Multiplicity of Flows in Coating Dies

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Abstract

Newtonian slow flow in a simple coating die is examined. It is shown that there is a multiplicity of flows satisfying the relevant equations of motion. The implication of this result for uniformity of coatings is discussed.

Introduction

The problem of finding the distribution of flow in an end-fed die has been examined by many authors.[1,2] In this work we follow one formulation of the problem.[1] The point of departure from other treatments is the fact that the 2-D nature of the flow in the metering slot is taken into account.

Mathematical Model

We assume that the dimensions of the cavity are large and the metering slot is narrow. We also assume that the velocity profile in the slot is parabolic, that inertial terms can be neglected and that Poisseuile flow exists in the cavity.

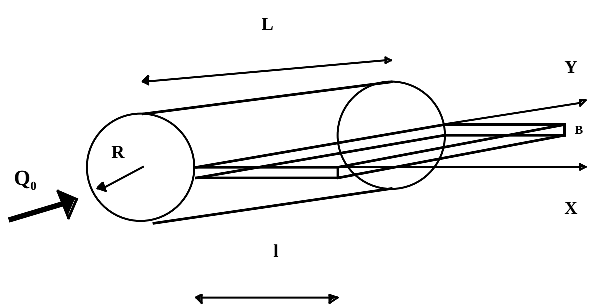

Figure 1. Schematic diagram and coordinate system for coating die.

Cavity

$$\frac{dQ}{dy} = Q_{source} - q_x(0, y) \tag{1}$$

$$Q(L) = 0 \tag{1a}$$

$$Q_{source} = 2Q_0\delta(y) = \frac{Q_0}{L} + \frac{2Q_0}{L}\sum_{n=1}^{\infty}\cos\frac{n\pi y}{L} \tag{2}$$

$$\frac{dP_c}{dy} = -\alpha^2 Q \tag{3}$$

where

$$\alpha^2 = \frac{8\mu}{\pi R^4} \tag{3a}$$

Metering Slot:

$$q_x = -\beta^2\frac{\partial p}{\partial x} \tag{4a}$$

$$q_y = -\beta^2\frac{\partial p}{\partial y} \tag{4b}$$

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0 \tag{5}$$

where

$$\beta^2 = \frac{B^3}{12\mu} \tag{6}$$

$$p(l, y) = 0 \tag{7}$$

$$\frac{\partial p}{\partial y} = 0 \text{ at } y = 0 \text{ and } y = L \tag{8}$$

One solution to this problem can be obtained in the following manner; consider the case where:

$$\frac{dQ}{dy} = Q_{source} - q_x(0, y) = 0 \quad (9)$$

i.e. $Q(y) = 0$ (10)

and $P_c(y) =$ constant (11)

Then the metering slot has in addition to the (4) to (8) a further boundary condition:

$$q_x(0, y) = Q_{source} = \frac{Q_0}{L} + \frac{2Q_0}{L}\sum_{n=1}^{\infty}\cos\lambda_n y \quad (12)$$

$$\text{where } \lambda_n = \frac{n\pi}{L} \quad (13)$$

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If we assume that a solution of the form:

$$p_1 = A_0(x - l) + \sum_{n=1}^{\infty} A_n \sinh\lambda_n(x - l)\cos\lambda_n y \quad (14)$$

exists, it will satisfy the boundary conditions (7) and (8) as well as equation (5).

$$\text{At } x = 0, \ \frac{\partial p_1}{\partial x} = A_0 + \sum_{n=1}^{\infty} A_n\lambda_n \cosh\lambda_n l \cos\lambda_n y \quad (15$$

Substituting in (12) and using (4a) we get:

$$-\beta^2\frac{\partial p_1}{\partial x} = -\beta^2 A_0 + \sum_{n=1}^{\infty}(-\beta^2 A_n\lambda_n\cosh\lambda_n l)\cos\lambda_n y$$

$$= \frac{Q_0}{L} + \frac{2Q_0}{L}\sum_{n=1}^{\infty}\cos\lambda_n y \quad (16)$$

Henc :

$$A_0 = -\frac{Q_0}{\beta^2 L}, \ A_n = \frac{(-2Q_0/L)}{\beta^2\lambda_n\cosh\lambda_n l} \quad (17)$$

$$\text{i.e. } p_1 = \frac{Q_0(l - x)}{\beta^2 L} + \sum_{n=1}^{\infty}\frac{(-2Q_0/L)\sinh\lambda_n(x - l)\cos\lambda_n y}{\cosh\lambda_n l} \quad (18)$$

The flow at the exit of the slot is given by:

$$-\beta^2\frac{\partial p_1}{\partial x}\Big|_{x=l} = \frac{Q_0}{L} + \sum_{n=1}^{\infty}\frac{(-2Q_0/L)\cos\lambda_n y}{\cosh\lambda_n l} \quad (19)$$

A second solution may be obtained by assuming that:

$$p(0, y) = P_c(y) \quad (20)$$

In this case, if we assume that a solution of the form:

$$p_2 = B_0(x - l) + \sum_{n=1} B_n\sinh\lambda_n(x - l)\cos\lambda_n y \quad (21)$$

exists, then equations (1) to (3) for the cavity yield the condition at x = 0

$$-\frac{1}{\alpha^2}\frac{\partial^2 p_2}{\partial y^2} = \beta^2\frac{\partial p_2}{\partial x} + \frac{Q_0}{L} + \frac{2Q_0}{L}\sum_{n=1}^{\infty}\cos\lambda_n y \quad (22)$$

Substituting (21) in (22) gives :

$$B_0 = -\frac{Q_0}{\beta^2 L}, \ B_n = \frac{(-2Q_0/L)}{\lambda_n(\beta^2\cosh\lambda_n l + \frac{\lambda_n}{\alpha^2}\sinh\lambda_n l)} \quad (23)$$

$$\text{i.e. } p_2 = \frac{Q_0(l - x)}{\beta^2 L} + \sum_{n=1}^{\infty}\frac{(-2Q_0/L)\sinh\lambda_n(x - l)\cos\lambda_n y}{\lambda_n(\beta^2\cosh\lambda_n l + \frac{\lambda_n}{\alpha^2}\sinh\lambda_n l)} \quad (24)$$

The flow at the exit of the slot is given by :

$$-\beta^2\frac{\partial p_2}{\partial x}\Big|_{x=l} = \frac{Q_0}{L} + \sum_{n=1}^{\infty}\frac{(-2Q_0/L)\cos\lambda_n y}{(\cosh\lambda_n l + \frac{\lambda_n}{\alpha^2\beta^2}\sinh\lambda_n l)} \quad (25)$$

Equations (18) and (24) give two independent solutions to the problem in question. Hence any linear combination:

$$p(x, y) = t\, p_1(x, y) + (1 - t)\, p_2(x, y) \quad (26)$$

is also a solution. Note also that:

$$\phi(x, y) = p_1(x, y) - p_2(x, y) \quad (27)$$

provides a solution to the problem (1)-(8) with $Q_{source} = 0$. Thus solutions of this type with arbitrary wavelength L can also be added to any solution with any origin. It is therefore clear that multiple steady solutions exist for the problem of flow in a coating die. No conclusions can be drawn about the stability of these solutions with this simple model. However, the fact that multiplicity is observed in the case of real coating dies would encourage the belief that some of these solutions (27) corresponding to wavelengths L are indeed neutrally stable. In this connection, we may also note that such disturbances are neutrally stable for films issuing from slits as reported in reference [3]. We may thus expect the free surfaces of films to exhibit longitudinal bands. Such streaks and bands are routinely observed in coatings.

Conclusions

The problem of flow in a coating die has no unique solution at low flow rates and hence must have a multiplicity of solutions under all practical coating conditions.

References

1. S. Middleman, *Fundamentals of Polymer Processing*, McGraw Hill (1977)p 107.
2. J. M. McKelvey, and K. Ito, Uniformity of Flow from Sheeting Dies, *Polym. Eng. Sci.*, vol **11**, No.3, p 258 (1971).
3. V. V. Gokhale, Longitudinal Instabilities in Exit Flow from a Slit, *Rheologica Acta*, vol **18**, p 335 (1979).

Nomenclature

$\alpha = \sqrt{\dfrac{8\mu}{\pi R^4}}$

$\beta = \sqrt{\dfrac{B^3}{12\mu}}$

$\lambda_n = \dfrac{n\pi}{L}$

μ = viscosity, Poise

B = slot opening, cm

L = width of die, cm

P_c = pressure in cavity, dyne/ cm^2

Q = flow rate in cavity, cm^3 / sec

Q_0 = half the total flow entering the die, cm^3 / sec

Q_{source} = source function, cm^3 / sec

R = radius of cavity, cm

l = length of metering slot, cm

p, p_1, p_2 = pressure in metering slot, dyne/cm^2

q_x, q_y = flow rate in metering slot in x and y direction, cm^2 /sec

x = x coordinate, cm

y = y coordinate, cm