

How Many Photons are There?

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Abstract

For many digital imaging systems, it is important to know or model the absolute illumination levels for a scene, light source, etc. Often this information is used to select sensors, lenses, and light sources. Because the signal-to-noise ratio of an acquired image is ultimately controlled by the quantum nature of light, this exposure is best quantified in terms of number of available quanta. Even if all the electronic sources of noise have been reduced to undetectable levels, the noise caused by "photon shot noise" or quantum noise will remain.

This tutorial paper illustrates the calculation of the number of photons per unit area falling on a photographic sensor based on the ISO speed of the sensor. Camera design parameters are chosen based on the above analysis. In addition, it will be shown how the calculated exposure can be compared with the number of actual detected photons, thus indicating the limits of potential improvements.

Introduction

If the number of photons per second falling on unit area is known then the photons falling on a pixel is given by the equation below.

$$NP = kpps * A * T \quad (1)$$

<p>T = exposure time, seconds NP = number of photons kpps = photons per second per unit area A = pixel area</p>
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For photographic systems using the ISO standard for photographic speed the exposure for the average scene luminance is (10/ISO) lux seconds. The average scene luminance is assumed equal to the luminance of an 18% gray reflector.

$$H = \frac{10}{ISO} \quad (2)$$

<p>H = exposure in lux-seconds for an 18% gray = (lumen/meter²) second ISO = photographic speed based on the ISO standard</p>
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If the ISO speed for an image is specified then the number of photons per unit area due to an 18% gray (average scene luminance) can be determined. To determine the signal-to-noise due to "photon shot noise" the number of photons/pixel has to be determined. This can be calculated from the number of photons per meter², which leads to the final question: how many photons are in a lux-second? A lux second is a (lumen second)/meter² a photometric unit which somewhat complicates the calculation. A lumen is the photometric counterpart of the radiometric watt. The number of photons in a lumen-second will vary depending on the spectral distribution of the light source. For sources with a spectral distribution that can be described as a blackbody radiator the number of photons per lumen-second tends to vary a small amount with color temperature but the distribution of the photons changes with a higher proportion of red photons at lower color temperatures. For monochrome, panchromatic materials 1.1 x 10¹⁶ photons/(lumen-sec) is a reasonable value from 2400 K to 6500 K color temperature and a 400 to 700 nm bandwidth. For color sensors, where each color channel has to be considered separately the change in the proportion of red, green, and blue photons is important to consider.

The number of photons in a lumen-second can be calculated in a spreadsheet. Before dropping into the abyss of photometric units, the calculation will be done in radiometric units in Table 1. The calculation in the first row is repeated in every row, so the first row calculation will be used to illustrate the calculation in the remaining rows. Column B is a measured spectral irradiance. At 340 nm (column A) there are 8.09 x 10⁻⁶ watts per meter² falling on the surface per nanometer of bandwidth.

The energy in a photon at 340 nm is calculated² in column C using E = hv. The equation has to be modified because wavelength (λ) is known rather than frequency (ν). The frequency in Hertz is the number of waves that occur in the distance light travels in a second, just divide the speed of light in a distance metric per second by the length of one wave in the same units as shown in equation 4 to get the frequency. To get the number of photons/(second meter² nm) divide column B by column C. Watts/(meter² nm) are divided by watt-sec/photon to get photons/(second meter² nm) in column D. These calculations are repeated in each row.

$$E = h * \nu \quad (3)$$

$$v = \frac{c}{\lambda} \tag{4}$$

$$E = \frac{h * c}{\lambda} \tag{5}$$

E = energy in 1 photon in watt seconds (Joules)
 h = Plank's constant (6.62517x10⁻³⁴ Joule sec)
 v = frequency in Hertz (seconds⁻¹)
 c = speed of light (3x10⁸ meters/second)
 λ = wavelength in meters

To calculate the total number of photons/ (sec meter²) falling on the surface from 340 to 800 nm the number of photons at the missing wavelengths (341 to 349 nm, 351 to 359 nm etc.) has to be calculated. Remember there are 10 nm from 340 to 350 nm but each row in column D only indicates the number of photons in a 1 nm bandwidth. The simplest thing to do is multiply the number of photons in first row by 10, this will be close to the number of photons in the "bin" from 340 to 350 nm. To some extent this will cancel the unit "per nm" so the result will be photons/(sec meter²) from 340 to 350 nm or more accurately photons/(sec meter²10 nm). To be more precise a trapezoidal rule or Simpson's rule can be used to "integrate" between values but if the increments are small, the difference between the simple multiplication and the more sophisticated methods will be small. To simplify things a little further the distributive rule of multiplication can be used. Just add the values in column D and multiply the result by 10. This result, the number of photons/(sec meter²), due to the source spectral distribution in column B is shown at the bottom of column D. The resulting units are photons/(sec meter²) from 340 to 800 nm.

To calculate the number of photons/second in a lumen the power per cm² of source per nanometer emitted by a blackbody radiator can be modeled³ with Eq. 6.

$$P_{\lambda} = \frac{c1}{\left(\frac{c2}{e^{\lambda * T}} - 1 \right) \cdot \lambda^5} \tag{6}$$

P_λ = power in watts/(cm² of source nm)
 λ = wavelength in nanometers
 T = temperature in degrees Kelvin
 c1 = 3.7405*10¹⁶
 c2 = 1.43879*10⁷

This equation is evaluated for a 5500 K source in column B, Table 2. The purpose of this calculation is to get data with the proper spectral distribution that can be scaled to correspond to 1 lumen. Equation 7 is used to convert this

spectral distribution to lumens.⁴ This equation is numerically integrated at the bottom of column D. Column C is the eye visibility curve V[λ]. Column D is the product of V[λ] and P[λ]. The increment dλ becomes a Δλ of 10 nm because the power levels were calculated per nanometer but the steps are 10 nm.

$$F = 683 * \int_{\lambda 1}^{\lambda 2} V[\lambda] * P[\lambda] * d\lambda \tag{7}$$

λ = wavelength in nanometers
 V[λ] = eye visibility curve
 P[λ] = spectral power distribution
 dλ = wavelength bandwidth

The result of the numerical integration of this data is at the bottom of column D. At 5500 K, a 1 cm² source emits 462220 lumens. Column E is Column B multiplied by an area of 1/462220 cm². A source with this size and spectral power will emit 1 lumen as confirmed by the numerical integration at the bottom of column F. Column F repeats the calculation in column D, (V[λ] * P[λ]) except column E, the scaled data, is used for P[λ].

At this point column E is a source spectral power distribution with a color temperature of 5500 K that results in 1 lumen. Column H is the calculation of the energy in a photon at a given wavelength, column I is the power at each wavelength (column E) divided by the number of watts-sec in at photon at that wavelength. The result is the number of photons in a 1nm bandwidth at each wavelength. At the bottom of the page the integral of P[λ]dλ in photons/second is calculated over the interval from 400 to 700 nm. There are 1.12 x 10¹⁶ photons per second produced by a 1-lumen source over the interval from 400 to 700 nm or the units can be taken to be 1.12 x 10¹⁶ photons per (lumen-second). The number of photons per lumen-second over the intervals associated with arbitrary red, green, and blue bandwidths are shown at the bottom of column I.

Once the number of photons/second are calculated as a function of wavelength the result can be cascaded with the filtration in the optical system including the color filter array (CFA) on an electronic sensor or the spectral sensitivity of each layer in a silver halide photographic system. The quantum yield of the electronic system can also be included in this calculation. This is actually a good place to include the effect of quantum yield because the units are already in photons. Applying the quantum yield to a spectral distribution in watts is a common error. The units on quantum yield are actually electrons/photon. The system will have a quantum yield of 1 if the sensor produces 1 electron for every photon; real sensors have quantum yields less than 1. If the quantum yield is applied to a spectral distribution in watts it has to be converted to electrons/watt using E = hv.

The sensor infra-red cut filter and the quantum yield are in columns J and K. Columns M, N, and O are the red,

green, and blue spectral transmission of the CFAs that are placed over each pixel depending on the pixel color channel. The IR cut and CFA filters are not typical of real products, they are used to illustrate the calculation. The numbers of photons/(sec nm) are cascade with the IR cut filter, the quantum yield for the sensor, and one of the CFA colors. The results are in columns P, Q, and R. These are the number of photons/(sec nm) in each color channel for 1 lumen of illumination. Columns P, Q, and R are then numerically integrated over the wavelength band from 340 to 800 nm and the result placed at the bottom of each column. Notice there are substantially fewer photons/(lumen-second) in each channel when the transmission of the CFA, the IR cut, and the quantum yield are considered.

Once the number of photons/(lumen-second) (kpls) are known for a given quantum yield and filtration the number of photons/meter² (ppm2) can be determined as shown in Eq. 8. Multiply the number of photons per meter² by the area of a pixel in meters and the result is the number of photons a pixel collects in an 18 percent gray area.

$$ppm2 = H * kpls \quad (8)$$

ppm2 = photons/meter²
 H = exposure in lux-seconds or (lumens/meter²)-second
 kpls = photons/(lumen-second)

For an ISO photographic speed rated system, insert the equation for exposure due to an 18% gray (Eq. 2):

$$ppm2 = \frac{10}{ISO} \bullet kpls \quad (9)$$

kpls for the worked example is 1.13 x 10¹⁴ photons/lumen-second in the blue channel.

The number of photons per meter² as a function of ISO can be used to determine the best possible signal-to-noise for a system that is signal-to-noise limited only by "photon shot noise". The signal-to-noise is the square root of the average number of photons falling on a pixel⁶:

$$S / N = \sqrt{ppm2 * A} \quad (10)$$

Substituting for A and ppm2:

$$S / N = \sqrt{\frac{10 * kpls * pw^2}{ISO}} \quad (11)$$

solving for pw:

$$pw = \sqrt{\frac{ISO * (S / N)^2}{10 * kpls}} \quad (12)$$

For a design example, specify an ISO 100 system with a signal-to-noise of 40 in the blue channel. For this channel, kpls is 1.13 x 10¹⁴ photons/(lumen second). The calculated pixel width (pw) is 12 microns. To have the resolution of the eye each pixel should subtend 1 minute of arc. The normal lens has an field angle of about 45 degrees so the imager should be 2700 pixels wide and using a 35 mm still frame aspect ratio the height should be 1800 pixels, these values correspond to a 32 mm by 22 mm imager. The normal lens has a focal length equal to the diagonal of the format so the lens focal length should be about 39 mm.

Conclusion

The average number of photons collected by a pixel determines the best possible signal-to-noise ratio that an imager can produce. This data can be used to choose a pixel size based on the desired signal-to-noise and ISO imager speed. Once the pixel size is chosen, the remaining system specifications can be calculated in a straightforward way.

References

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3. W. Smith, Modern Optical Engineering, second edition, McGraw-Hill, New York, 1990, pg. 217.
4. R.W.G. Hunt, Measuring Colour, second edition, Ellis Horwood, New York, 1991, pg. 259.
5. R. Palum, The Advantages and Disadvantages of Small Pixels, *Proc. PICS*, pg. 31. (2000).
6. Ibid.
7. W. Smith, Modern Optical Engineering, pg. 122.

Biography

Russ Palum received a BS and MS in photographic science in 1979, and an MS in electrical engineering in 1988, both from the Rochester Institute of Technology. He joined Eastman Kodak Company in 1977 and has worked on process development for molded glass optics, asphere metrology, scanner light source design, lens design, antialiasing filter design and most recently image data path software for small CMOS image arrays.

Col A	Col B	Col C	Col D
wavelength in nm	watts/ (meter ² nm)	h-w watts-sec/ photon	photons/ (sec meter ² nm)
340	9.06390E-06	5.84674E-10	1.38161E+13
350	8.7254E-06	5.67872E-10	1.53644E+13
360	9.28974E-06	6.50006E-10	1.88087E+13
370	9.90335E-06	5.37175E-10	1.84036E+13
380	1.04424E-05	5.2504E-10	1.98048E+13
390	1.0944E-05	5.09629E-10	2.14749E+13
400	1.14074E-05	4.96985E-10	2.29571E+13
410	1.18295E-05	4.84769E-10	2.44531E+13
420	1.2212E-05	4.73225E-10	2.56658E+13
430	1.25554E-05	4.62221E-10	2.71693E+13
440	1.28551E-05	4.51715E-10	2.84584E+13
450	1.31178E-05	4.41679E-10	2.96622E+13
460	1.33418E-05	4.32075E-10	3.05784E+13
470	1.35295E-05	4.22963E-10	3.11626E+13
480	1.36833E-05	4.14079E-10	3.20435E+13
490	1.38019E-05	4.05623E-10	3.40264E+13
500	1.3881E-05	3.97611E-10	3.40426E+13
510	1.39457E-05	3.89716E-10	3.52919E+13
520	1.39979E-05	3.82221E-10	3.5575E+13
530	1.39654E-05	3.75114E-10	3.72920E+13
540	1.39658E-05	3.68255E-10	3.75466E+13
550	1.39264E-05	3.61739E-10	3.8538E+13
560	1.38663E-05	3.5432E-10	3.80687E+13
570	1.37876E-05	3.48039E-10	3.95407E+13
580	1.36922E-05	3.42811E-10	3.85661E+13
590	1.35818E-05	3.38573E-10	4.03171E+13
600	1.34577E-05	3.31229E-10	4.12626E+13
610	1.33214E-05	3.25629E-10	4.09851E+13
620	1.31745E-05	3.20573E-10	4.10967E+13
630	1.30179E-05	3.15444E-10	4.12627E+13
640	1.28529E-05	3.10555E-10	4.13871E+13
650	1.26707E-05	3.05777E-10	4.14700E+13
660	1.25212E-05	3.01144E-10	4.15154E+13
670	1.25102E-05	2.96549E-10	4.15245E+13
680	1.21299E-05	2.92287E-10	4.14995E+13
690	1.19378E-05	2.88351E-10	4.14435E+13
700	1.17428E-05	2.83936E-10	4.13574E+13
710	1.15456E-05	2.79379E-10	4.12436E+13
720	1.13467E-05	2.75048E-10	4.11015E+13
730	1.11467E-05	2.72287E-10	4.09404E+13
740	1.09462E-05	2.68588E-10	4.07545E+13
750	1.07454E-05	2.65077E-10	4.05479E+13
760	1.05454E-05	2.61525E-10	4.03222E+13
770	1.03463E-05	2.58134E-10	4.00779E+13
780	1.01480E-05	2.54814E-10	3.98197E+13
790	9.94921E-06	2.51589E-10	3.95456E+13
800	9.75396E-06	2.48444E-10	3.92579E+13
photons/(sec m ²) = 10nm ² sum:(photons/(sec m ² nm))			1.63206E+16

table 1

Col G	Col H	Col I	Col J	Col K
wavelength nm	energy/photon watt-seconds/photon	photons/(sec nm)	lit. cut	quantum yield in electrons/photon
340	5.84574E-10	1.38451E+13	0	0.08616438
350	5.67272E-10	1.53644E+13	0	0.078513335
360	5.82088E-10	1.88087E+13	0	0.066527554
370	5.37176E-10	1.84036E+13	0.000666	0.114109959
380	5.2304E-10	1.99648E+13	0.12 17	0.131763471
390	5.09428E-10	2.14749E+13	0.300255	0.148591569
400	4.96882E-10	2.29571E+13	0.536071	0.164594966
410	4.84769E-10	2.44531E+13	0.71905	0.179179652
420	4.73225E-10	2.56658E+13	0.83815	0.194146980
430	4.62221E-10	2.71693E+13	0.97108	0.207698854
440	4.51715E-10	2.84584E+13	0.97168	0.220441342
450	4.41679E-10	2.96622E+13	0.89904	0.232370382
460	4.32075E-10	3.05784E+13	0.71412	0.243501895
470	4.22833E-10	3.11626E+13	0.19936	0.25328802
480	4.14079E-10	3.20435E+13	0.72124	0.263520293
490	4.05623E-10	3.40264E+13	0.75204	0.27299940
500	3.97611E-10	3.40426E+13	0.92712	0.283015394
510	3.89716E-10	3.52919E+13	0.78103	0.281188785
520	3.82221E-10	3.5575E+13	0.79866	0.289361447
530	3.75114E-10	3.72920E+13	0.81364	0.29905828
540	3.68255E-10	3.75466E+13	0.83269	0.303873508
550	3.61739E-10	3.8538E+13	0.83268	0.307882735
560	3.5432E-10	3.90687E+13	0.83664	0.311115624
570	3.48039E-10	3.95407E+13	0.85964	0.31375115
580	3.42811E-10	3.95661E+13	0.85177	0.315284429
590	3.38573E-10	4.03171E+13	0.86339	0.315885887
600	3.31229E-10	4.12626E+13	0.88366	0.315043409
610	3.25629E-10	4.09851E+13	0.89469	0.315793516
620	3.20573E-10	4.10967E+13	0.8816	0.314328803
630	3.15444E-10	4.12627E+13	0.89587	0.312284271
640	3.10555E-10	4.13871E+13	0.9079	0.309581789
650	3.05777E-10	4.14700E+13	0.88525	0.307039217
660	3.01144E-10	4.15154E+13	0.73138	0.301389563
670	2.96549E-10	4.15245E+13	0.48908	0.292627272
680	2.92287E-10	4.14995E+13	0.50521	0.290429000
690	2.88351E-10	4.14435E+13	0.18101	0.288327140
700	2.83936E-10	4.13574E+13	0.16819	0.276488511
710	2.79379E-10	4.12436E+13	0.05309	0.269438888
720	2.75048E-10	4.11015E+13	0.033064	0.269458880
730	2.72287E-10	4.09404E+13	0.0274	0.255128345
740	2.68588E-10	4.07545E+13	0.02087	0.234868857
750	2.65077E-10	4.05479E+13	0.01641	0.223994240
760	2.61525E-10	4.03222E+13	0.010788	0.217544520
770	2.58134E-10	4.00779E+13	0.00983	0.204843437
780	2.54814E-10	3.98197E+13	0.00513	0.191744289
790	2.51589E-10	3.95456E+13	0.00432	0.177255086
800	2.48444E-10	3.92579E+13	0.00355	0.163403472
White		1.12448E+16	400-700nm	
Blue		2.88425E+15	400-480nm	
Green		3.79669E+15	500-590nm	
Red		4.54988E+15	600-700nm	

table 3

Col A	Col B	Col C	Col D	Col E	Col F
wavelength nm	watts/ m ² nm	photonic curve	h-w 683 ² sum V _A ² /(L _A) ² W _A	apply a constant with units cm ² to produce watts/m ² for 1 lumen	h-w 683 ² sum V _A ² /(L _A) ² W _A
340	3.74097970	0	0	0	0
350	4.01307366	0	0	0	0
360	4.52797271	0	0	0	0
370	4.57783068	0	0	0	0
380	4.82667933	0	0	0	0
390	5.05864348	0.0001	0.000000864	1.09442E-09	1.09442E-09
400	5.27292502	0.0004	0.002106034	1.14271E-09	4.56289E-09
410	5.42797602	0.0012	0.006306188	1.16929E-09	1.41529E-09
420	5.14401577	0.004	0.020710375	1.22121E-09	4.81048E-09
430	5.18250602	0.0116	0.067308308	1.25336E-09	1.45222E-07
440	5.14110461	0.033	0.13866286	1.28551E-09	2.06688E-07
450	6.05316818	0.038	0.230400391	1.31176E-09	4.98464E-07
460	6.19665205	0.26	0.37011724	1.33418E-09	8.90506E-07
470	6.25364364	0.051	0.56508177	1.35295E-09	1.25118E-06
480	6.32425678	0.139	0.875071689	1.36833E-09	1.90185E-06
490	6.57953862	0.208	1.320997792	1.38019E-09	2.87079E-06
500	6.42024975	0.329	2.02749067	1.3881E-09	4.48948E-06
510	6.44753989	0.573	3.24420215	1.38467E-09	7.07018E-06
520	5.46171184	0.71	4.582083065	1.38072E-09	0.92681E-05
530	5.464222	0.892	6.57215054	1.38052E-09	1.20522E-05
540	6.45574573	0.054	8.15878148	1.35066E-09	1.33243E-05
550	6.43718817	0.098	8.401962431	1.32666E-09	1.38368E-05
560	6.40953583	0.096	8.377217217	1.30963E-09	1.37966E-05
570	6.37338632	0.052	8.065992778	1.37191E-09	1.31348E-05
580	6.32881746	0.87	5.506071193	1.36322E-09	1.91222E-05
590	6.2778202	0.757	4.75228585	1.38818E-09	1.02814E-05
600	6.22042725	0.031	3.920066596	1.34577E-09	0.48188E-06
610	6.15747179	0.028	3.87920081	1.32915E-09	0.70372E-06
620	6.08814584	0.021	3.821001825	1.31791E-09	0.70348E-06
630	6.01718862	0.026	1.694441734	1.30179E-09	3.44937E-06
640	5.94088444	0.176	1.033664779	1.28282E-09	6.24262E-06
650	6.86128814	0.107	0.627184621	1.26307E-09	1.35893E-06
660	5.77828614	0.06	0.35402466	1.25212E-09	7.65229E-07
670	5.69373132	0.032	0.181198402	1.24218E-09	3.94143E-07
680	5.50606642	0.017	0.095913329	1.21296E-09	2.06206E-07
690	5.5170364	0.0029	0.0424081	1.18376E-09	3.76301E-08
700	5.42778979	0.004	0.0217111979	1.17491E-09	4.889714E-08
710	5.38931291	0.029	0.013672295	1.14848E-09	2.381126E-08
720	5.34438933	0.021	0.024824863	1.13492E-09	1.24847E-08
730	5.1824271	0.0005	0.000267121	1.11467E-09	5.67336E-09
740	5.056354	0.0003	0.0001517861	1.09462E-09	3.26385E-09
750	4.96677171	0	0	1.07456E-09	0
760	4.87457124	0	0	1.05451E-09	0
770	4.7818259	0	0	1.03456E-09	0
780	4.6893753	0	0	1.01461E-09	0
790	4.5982907	0	0	9.94321E-10	0
800	4.5081253	0	0	9.75339E-10	0
h-w 683 ² sum V _A ² /(L _A) ² W _A lumens				482220	h-w 683 ² sum V _A ² /(L _A) ² W _A lumens

table 2

Col L	Col M	Col N	Col O	Col P	Col Q	Col R
wavelength nm	CFA R	CFA G	CFA B	R	G	B
340	0.001	0	0	0.0000E+00	0.0000E-00	0.0000E+00
350	0.001	0	0	0.0000E+00	0.0000E-00	0.0000E+00
360	0.001	0	0.0005	0.0000E+00	0.0000E-00	0.0000E+00
370	0.001	0	0.0005	1.8218E+06	0.0000E-00	8.1091E+05
380	0.001	0	0.012	3.1875E+06	0.0000E-00	3.8250E+06
390	0.001	0	0.162	1.5972E+06	0.0000E-00	1.2657E+11
400	0	0	0.20255	0.0000E+00	0.0000E-00	4.16828E+11
410	0	0	0.32025	0.0000E+00	0.0000E-00	8.9477E+11
420	0	0	0.46	0.0000E+00	0.0000E-00	1.4707E+12
430	0	0	0.488	0.0000E+00	0.0000E-00	1.8611E+12
440	0	0	0.4988	0.0000E+00	0.0000E-00	1.9382E+12
450	0	0	0.5766	0.0000E+00	0.0000E-00	