Reconstructing Spectral and Colorimetric Data
Using Trichromatic and Multi-channel Imaging

Daniel Nyström
Dept. of Science and Technology (ITN), Linköping University
SE-60174, Norrköping, Sweden
danny@itn.liu.se

Abstract
The aim of the study is to reconstruct spectral and colorimetric data, using trichromatic and multi-channel imaging. An experimental image acquisition system is used, which besides trichromatic RGB filters also provides the possibility of acquiring multi-channel images, using 7 narrowband filters. To derive mappings to colorimetric and multispectral representations, two conceptually different approaches are used. In the model-based approach, the physical model describing the image acquisition process is inverted, to reconstruct spectral reflectance from the recorded device response. A priori knowledge on the smooth nature of spectral reflectances is utilized, by representing the reconstructed spectra as linear combinations of basis functions, using Fourier basis and a database of real reflectance spectra. In the empirical approach, the characteristics of the individual components are ignored, and the functions are derived by relating the device response for a set of training colors to the corresponding colorimetric and spectral measurements. Beside colorimetric regression, mapping device values directly to CIEXYZ and CIELAB, experiments are also made on reconstructing spectral reflectance, using least squares regression techniques.

The results indicate that for trichromatic imaging, accurate colorimetric mappings can be derived by the empirical approach, using polynomial regression to CIEXYZ and CIELAB. Because of the media-dependency, the characterization functions must be derived for each combination of media and colorants. However, accurate spectral reconstructions require for multi-channel imaging, using model-based device characterization. Moreover, the model-based approach is general, since it is based on the spectral characteristics of the image acquisition system, rather than the characteristics of a set of color samples.

Introduction
The trichromatic principle of representing color has for a long time been dominating in color imaging. The reason is the trichromatic nature of human color vision, but as the characteristics of typical color imaging devices are different from those of human eyes, there is a need to go beyond the trichromatic approach. The interest for multi-channel imaging, i.e. increasing the number of color channels, has made it an active research topic with a substantial potential of application.

To achieve consistent color imaging, one needs to map the imaging-device data to the device-independent colorimetric representations CIEXYZ or CIELAB. As the color coordinates depend not only on the reflective spectrum of the object but also on the spectral properties of the illuminant, the colorimetric representation suffers from metamerism, i.e. objects of the same color under a specific illumination may appear different when they are illuminated by another light source. Furthermore, when the sensitivities of the imaging device differ from the CIE color matching functions, two spectra that appear different for human observers may result in identical device response. In multispectral imaging, color is represented by the object’s spectral reflectance, which is illuminant
independent. With multispectral imaging, different spectra are readily distinguishable, no matter they are metameric or not. The spectrum can then be transformed to any color space and be rendered under any illumination.

The focus of the study is colorimetric and multispectral image acquisition, which requires methods for computing colorimetric and spectral data from the recorded device signals. Experiments are performed using trichromatic imaging as well as multi-channel imaging, using an experimental image acquisition system. Two conceptually different approaches for device characterization are evaluated: model-based and empirical characterization. In the model-based approach, the physical model describing the process by which the device captures color is inverted to reconstruct spectral reflectance. In the empirical approach, the device characteristics are ignored and the mappings are derived by correlating the device response for a set of reference colors to the corresponding colorimetric and spectral measurements, using least-squares regression.

The aim of this comparative study is to answer which colorimetric and spectral accuracy that can be achieved by employing the different methods for device characterization, for trichromatic and multi-channel imaging, respectively. Is the conventional trichromatic principle of image acquisition sufficient, or is multi-channel imaging required to reconstruct spectral and colorimetric data of high accuracy?

**Model-based Characterization**

The linear model for the image acquisition process, describing the device response to a known input, is given in Eq.1. The device response, \(d_k\), for the \(k\)th channel is, for each pixel, given by:

\[
d_k = \int_{\lambda \in V} I(\lambda) F_k(\lambda) R(\lambda) S(\lambda) d\lambda + \epsilon_k
\]

where \(I(\lambda)\) is the spectral irradiance of the illumination, \(F_k(\lambda)\) is the spectral transmittance of filter \(k\), \(R(\lambda)\) is the spectral reflectance of the object, \(S(\lambda)\) is the spectral sensitivity function for the camera, \(\epsilon_k\) is the measurement noise for channel \(k\), and \(V\) is the spectral sensitivity region of the device. The model requires for a linear response of the CCD sensor with respect to the incoming light, which has been verified.

The spectral characteristics of the illumination and the filters have been derived from direct measurements, using a spectroradiometer. The spectral sensitivity of the CCD camera has been estimated in previous work, by relating the device response to the known spectral reflectance for a set of carefully selected color samples, using least-squares regression techniques. The spectral properties of the components in the image acquisition system are displayed in Fig. 1.

![Spectral properties of components](image)

*Figure 1. Spectral power for the illuminant (a), estimated spectral sensitivity for the camera (b) and spectral transmittance for the RGB filters (c) and the 7 multi-channel filters (d).*

Having obtained the forward characterization function of all the components in the image acquisition system, the known spectral characteristics of the system can be represented by a spectral transfer function. The spectral transfer function, \(W_k(\lambda)\), describes the spectral characteristics for each channel, \(k\), as:

\[
W_k(\lambda) = I(\lambda) F_k(\lambda) S(\lambda)
\]

Denote the spectral signal as a discrete \(N\)-component vector, sampled at wavelengths \(\lambda_1, \ldots, \lambda_N\), and let \(W\) be the \(N \times K\) matrix in which each column describes the spectral transfer function of channel \(k\). Then the device response vector, \(d\), for a sample with spectral
reflectance $\mathbf{r}$ is given by:

$$\mathbf{d} = \mathbf{W}^T \mathbf{r}$$  \hspace{1cm} (3)

When inverting the model, we seek the $N \times K$ reconstruction matrix $\mathbf{M}$ that reconstructs the spectral reflectance, $\tilde{\mathbf{r}}$, from the camera response $\mathbf{d}$, as:

$$\tilde{\mathbf{r}} = \mathbf{M} \mathbf{d}$$  \hspace{1cm} (4)

The most straightforward approach to derive the reconstruction matrix is to simply invert Eq. 3, using the pseudo-inverse approach, giving the spectral reconstruction operator:

$$\mathbf{M}_0 = (\mathbf{W} \mathbf{W}^T)^{-1} \mathbf{W} = (\mathbf{W}^T)^{-1} \mathbf{W}$$  \hspace{1cm} (5)

where $(\mathbf{W}^T)^{-1}$ denotes for the More-Penrose pseudo-inverse of $\mathbf{W}$. Generally, the pseudo-inverse (PI) reconstruction is sensitive to noise, which makes the approach not always useful in practices. When $K < N$, i.e. the number of color channels $K$ is less than the number of spectral sampling points, $N$, the matrix $\mathbf{W}$ is of insufficient rank and the algebraic equations are underdetermined. Further more, this method minimizes the Euclidian distance in the camera response domain (i.e. between $\mathbf{d}$ and $\mathbf{W}^T \tilde{\mathbf{r}}$), which does not necessarily mean that the reconstructed spectrum will be close to the real spectrum.

Another approach is to instead seek another reconstruction matrix, $\mathbf{M}_1$, which minimizes the Euclidian distance between the reconstructed spectrum and the original spectrum. By exploiting the a priori information that the vast majority of reflectance spectra for real and man-made surfaces are smooth functions of wavelength, it can be assumed that the spectrum can be represented by a linear combination of a set of smooth basis functions, $\mathbf{B} = [\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_p]$. This gives the reconstruction operator $\mathbf{M}_1$, which minimizes the RMS spectral difference of the reconstructed spectrum, as:

$$\mathbf{M}_1 = \mathbf{B} \mathbf{B}^T \mathbf{W} \left( \mathbf{W}^T \mathbf{B} \mathbf{B}^T \mathbf{W} \right)^{-1}$$  \hspace{1cm} (6)

The base functions, $\mathbf{B}$, can consist of a set of real, measured spectral reflectances, which then should be representative to the reflectance of samples that is likely to be encountered in the image acquisition system. An alternative to spectral basis is to simply let $\mathbf{B}$ consist of a set of Fourier basis functions.

**Empirical Characterization**

In empirical characterization, colorimetric and spectral data are derived using a “black box” approach, i.e. without explicitly modeling the device characteristics. By correlating the device response for a training set of color samples to the corresponding colorimetric or spectral values, the characterization functions are derived using least squares regression techniques.

The characterization functions derived using empirical approaches will be optimized only for a specific set of conditions, including the illuminant, the media and the colorant. Once the conditions change, e.g. a different substrate or a different print mechanism, the characterization has to be re-derived in order to obtain good accuracy. The dependency on the illuminant is not an issue when the light source is fixed and can be considered as a property of the system. However, the fact that the characterization function is also media- and colorant dependent is a major drawback, preventing the characterization function from being applied to arbitrary combinations of media and colorants.

**Spectral Regression**

Even though empirical approaches are mainly used to derive mappings directly to the colorimetric representations, CIEXYZ or CIELAB, there have been attempts to reconstruct spectral reflectance. The spectral reconstruction matrix, $\mathbf{M}_s$, is now derived entirely based on the recorded device response to a set of training samples, i.e. ignoring the spectral characteristics of the imaging system. If the spectral reflectance for a set of $T$ training samples are collected into a $T \times N$ matrix $\mathbf{R} = [\mathbf{r}_1, \ldots, \mathbf{r}_T]$ and the corresponding device responses into a $T \times K$ matrix $\mathbf{D} = [\mathbf{d}_1, \ldots, \mathbf{d}_T]$, then the linear relationship is given by:

$$\mathbf{R} = \mathbf{D} \mathbf{M}_s$$  \hspace{1cm} (7)
and the optimal $K \times N$ spectral reconstruction matrix $M$ is then given by:

$$M = (D'D)^{-1}D'R = (D)^{-1}R$$  \hspace{1cm} (8)$$

In a similar way as for the model-based approach, the reconstructed spectra can be represented as linear combinations of a set of basis functions.

**Colorimetric Regression**

A common approach to derive colorimetric data is to use polynomial regression from device values directly to CIEXYZ. Polynomial regression is a special case of least squares regression, where the characterization function, relating the device data to the colorimetric representations, is approximated by a polynomial, as:

$$c = pA$$  \hspace{1cm} (9)$$

where $c$ is the colorimetric output vector (XYZ or L*a*b*), $p$ is the $Q$-component vector of polynomial terms derived from the device data, $d$, and $A$ is the $Q \times n$ matrix of polynomial weights. After arranging the input device data, $d$, into the polynomial vector, $p$, the polynomial regression is reduced into a linear least squares regression problem, with the optimal matrix of polynomial weights, $A$, given by:

$$A = (P'P)^{-1}P'C = (P)^{-1}C$$  \hspace{1cm} (10)$$

The drawback with using regression to CIEXYZ is that the RMS error in XYZ color space, which is minimized in the regression, is not closely related to the perceived color difference. If the final aim is to derive data in CIELAB color space, it could therefore preferable to use regression directly in the CIELAB domain, i.e. to minimize the CIE 1976 color difference $\Delta E_{ab}$, which provides a better correspondence to the visual color difference. Since the relationship between device data and CIELAB is not linear, a non-linear pre-processing step of the device values, using a cubic root function has been proposed, i.e. using $R^{1/3}$, $G^{1/3}$, $B^{1/3}$ in the regression. The cubic root function originates from the CIELAB transformation, which involves a cubic root function of the XYZ tristimulus values.

We propose an alternative approach, using polynomial regression from device data to CIEXYZ color space, but minimizing the perceptually more meaningful $\Delta E_{ab}$ color difference in the process. This cannot be achieved using the basic pseudo-inverse solution, but requires for some optimization software. The parameters derived according to Eqs. 9 and 10, i.e. the characterization function minimizing $\Delta XYZ$, are used as initial conditions. Then, the new parameters are derived by an unconstrained nonlinear optimization, using the Optimization toolbox for Matlab, with the mean $\Delta E_{ab}$ color difference for the training set, as a cost function. The algorithm used is the BFGS Quasi-Newton method, with a mixed quadratic and cubic line search procedure. The result is a function relating device data to CIEXYZ in a way that minimizes the CIE 1976 color difference, $\Delta E_{ab}$.

**The Image Acquisition System**

The image acquisition system is an experimental system, with great flexibility to control and alter the setup. While possible to capture images of arbitrary objects, the primary usage is to capture macro images of flat objects, such as substrates. The main use is to acquire high resolution images of prints and substrates, to study properties of color printing on a micro-scale level.

The images are captured using a monochrome CCD camera with 12 bit dynamic range, specially designed for scientific imaging. Macro optics allow for a maximal resolution corresponding to 1.2 $\mu$m/pixel. The illumination is provided using a tungsten halogen lamp through optical fibers, which offers an adjustable angle of incidence, as well as the possibility of using a backlight setup. Color images are sequentially captured, using filters mounted in a filter wheel in front of the light source. By using this color sequential method, there is no need for any interpolation or de-mosaicing scheme, as is the case for conventional digital cameras, using color filter arrays. Besides the trichromatic RGB-filters, the filter wheel also contains a set of 7 interference filters, allowing for the acquisition of multi-channel images. The interference filters have been selected to cover the visible spectrum with equally spaced pass bands, see Fig. 1.
Since the accuracy of the characterization will always be limited by the stability and uniformity of a given device, the characterization process has been preceded by a thorough calibration of the system. All the components have been controlled with respect to linearity, temporal stability and spatial uniformity.

**Experimental Setup**

The evaluation of the spectral and colorimetric reconstructions requires for the acquisition of spectral and colorimetric data for a set of test colors, along with the corresponding device response. Spectral measurements of the color-patches are performed using a spectroradiometer, placed in the same optical axis as the CCD-camera, using the $45^\circ / 0^\circ$ measurement geometry. The spectral data obtained are in the interval 380 to 780 nm, sampled at 4 nm intervals. For each color patch, the mean reflectance spectrum from 5 sequential measurements is computed, to account for random noise. The colorimetric data have been computed using standard formulae, under the D65 standard illuminant. Correspondingly, the camera response values have been acquired under identical conditions. Before the mean values are computed, the images are corrected for dark current and CCD gain.

For reference colors to evaluate the results of the model-based spectral reconstruction, 25 color patches from NCS are used. For the empirical characterization, a training set of 50 printed test colors are used to derive the characterization functions. For the evaluation, 50 independent colors are used, printed using the same substrate and conditions as the training set. Since characterization functions derived by least squares regression will always be optimized for the specific training set, it is important to use an independent set of evaluation colors to guard against a model that overfits the training set, giving unrealistically good results.

As basis functions we evaluate spectral basis, using a database of real spectra available from NCS, as well as Fourier basis. Five basis functions are used, corresponding to the first five Fourier basis functions and to the singular vectors corresponding to the five most significant singular values in the spectral autocorrelation function of the spectral database, using the principle eigenvector method.

**Experimental Results**

**Spectral Reconstruction**

Spectral data has been reconstructed from the recorded device response, using the pseudo-inverse (PI) method, as well as using spectral and Fourier basis, for the model-based and empirical approaches, respectively. The results are evaluated using the spectral RMS error, corresponding to the Euclidian distance in spectral reflectance space, between the original and the reconstructed spectra. The CIE 1976 color difference $\Delta E_{ab}$ is also computed, to provide a better measure of the perceived color difference between the spectra.

Table 1 lists the mean and maximum reconstruction errors for the different characterization methods, using trichromatic RGB imaging, as well as multi-channel imaging. Examples of reconstructed spectra, compared to the corresponding measured spectra, are displayed in Figures 2 and 3, for the model-based and empirical characterization, respectively.

<table>
<thead>
<tr>
<th>Data</th>
<th>Method</th>
<th>RMS Max</th>
<th>Mean</th>
<th>$\Delta E_{ab}$ Max</th>
<th>Mean</th>
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<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>RGB</td>
<td>PI</td>
<td>0.0706</td>
<td>0.0230</td>
<td>24.35</td>
<td>14.80</td>
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<td></td>
<td>Spectral</td>
<td>0.0041</td>
<td>0.0014</td>
<td>15.87</td>
<td>4.170</td>
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<td></td>
<td>Fourier</td>
<td>0.0155</td>
<td>0.0049</td>
<td>18.75</td>
<td>8.287</td>
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<tr>
<td>Multi</td>
<td>PI</td>
<td>0.0092</td>
<td>0.0030</td>
<td>4.364</td>
<td>1.529</td>
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<tr>
<td></td>
<td>Spectral</td>
<td>0.0039</td>
<td>0.0012</td>
<td>4.218</td>
<td>1.816</td>
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<tr>
<td></td>
<td>Fourier</td>
<td>0.0040</td>
<td>0.0011</td>
<td>7.271</td>
<td>2.112</td>
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<td>Empirical</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RGB</td>
<td>PI</td>
<td>0.0082</td>
<td>0.0023</td>
<td>13.22</td>
<td>7.532</td>
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<td></td>
<td>Spectral</td>
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<td>0.0031</td>
<td>12.49</td>
<td>7.444</td>
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<td>Fourier</td>
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<td>0.0035</td>
<td>13.81</td>
<td>6.897</td>
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<tr>
<td>Multi</td>
<td>PI</td>
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<td>0.0004</td>
<td>6.899</td>
<td>3.908</td>
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<tr>
<td></td>
<td>Spectral</td>
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<td>0.0018</td>
<td>9.320</td>
<td>5.525</td>
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<tr>
<td></td>
<td>Fourier</td>
<td>0.0052</td>
<td>0.0023</td>
<td>13.90</td>
<td>6.085</td>
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</table>
The results show that for the model-based approach, trichromatic imaging is not sufficient to achieve spectral or colorimetric accuracy. A mean color difference of 4.2 $\Delta E_{ab}$ and a maximal of almost 16 $\Delta E_{ab}$, is too high to be considered as useful. For the multi-channel images, the results improve dramatically. Spectral basis and Fourier basis lead to equivalent results in terms of the RMS difference, while the colorimetric results are in favor of the spectral basis. The pseudo-inverse solution is somewhat noisy and suffers from larger RMS difference. However, the general shapes of the reconstructed spectra follow the real spectra well, resulting in small colorimetric errors. Clearly, the PI-method produces spectral reconstructions that are close to metameric matches to the real spectra.

For the empirical characterization using trichromatic imaging, the pseudo-inverse method is superior to the corresponding model-based results. However, the improvement when applying the different basis functions is not as evident, and the best results for the model-based approach could not be achieved. The results for the multi-channel images are comparable to the corresponding model-based approach in terms of spectral RMS difference, but produces larger colorimetric errors.

**Colorimetric Regression**

For the polynomial regression to CIEXYZ and CIELAB, there are numerous ways to build the approximation functions, $p$, and the number of terms, $Q$, increases rapidly for higher order polynomials. The results presented here are based on second order polynomials. For a detailed description on the experiments and the results using different polynomial functions, we refer to 12.

Table 2 lists the results for the colorimetric regression to CIEXYZ and CIELAB, using trichromatic RGB images, as well as multi-channel images. The regression to CIEXYZ has been carried out using the pseudo-inverse approach according to Eqs. 9 and 10, as well as using the non linear optimization, minimizing $\Delta E_{ab}$. The regression directly to CIELAB has been carried out with the unprocessed device response, as well as by using the cubic root function as a pre-processing (pp) step.

<table>
<thead>
<tr>
<th>Data</th>
<th>Regression</th>
<th>$\Delta XYZ$</th>
<th>$\Delta E_{ab}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max</td>
<td>Mean</td>
<td>Max</td>
</tr>
<tr>
<td>RGB</td>
<td>XYZ</td>
<td>3.453</td>
<td>0.904</td>
</tr>
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<td></td>
<td>XYZ ($\Delta E_{ab}$)</td>
<td>4.214</td>
<td>0.890</td>
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<td></td>
<td>LAB</td>
<td>5.199</td>
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<td></td>
<td>LAB (pp)</td>
<td>4.317</td>
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<tr>
<td>Multi</td>
<td>XYZ</td>
<td>3.240</td>
<td>0.945</td>
</tr>
<tr>
<td></td>
<td>XYZ ($\Delta E_{ab}$)</td>
<td>3.490</td>
<td>0.972</td>
</tr>
<tr>
<td></td>
<td>LAB</td>
<td>8.439</td>
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</tr>
<tr>
<td></td>
<td>LAB (pp)</td>
<td>3.846</td>
<td></td>
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</table>
The results show that colorimetric regression directly to CIEXYZ and CIELAB gives a good colorimetric accuracy. When only colorimetric data is required, it is clearly an advantage to use colorimetric regression directly to CIEXYZ or CIELAB, compared to using regression to reconstruct spectral reflectance. The best results were achieved with the approach of using regression to CIEXYZ, with the functions derived to minimize the $\Delta E_{ab}$ color difference in CIELAB color space.

Noticeable is that the results using trichromatic imaging are comparable to the multi-channel results, an obvious difference to the results for the model-based approach. Clearly, there is nothing to gain by increasing the number of channels from 3 to 7, when using the colorimetric regression approach. This can be explained by the fact that the system of equations that is inverted, now is based on the number of samples in the training set, not on the number of channels in the image acquisition system, as is the case for the model-based characterization.

Experiments on cross-media characterization, i.e how the derived functions perform when using color samples of different media and colorants, has shown, that the reconstruction errors increase dramatically. This illustrates the strong media dependency of the empirical characterization approach, and the necessity to derive the functions once the conditions changes.

**Summary and Conclusions**

The focus of this study has been colorimetric and multispectral image acquisition, comparing the performance of trichromatic and multi-channel imaging. To reconstruct colorimetric and spectral data from the recorded device response, two conceptually different approaches have been investigated: model-based and empirical characterization. In the model-based approach, the spectral model of the image acquisition system is inverted to reconstruct spectral reflectance data. A priori knowledge on the smooth nature of typical reflectance spectra was utilized by representing the reconstructed spectra as linear combinations of basis functions, using Fourier basis and a set of real reflectance spectra. In the empirical approach, the spectral characteristics of the system are ignored and the mappings are derived by relating the recorded device response to colorimetric and spectral data for a set of training colors, using least squares regression techniques.

The results have showed that when only trichromatic imaging is available, a good colorimetric accuracy can be obtained, using polynomial regression to CIEXYZ or CIELAB. The best results were obtained using regression to CIEXYZ, but minimizing the CIE1976 $\Delta E_{ab}$ color difference. Noticeable is that the performance for trichromatic imaging was equally good as for multi-channel imaging, when using the colorimetric regression. However, because of the media-dependency, this approach requires for the characterization functions to be derived for each combination of media and colorants. A satisfactory accuracy for spectral reflectance reconstruction could not be obtained for trichromatic imaging.

For multispectral imaging, i.e. reconstructing the spectral reflectance of objects, multi-channel images are required to obtain the highest accuracy. The best results were obtained with the model-based approach, using multi-channel images combined with spectral basis. The model-based approach provides the additional advantage of being general, since it is derived based on the spectral characteristics of the image acquisition system, rather than on the characteristics of a set of color samples. However, the model-based approach, i.e. inverting the spectral transfer function of the imaging system, requires for multi-channel imaging to obtain a satisfactory spectral or colorimetric accuracy.

**References**

3. J.Y. Hardeberg, Acquisition and Reproduction of Color Images: Colorimetric and


