Spectral Reproduction using LabPQR: Inverting the Fractional-Area-Coverage-to-Spectra Relationship

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Abstract

*The tuning of parameters used in inverting the spectral characterization of a six-color inkjet printer was performed. This approach was necessary for building lookup tables for use in spectral color management. First, spectra were converted to a low-dimensional analog known as an Interim Connection Space (ICS). LabPQR was used as the ICS. LabPQR was defined with three colorimetric dimensions (L*a*b*) plus three dimensions describing a metameric black (PQR). Once converted to ICS units, the spectral characterization related printed fractional area coverages to LabPQR. The inversion process minimized the weighted sum of CIEDE2000 and a Euclidian distance in PQR coordinates. A weight series was performed to find the optimal trade-off between the colorimetric and spectral error. A 1:50 weighting ratio, CIEDE2000 to PQR difference, was deemed best.*

Introduction

An important goal of spectral color management is to reproduce images that match originals under arbitrary illuminants. Spectral reproduction requires new approaches including spectral profiling of devices, Spectral Profile Connection Spaces, spectral image processing and new quality metrics. Spectral color management will take advantage of all these concepts and require transformation chains that deliver high-quality results quickly.

In previous work [1]-[3], a spectral reproduction workflow from scene to hardcopy was proposed. One of the difficulties associated with spectral reproduction is its high dimensionality since more information is necessary for reproducing samples with illuminant-independence than needed for more traditional colorimetric reproduction. The proposed workflow included a step where spectra of high dimensionality were converted to a lower-dimensional encoding known as an Interim Connection Space (ICS) [3][4].

Recently, Derhak and Rosen proposed an ICS called LabPQR [5]-[7]. LabPQR is a six-dimensional ICS that has three colorimetric axes (L*a*b*) plus three spectral reconstruction axes (PQR). PQR describes a stimulus' metameric black [8]. The spectral characterization of a printer [9] yields the forward relationship from fractional area coverage to spectra. Unfortunately, spectra are typically 31 or more dimensional values. For the purposes of spectral color management, the spectra are then converted to the lower-dimensional ICS, in this case LabPQR.

An inversion of the printer characterization is necessary so fractional area coverages can be chosen for a requested spectrum. Spectral gamut mapping [5]-[7] is necessary when considering the problem of spectral color management because an answer must be delivered for any arbitrary spectral request. How to choose appropriate printer values for an out-of-spectral-gamut request is considered in this paper.

Theory

LabPQR

LabPQR [5]-[7] is a six-dimensional ICS. The first three dimensions are CIELAB values under a particular viewing condition, and the last three are spectral reconstruction dimensions describing a metameric black (PQR).

The reconstituted spectra from LabPQR is expressed as:

$$\hat{\mathbf{R}} = \mathbf{T}\mathbf{N}_c + \mathbf{V}\mathbf{N}_p \,, \tag{1}$$

where $\mathbf{T}$ is a n by 3 transformation matrix, $\mathbf{V}$ is a n by 3 matrix describing PQR bases, $\mathbf{N}_c$ is a 3 by 1 tristimulus vector, and $\mathbf{N}_p$ is a 3 by 1 vector of PQR values (n counts wavelengths). Note that $\mathbf{T}$ is applied to tristimulus values converted from CIELAB values. Using a set of the tristimulus vectors, $\mathbf{T}$ is determined by a matrix calculation using least square analysis:

$$\mathbf{T} = \mathbf{R}\mathbf{N}_{c,m}^T\left(\mathbf{N}_{c,m}\mathbf{N}_{c,m}^T\right)^{-1} , \tag{2}$$

where $\mathbf{R}$ is a n by m matrix of spectra of m training samples.

The PQR bases $\mathbf{V}$ is derived from Principal Component Analysis (PCA) on a set of spectral differences between the original spectra and the reconstructed spectra through an inverse transformation with $\mathbf{T}$ from $\mathbf{N}_{c,m}$. This spectral difference is expressed as:

$$\mathbf{E} = \mathbf{R} - \mathbf{T}\mathbf{N}_{c,m} \,. \tag{3}$$

Only the first three eigenvectors are preserved as the PRQ bases:

$$\mathbf{V} = (\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3) \,, \tag{4}$$

where $\mathbf{v}_i$ are eigenvectors in a set of the spectral difference.

Spectral Gamut Mapping

Spectral gamut mapping has two aspects to it: colorimetric and spectral. In the current approach, the two are combined and considered simultaneously. Fractional area coverages of an inkjet printer for arbitrary requested spectra are computed by minimizing a single objective function: the weighted sum of CIEDE2000 color difference and normalized Euclidian distance in PQR, defined as:

$$\text{ObjFunc1} = \text{Minimize}(\text{CIEDE2000} + k\,\Delta PQR\big/\sqrt{n}) \,, \tag{5}$$

where n is the number of samples in a spectrum (i.e., wavelengths) and k is a weighting that may be empirically fitted.

Eq. (5) can be globally utilized regardless of whether the requested stimuli are within the colorimetric or spectral response gamuts. Tuning the magnitude of k allows one to choose an optimal trade-off between the colorimetric and spectral error. Choosing smaller values of k increases the relative importance of the colorimetric matching.

Eq. (5) is equivalent to minimizing spectral RMS error if the requested stimuli are within the colorimetric response gamut, because the Euclidian distance in PQR between a metameric pair is proportional to spectral RMS error.

To show this, let $\hat{\mathbf{R}}_1$ and $\hat{\mathbf{R}}_2$ denote reconstructed spectra of a metameric pair with the identical tristimulus values $\mathbf{N}_c^*$. From Eq. (1) we obtain

$$\hat{\mathbf{R}}_i = \mathbf{T}\mathbf{N}_c^* + \mathbf{V}\mathbf{N}_{p,i} , \tag{6}$$

where

$$\mathbf{N}_{p,i}^T = (p_i, q_i, r_i) . \tag{7}$$

The spectral RMS error between the metameric pair is defined as:

$$\begin{aligned} sRMS &= \sqrt{\left(\hat{\mathbf{R}}_1 - \hat{\mathbf{R}}_2\right)^2 / n} \\ &= \sqrt{\left(\mathbf{T}\mathbf{N}_c^* + \mathbf{V}\mathbf{N}_{p,1} - \mathbf{T}\mathbf{N}_c^* - \mathbf{V}\mathbf{N}_{p,2}\right)^2 / n} \\ &= \sqrt{\left\{(p_1 - p_2)\mathbf{v}_1 + (q_1 - q_2)\mathbf{v}_2 + (r_1 - r_2)\mathbf{v}_3\right\}^2 / n} \end{aligned} , \tag{8}$$

where n is the number of samples in a spectrum. Since $\|\mathbf{v}_i\| = 1$ and $\langle \mathbf{v}_i, \mathbf{v}_j \rangle = 0$ for i ≠ j, Eq. (8) may be rewritten as:

$$\begin{aligned} sRMS &= \sqrt{\left\{(p_1 - p_2)^2 + (q_1 - q_2)^2 + (r_1 - r_2)^2\right\} / n} \\ &= \Delta PQR / \sqrt{n} \end{aligned} . \tag{9}$$

Thus, Eq. (9) shows that the Euclidian distance of PQR is proportional to spectral RMS error for a metameric pair.

If the requested stimuli are within the colorimetric response gamut, the CIEDE2000 portion of Eq. (5) will vanish and the only active portion of the objective function will be the spectral RMS error. On the other hand, Eq. (5) will attend to both the colorimetric and spectral differences if the requested stimuli are outside the colorimetric gamut.

Fractional-Area-Coverage Difference

For a six-color printer, the difference in a set of fractional area coverages is calculated as:

$$\sqrt{\sum_{i=1}^{6} (a_i - \hat{a}_i)^2 / 6} , \tag{10}$$

where a_i and $\hat{a}_i$ represent the fractional area coverages of the input and estimated ones derived by Eq. (5), respectively. The fractional area coverage varies between 0 and 1.

Experimental

A Canon i9900 dye-based inkjet printer with customized control driver was employed in this research. This printer had capability of an eight-ink set, but only six were utilized: cyan, magenta, yellow, black, red, and green. Spatial addressability of the inkjet printer was 1200 by 2400 dpi. All samples were printed on Canon Photo Paper Pro (PR-101). A GretagMacbeth SpectroScan spectrophotometer was used for spectral measurements of the printed samples. Spectral reflectance factor in the range between 400 and 700 nm in 10 nm intervals were measured. CIELAB was calculated under illuminant D50 and for the CIE 1931 2-degree standard observer. From these data, the coefficients for the spectral printer model [9] were derived.

In Fig. 1, the steps for building the forward mapping from fractional area coverage to LabPQR are shown. This mapping associates an LabPQR value with every fractional area coverage value in a full factorial sampling of fractional area coverage space. For the use of spectral color management, this mapping must be inverted. Figure 2 shows that the inversion relies on the spectral printer model and the choice of the weighting factor *k*. Within the inversion process of Fig. 2, a fractional area coverage value is chosen for each LabPQR such that the objective function [Eq. (5)] is minimized for each.

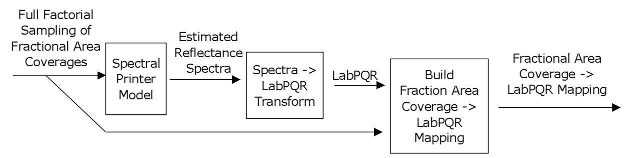

***Figure 1**.Schematic diagram for creating the forward mapping.*

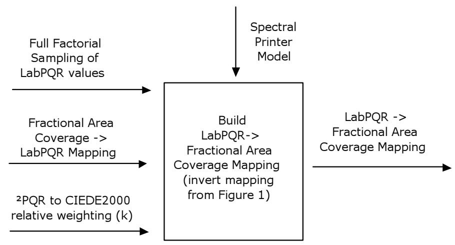

***Figure 2**.Inputs to the inversion process.*

Spectral Printer Model

The CMYKRG inkjet printer was spectrally characterized. In this study, the Cellular Yule-Nielsen Spectral Neugebauer model was adopted. This printer model was built in a similar fashion to Chen, Berns, and Taplin's approach [9]. For 729 print patches randomly distributed in the CIELAB space, this printer model achieved sufficient prediction accuracies: an average CIEDE2000 of 1.25 and average spectral RMS error of 0.63%.

Reconstruction Matrices

Using virtual spectral samples generated by the printer model, the reconstruction matrices **T** and **V** were derived with the so-called "XYZ+" method [5][7]. The training dataset consisted of 729 samples randomly distributed in the CIELAB plus 6,620 samples uniformly spaced in fractional area coverages from 0 to 1 in 5 steps. This dataset excluded samples of which fractional area coverages exceeded the maximum ink limitation for the selected substrate. For 805,355 verification samples uniformly spaced in fractional area coverages in 11 steps, reconstruction accuracies through the LabPQR transformation resulted in average CIEDE2000 and spectral RMS error of 0.00 and 0.43%, respectively.

Datasets

For finding optimal *k* for Eq. (5) two different types of datasets were prepared:

1. 729 patches printed by the CMYKRG inkjet printer.
2. 1,000 randomly selected feasible LabPQR values under a constraint that bounded reconstructed spectral reflectance factor between 0 and 1.

Results and discussion

Finding an Optimal Weighting

The spectral gamut mapping accuracies for Datasets 1 and 2 are shown in Fig. 3 and 4, respectively, indicating the trade-off between the colorimetric and spectral differences. It is clear from Fig. 3 that the spectral RMS errors decrease with increasing *k* while the CIEDE2000s increase. Dataset 2 included 240 samples outside the colorimetric gamut of the printer, so average colorimetric and spectral differences of Dataset 2 were larger than those of Dataset 1. Interestingly, for k over approximately 200, the spectral RMS errors increase. Perhaps this is due to the fact that reconstructed spectra have residual error and colorimetric matching must at some level correct for that. From these results, it is reasonable to choose k between 20 and 50. In this study, k was set to 50, thereby attaching importance to spectral differences.

As shown in Eq. (9) and Fig. 3, Eq. (5) is equivalent to Eq. (11) for a metameric pair, but not for sample pair with different colorimetric values. To explore efficiency of the six-dimension approach, an additional objective function was evaluated. This objective function minimized differences in the full 31-dimensional reflectance space:

$$\mathrm{ObjFunc2} = \mathrm{minimize}\,(\mathrm{CIEDE2000} + k\ \mathrm{sRMS})\,. \tag{11}$$

Shown in Fig. 5 are the spectral gamut mapping accuracies for Dataset 1. As expected the spectral RMS error is strictly monotonic with respect to k. At k of 50 the spectral RMS is in substantial agreement with the PQR difference. Statistics of the mapping performances are summarized in Table 1. There was no significant difference between the objective functions. The proposed approach achieved the equivalent level of spectral matching accuracies to the full 31-diminsional approach.

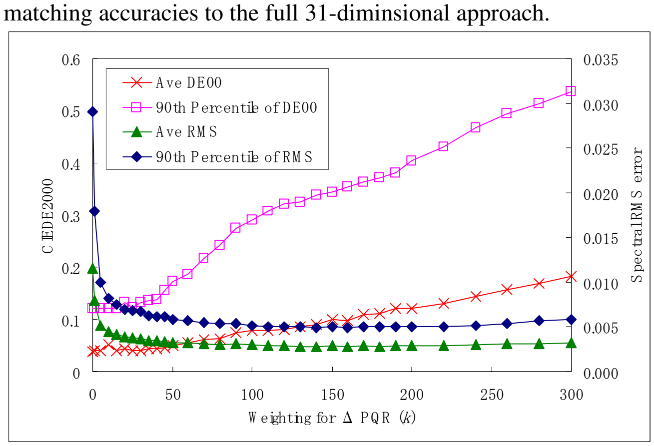

***Figure 3**. Spectral gamut mapping accuracies for 729 print patches (Dataset 1) in a series of weighting (k).*

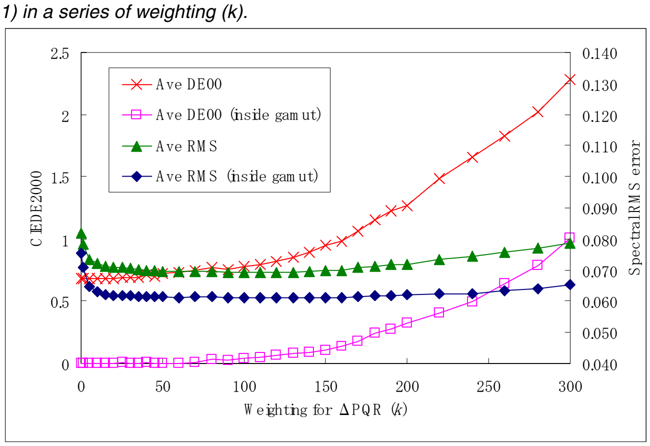

***Figure 4**. Spectral gamut mapping accuracies for 1,000 randomly selected LabPQR values (Dataset 2) in a series of weighting (k).*

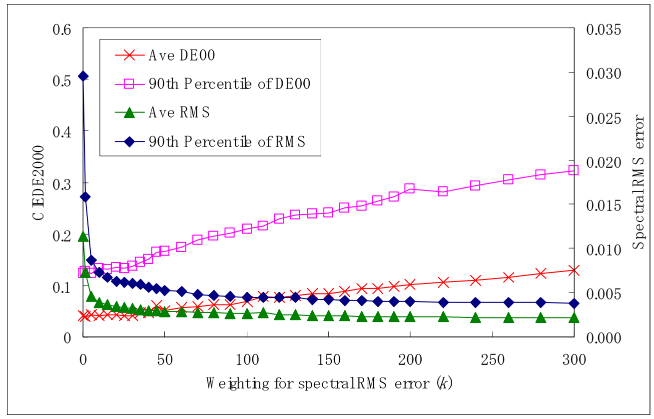

***Figure 5**. Spectral gamut mapping accuracies using Eq. (11) for 729 print patches (Dataset 1) in a series of weighting (k).*

Table 1: Spectral gamut mapping accuracies for Dataset 1 at the optimal weighting (k) of 50, for LabPQR and the full 31-dimensional reflectance space

Color Space	CIEDE2000 (D50, 2-degree)			
	Ave.	Max.	Std. Dev.	90%
LabPQR	0.05	1.13	0.13	0.17
Reflectance	0.05	1.11	0.14	0.17
Color Space	Spectral RMS error (%)			
	Ave.	Max.	Std. Dev.	90%
LabPQR	0.33	1.42	0.21	0.58
Reflectance	0.29	1.22	0.18	0.53

Fractional-Area-Coverage Difference

Using Dataset 1 the fractional-area-coverage differences were calculated and grayscale-coded into CIELAB plots, as shown in Fig. 6. Excellent estimation of the fractional area coverage was provided except lower lightness, red and green color regions. These areas of lower accuracy are not surprising. This is because our inkjet printer was equipped with green and red inks, making for likely spectral redundancy [10] in those color regions. One thing to note is that there is no large difference around boundaries of the gamut. This indicates that few metameric pairs exist in those regions. Average and maximum of the fractional-area-coverage differences for the overall samples were 7.43 and 43.97%, respectively.

Feasibility of the Spectral Gamut Mapping

Using the GretagMacbeth ColorChecker (CC) and ColorChecker DC (CCDC), the feasibility of the proposed objective function [Eq. (5)] was explored in comparison to a full 31-dimensional approach [Eq. (11)] and also a colorimetric-only mapping.

Tables 2 and 3 summarize the performances. Since the CCDC includes several samples exceeding the colorimetric gamut of the inkjet printer, its performance was worse. Colorimetrically, there was little difference between the approaches. Spectrally, the LabPQR and full spectral approaches were equivalent. In other words, the proposed approach with only six-dimensional LabPQR was able to yield an effective mapping for the CMYKRG inkjet printer in terms of both the colorimetric and spectral matching.

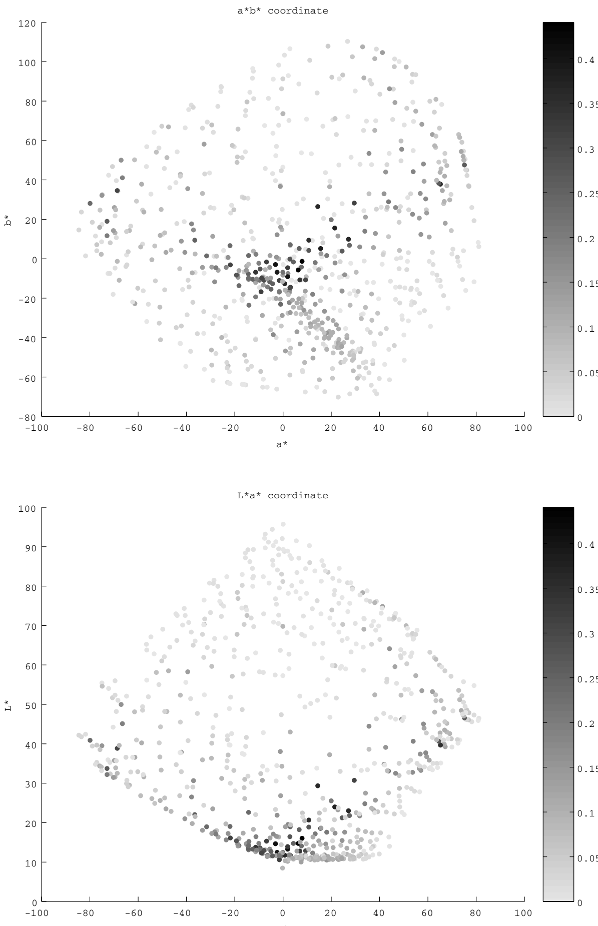

Figure 6*. Fractional-area-coverage difference projected onto a*b* coordinate (upper) and onto L*a* coordinate (lower).*

Table 2: Spectral gamut mapping accuracies for the CC, based on LabPQR, the full 31-dimensional reflectance space, and L*a*b*

Color Space	CIEDE2000 (D50, 2-degree)			
	Ave.	Max.	Std. Dev.	90%
LabPQR	0.03	0.73	0.15	0.00
Reflectance	0.03	0.65	0.13	0.00
L*a*b*	0.03	0.67	0.14	0.00
Color Space	Spectral RMS error (%)			
	Ave.	Max.	Std. Dev.	90%
LabPQR	4.18	7.02	1.35	6.07
Reflectance	4.12	7.08	1.34	6.03
L*a*b*	5.94	9.83	1.99	8.01

Conclusions

The spectral characterization of a six-color inkjet printer has been inverted using a spectral gamut mapping technique based on LabPQR. The spectral gamut mapping that minimizes the weighted sum of CIEDE2000 and Euclidian distance in PQR coordinate has been a successful approach for both colorimetric and spectral matching. Future work will evaluate capabilities of this spectral gamut mapping approach for a wide variety of out-of-gamut samples for building lookup tables.

Table 3: Spectral gamut mapping accuracies for the CCDC, based on LabPQR, the full 31-dimensional reflectance space, and L*a*b*

Color Space	CIEDE2000 (D50, 2-degree)			
	Ave.	Max.	Std. Dev.	90%
LabPQR	0.13	3.90	0.50	0.01
Reflectance	0.12	4.16	0.49	0.00
L*a*b*	0.11	3.37	0.43	0.00
Color Space	Spectral RMS error (%)			
	Ave.	Max.	Std. Dev.	90%
LabPQR	4.27	11.97	1.66	6.07
Reflectance	4.23	12.04	1.66	5.94
L*a*b*	6.46	15.15	2.55	10.05

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Author Biography

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