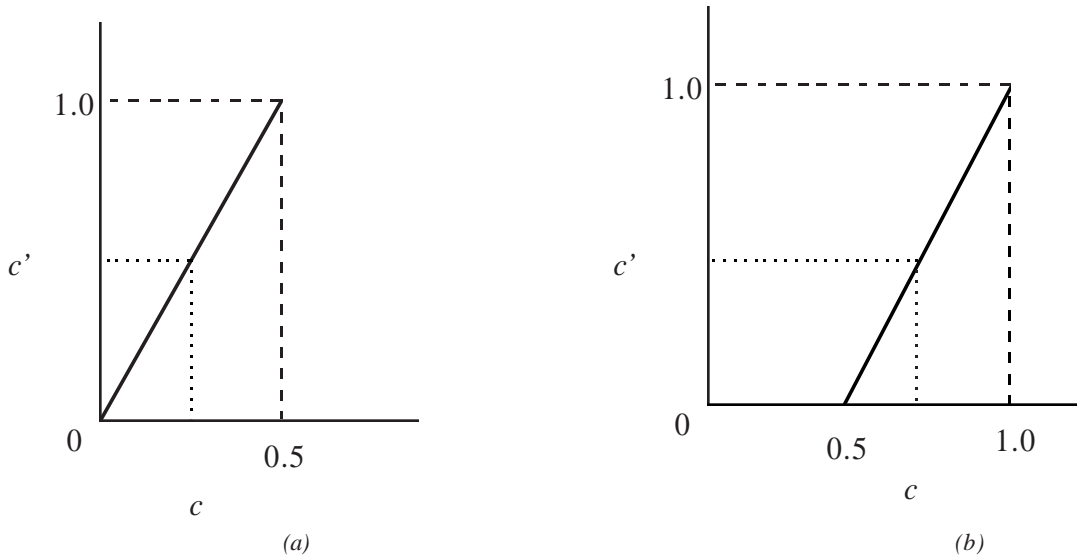


Figure 2. Re-scale cyan dot areas in the 81-primary model

The sixteen primaries and the eighty-one primaries were printed, and their spectral reflectances were measured to predict colorimetric value of any color using the 16-primary model or the 81-primary model.

Another set of color patches was printed to analyze color error  $\Delta E^*_{L^*a^*b^*}$  or  $\Delta E^*_{94}$ . This set of patches was designed to be distributed approximately uniformly in the whole color gamut so that the color errors are uniformly distributed. The Yule-Nielsen factor  $n$  was optimized by minimizing the average  $\Delta E^*_{L^*a^*b^*}$  or  $\Delta E^*_{94}$ .

#### 4.1 The 16-Primary Model

Dot areas were calculated with the Neugebauer CQF weighting by equations (3.1) and (3.2). Look-up-tables for CMYK dot areas with  $n = 9$  are shown in Figure 3. The

minimum average  $\Delta E^*_{L^*a^*b^*}$  and the minimum average  $\Delta E^*_{94}$  are 7.478 and 4.027, respectively. The Yule-Nielsen factor  $n$  is 9 by minimizing average  $\Delta E^*_{L^*a^*b^*}$  or 8 by minimizing average  $\Delta E^*_{94}$ . The difference of the average color errors for  $n = 9$  and  $n = 8$  is  $0.007 \Delta E^*_{L^*a^*b^*}$ . Figure 4 shows the average  $\Delta E^*$  versus  $n$ . Since the average color errors are almost the same for  $n$  optimized at 8 which was determined by minimizing average  $\Delta E^*_{L^*a^*b^*}$  or 9 which was determined by minimizing average  $\Delta E^*_{94}$ , the color errors by minimizing average  $\Delta E^*_{L^*a^*b^*}$  and by minimizing average  $\Delta E^*_{94}$  are almost the same. Therefore, it is not necessary to use the more complicated  $\Delta E^*_{94}$  equation to optimize  $n$ .

Dot areas were also calculated without using the Neugebauer CQF weights. In this case, dot areas were averaged in the visual spectrum. The average color errors

$\Delta E^*_{L^*a^*b^*}$  and  $\Delta E^*_{94}$  versus Yule-Nielsen factor  $n$  are shown in Figure 5. The minimum average  $\Delta E^*_{L^*a^*b^*}$  and  $\Delta E^*_{94}$  are 10.90 and 5.527, respectively. The Yule-Nielsen factor  $n$  is 12 by minimizing average  $\Delta E^*_{L^*a^*b^*}$  or 11 by minimizing average  $\Delta E^*_{94}$ . The average color error by this method is much greater than that with the Neugebauer CQF weights. Again, it was found that the average color errors from minimizing average  $\Delta E^*_{L^*a^*b^*}$  and from minimizing average  $\Delta E^*_{94}$  are almost the same.

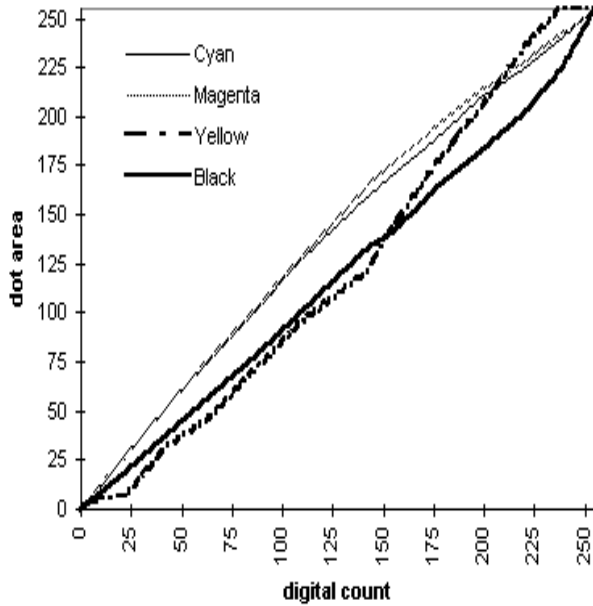


Figure 3. CMYK dot areas versus CMYK digital values for Yule-Nielsen factor  $n=9$ .

### 4.2 The 81-Primary Model with Neugebauer Weighting

The look-up-tables for looking CMYK dot areas from CMYK digital counts are the same as those with the 16-primary model. The average color error as a function of  $n$  is shown in Figure 6. The minimum average  $\Delta E^*_{L^*a^*b^*}$  and the minimum average  $\Delta E^*_{94}$  are 3.913 and 2.147, respectively, both with  $n$  being 10. The average color error is much less in the 81-primary model than that in the 16-primary model.

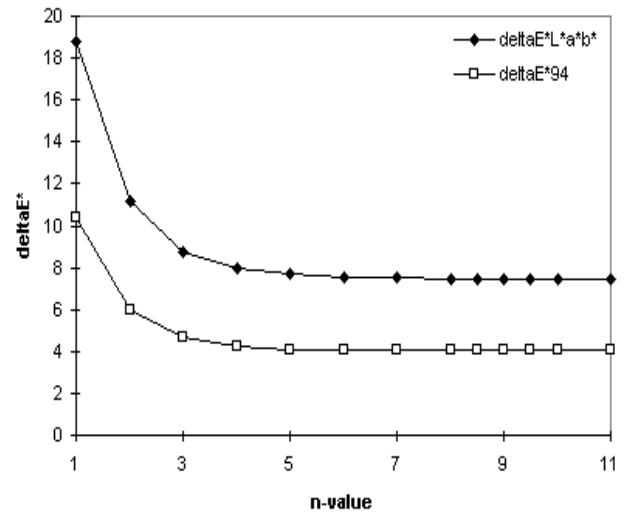


Figure 4. Average  $\Delta E^*_{L^*a^*b^*}$  and average  $\Delta E^*_{94}$  versus Yule-Nielsen factor  $n$  for the 16-primary model with Neugebauer CQF weighting.

### 4.3 Summary of the Experimental Results

The results of the three tested algorithms are summarized in Table 1, from the simplest Neugebauer color mixing model to the final cellular model with Neugebauer weighting.  $\Delta E^*_{L^*a^*b^*}$  and  $\Delta E^*_{94}$  in the table are average color errors. The cellular model with Neugebauer weighting results in smallest average color error. The average color errors from minimizing  $\Delta E^*_{L^*a^*b^*}$  and from minimizing  $\Delta E^*_{94}$  are almost the same.

A Novajet Pro wide-format color printer was used to make the experiment. The average random error after ten minutes of printing is about 1  $\Delta E^*_{L^*a^*b^*}$  in this type of printer<sup>18</sup>. Assuming  $\Delta E^*_{L^*a^*b^*}$  is additive, the color error would be about 2.9  $\Delta E^*_{L^*a^*b^*}$  by the cellular model with Neugebauer weighting after subtracting the printer random error. Recently, we modified the Neugebauer weighting which decreased average error for about 0.5  $\Delta E^*_{L^*a^*b^*}$ . Thus, the average color error in our printer model is about 2.4  $\Delta E^*_{L^*a^*b^*}$ .

Table 1 : Summary of Algorithms

MODEL	Minimize $\Delta E^*_{L^*a^*b^*}$		Minimize $\Delta E^*_{94}$	
	$\Delta E^*_{L^*a^*b^*}$	$\Delta E^*_{94}$	$\Delta E^*_{L^*a^*b^*}$	$\Delta E^*_{94}$
Neugebauer color mixing model, average dot area in the visual spectrum	10.900	5.527	10.910	5.527
Neugebauer color mixing model, Neugebauer weighting	7.478	4.030	7.485	4.027

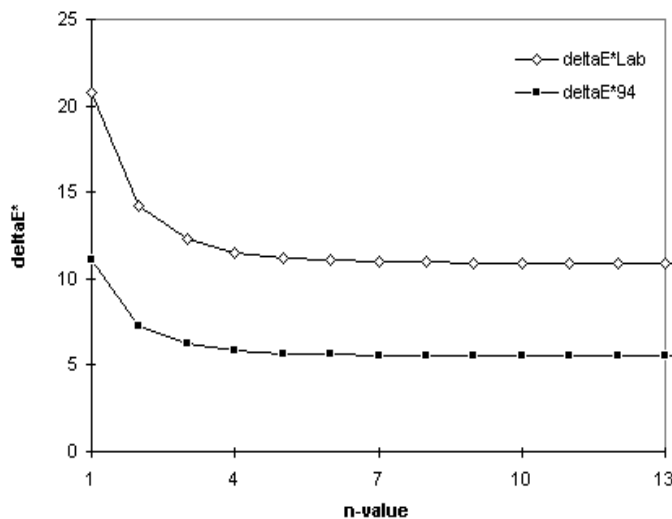


Figure 5. Average  $\Delta E^*_{L^*a^*b^*}$  and average  $\Delta E^*_{94}$  versus Yule-Nielsen factor  $n$  for the 16-primary model without Neugebauer CQF weighting.

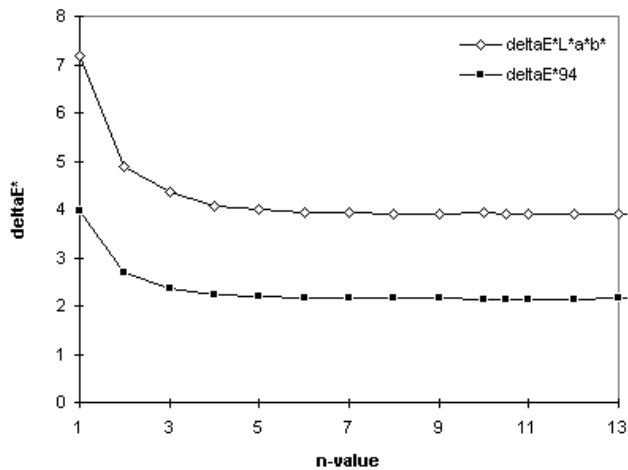


Figure 6. Average  $\Delta E^*_{L^*a^*b^*}$  and average  $\Delta E^*_{94}$  versus Yule-Nielsen factor  $n$  for the 81-primary model with Neugebauer CQF weighting.

## 5. Conclusions

A new model for printer color formation was presented. Neugebauer color mixing model was applied with modifications, and the Neugebauer CQF weights were implemented to optimize dot areas. We tested three models in a Novajet Pro wide-format ink jet printer and compared the results. The average color error  $\Delta E^*_{L^*a^*b^*}$  in the 81-primary model is smaller than that in the 16-primary model. The application of the Neugebauer CQF weighting decreases average color error significantly, and the computation is very simple. The cellular model with Neugebauer weighting results in smallest average color error. The average color errors from minimizing average  $\Delta E^*_{L^*a^*b^*}$  and from minimizing average  $\Delta E^*_{94}$  turn out almost the same results.

The simpler  $\Delta E^*_{L^*a^*b^*}$  equation is suggested to be used to optimize Yule-Nielsen factor  $n$ . The average color error after subtracting the printer random error is about  $2.4 \Delta E^*_{L^*a^*b^*}$  in our model.

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