Nonlinear Effect of Modulation on Image Quality

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Abstract

In most image quality metrics, a linear relation is assumed between modulation and perceived image quality. However, in practice it appears that this relation is nonlinear. It also appears that there is a linear relation between perceived image quality and the number of just-noticeable differences. We found that the number of just-noticeable differences is proportional to the square root of the modulation. The proportionality constant in this relation becomes independent of spatial frequency, if the modulation is divided by the threshold modulation. From measurements by Cannon (1985), we found that this square-root relation also holds for the perceived contrast of sinusoidal luminance patterns.

The above given principles are used in the SQRI, or square-root integral, which we proposed as a metric for the description of image quality. In the SQRI, furthermore, a logarithmic integration over spatial frequency is used, to account for the contribution of the different spatial frequency components of an image.

Various examples of measured image quality will be given to illustrate the practical use of the SQRI for the description of image quality.

Introduction

To obtain an objective measure for image quality, generally, a mathematical expression is used that contains a weighted combination of the physical parameters of the image and the psychophysical parameters of the human visual system. Such an expression is called a *metric*. In such a metric, the MTF of the imaging system is generally used as physical parameter, and the contrast sensitivity of the human eye as psychophysical parameter. Existing metrics differ from each other in the way these parameters are combined, and in the way the image quality contribution of the different spatial frequency components are taken into account.

Although the development of these metrics has contributed much to a better understanding of the effect of various parameters on image quality, they usually lack a good correlation with the subjectively perceived image quality. This is partly caused by the fact that in most metrics, it is assumed that the perceived image quality is linearly related with the MTF of the imaging system, which means that it is assumed that the perceived image quality is linearly related with the modulation of the spatial frequency components of the image. A linear relation may be valid for modulations at threshold level, but the largest part of an image consists of components with a modulation at suprathreshold level.

Granger & Cupery[1] (1972) noticed in an investigation of photographic pictures that there is a linear relation between the perceived image quality of these pictures and the number of just-noticeable differences or *jnds*. As the difference between the pictures mainly consisted of a difference of the modulations occurring in these pictures, it may be assumed that the image quality of these pictures is linearly related with the number of discriminable modulation levels.

Number of Discriminable Levels as a Function of Modulation

From contrast discrimination experiments by various authors, we found that the just-noticeable modulation difference Δm_t between two sinusoidal luminance patterns of equal spatial frequency can be described by the following equation[2]:

$$\Delta m_t = \sqrt{\frac{m_t^2 + 0.04k^2m^2}{1 + 0.004km/m_t} + m^2} - m \qquad (1)$$

where m is the modulation of one pattern, m_t is the threshold modulation, and k is a general constant for the signal to noise ratio of visual information. k is usually about 3. The results of this equation are shown in Figure 1 for k = 3, together with measurement data from Legge & Foley[3] (1980). The agreement between measurements and calculations shows that the equation gives a good description of the discrimination process. By using this equation for every possible modulation, starting from zero, and by adding the so obtained modulation difference to the previous modulation, etc., one can calculate the total number of discriminable levels that can be observed in a given modulation. The results of this calculation are shown in Figure 2 for k = 3. In this figure, the number of discriminable levels is plotted as function of the normalized modulation, defined by m/m_t. By plotting the calculated curve in this way, the curve is independent of spatial frequency and has a more general validity. Figure 3 shows the same curve, but now plotted as a function of the square root of the normalized modulation. From this figure, it appears that the number of discriminable levels increases about linearly with the square root of the normalized modulation. The dashed line through the origin represents the approximation by a square-root relation. Legge[4] proposed already in 1981 a power law for contrast discrimination, based on measurements at low and medium modulation levels. From Figure 3, where the modulation extends to much higher levels than he used, it appears that the power law is in fact approximately a square-root law.

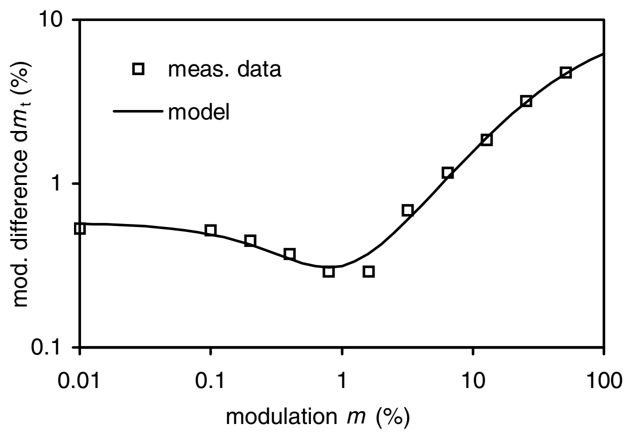

Figure 1. Contrast discrimination as a function of modulation calculated with Eq. 1 with measurement data from Legge & Foley[3] (1980) for a spatial frequency of 2 cycles/deg.

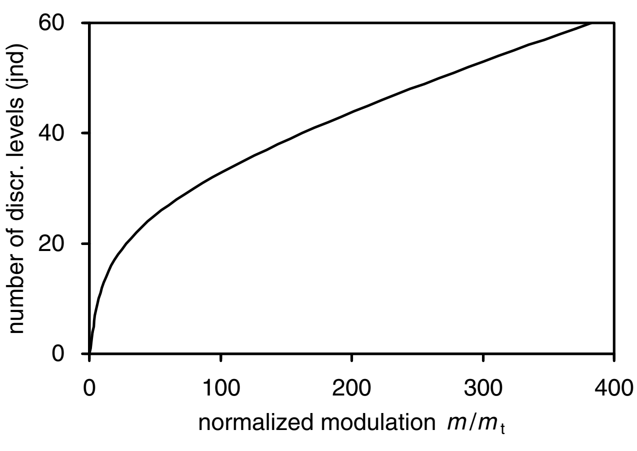

Figure 2. Number of discriminable levels as a function of normalized modulation, calculated by a step-by-step summation of Eq. 1.

The modulation of the spatial frequency components of an image generally extends over a large range of modulations. This means that the square-root relation forms a good basis for an image quality metric. A more precise description has no sense, as the modulation of the various spatial frequency components are arbitrarily distributed over the whole range, which cancels possible deviations. Therefore, the square-root relation will be used here as a practical method for the description of the nonlinear behavior of the visual system as the judgment of image quality.

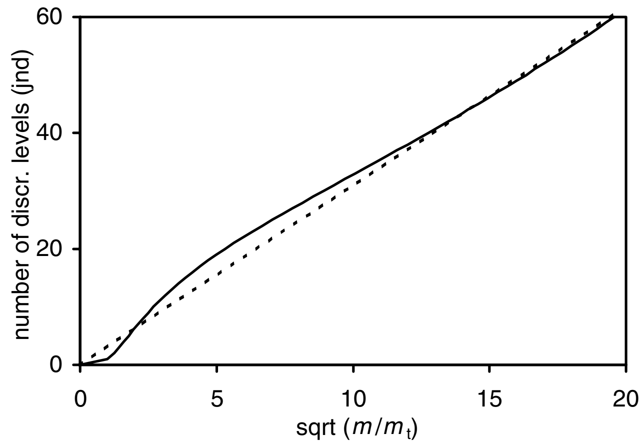

Figure 3. Solid curve: same as that of Figure 2, but plotted as a function of the square root of the normalized modulation. Dashed line: approximation with a linear relation.

Perceived Contrast as a Function of Modulation

The square-root dependence on modulation is also valid for the visually perceived contrast. This appears from measurements by Cannon[5] (1985) with sinusoidal luminance patterns of various spatial frequencies. The results are shown in Figure 4, where the perceived contrast is expressed in arbitrary units and is plotted as a function of the square root of the modulation. Although the modulation threshold is different for the different spatial frequencies, the measurement data nearly coincide with a straight line through the origin, which was obtained by a linear regression. The correlation is 98.3%. The perceived contrast appears to be about equal at the maximum modulation and about equal at the minimum modulation, where it approaches zero. This is in good agreement with measurements by Watanabe et al.[6] for equally perceived contrast at different spatial frequencies.

If the perceived contrast is plotted as a function of normalized modulation, the data for the different spatial frequencies would no longer coincide on a common curve, because of the difference in modulation threshold for these frequencies. However, they can be brought to a common curve again by also dividing the perceived contrast by the square root of the modulation threshold. This is shown in Figure 5, where the straight line is the regression line for this situation. The correlation is 98.5%. By plotting the curve in this way, the vertical scale unit is proportional to the number of dicriminable levels. This number is different for the different spatial frequencies at the maximum modulation of 100%. However, it is now proportional to the perceived image quality.

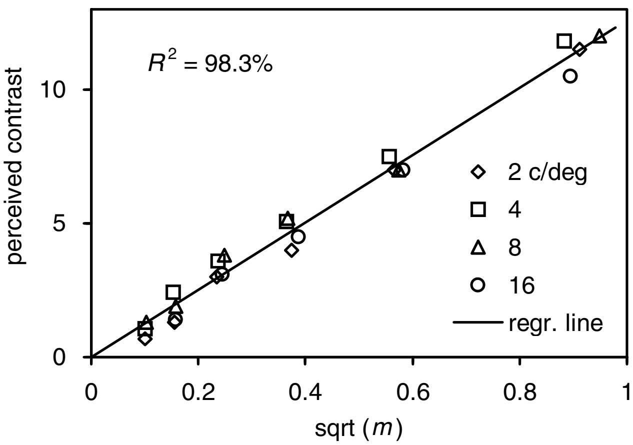

Figure 4. Measurements by Cannon[5] (1985) of perceived contrast measured in arbitrary units as a function of the square root of the modulation. The straight line through the origin represents a linear regression between both quantities. Correlation 98.3%.

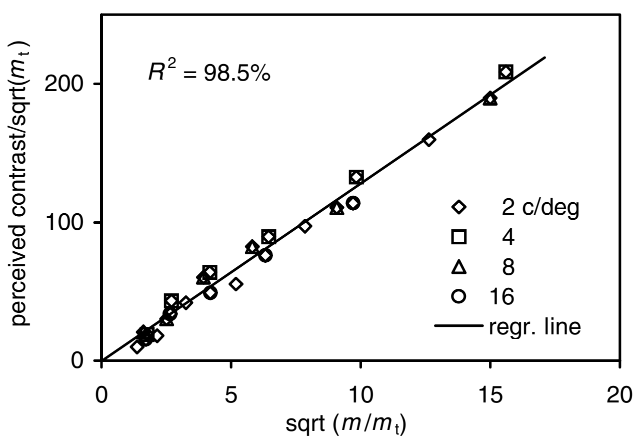

Figure 5. Same as Figure 4, but plotted with both axes divided by the square root of the modulation threshold. The straight line through the origin represents a linear regression between the quantities on both axes. Correlation 98.5%.

Image Quality Metric

Contrary to measurements where the object consists of a single sinusoidal luminance pattern, normal images consist of a combination of sinusoidal luminance patterns with different modulations and spatial frequencies. From the calculated curve in Figure 3 follows that the perceived contrast increases approximately linearly with the square root of the normalized modulation. As Granger and Cupery found a linear relation between perceived image quality and just-noticeable differences, it may be concluded from the foregoing that the perceived quality of an image is linearly related with the square root of the normalized modulation. The data given in Figure 5 show that the perceived contrast also increases linearly with the square root of the normalized modulation. They form an extra support for the use of the square root of the normalized modulation as functional parameter for the description of image quality. As the normalized modulation is the modulation divided by the threshold modulation, the threshold modulation and also its inverse, the contrast sensitivity, still plays an important role at suprathreshold level.

The modulations of the various spatial frequency components of an image are generally multiplied with the modulation transfer function or MTF of the imaging system. At low spatial frequencies, the MTF is usually about 1 and at high spatial frequencies, it decreases with spatial frequency. The contributions of the various spatial frequency components have to be integrated over the spatial frequency spectrum to obtain the total quality of an image. From practical experience we found, that a logarithmic integration over spatial frequency gives the best results. This work lead to the following equation for the description of perceived image quality[2]:

$$J = \frac{1}{\ln(2)} \int_{u_{min}}^{u_{max}} \sqrt{\frac{M(u)}{m_t(u)}}\, d(\ln u) \qquad (2)$$

where J is the perceived image quality, expressed in just-noticeable differences, u is the spatial frequency, expressed in angular units for the eye, $M(u)$ is the MTF of the display system, $m_t(u)$ is modulation threshold function of the eye, and u_{min} and u_{max} are the minimum and maximum spatial frequencies displayed. The constant 1/ln(2) in front of the integral was determined from a comparison with measurement data. in order to obtain that one unit of J corresponds with one just-noticeable difference. This metric is called *square-root integral* or SQRI. It is based on the square root of the normalized modulation.

Contribution of Different Spatial Frequency Components to Image Quality

For an analysis of the contribution of the different spatial frequency components of an image, Eq. (2) can be written in the following general form:

$$J = \int j(u)\, d(\ln u) \qquad (3)$$

where $j(u)$ is a distribution function that gives the image quality contribution per logarithmic spatial frequency interval d(ln$u$). This equation can also be applied to other image quality metrics. For the SQRI:

$$j(u) = \frac{1}{\ln(2)} \sqrt{\frac{M(u)}{m_t(u)}} \qquad (4)$$

If the focus condition of an image is varied in small steps of one jnd, the function $j(u)$ can be determined from two spatial frequencies u_1 and u_2 around u where the MTFs are 50% before and after the defocusing. If the MTFs are approximated by rectangular functions with a width given by u_1 and u_2:

$$j(u) = \frac{dJ}{d(\ln u)} \approx \frac{\Delta J}{\Delta(\ln u)} = \frac{1}{\ln u_1 - \ln u_2} \qquad (5)$$

For the spatial frequency u in $j(u)$, the geometric mean of u_1 and u_2 can be used.

Figure 6 shows the so obtained value of $j(u)$ from experimental data measured by Carlson & Cohen[7] (1980). They varied the focus condition of projected slides of various scenes by a just-noticeable change in sharpness. Although the data points show a considerable scattering, their general behavior is consistent and shows no systematic difference between the different types of pictures which were used The solid curve through the data points gives the SQRI prediction calculated with Eq. (4). This curve agrees very well with the average value of the data points.

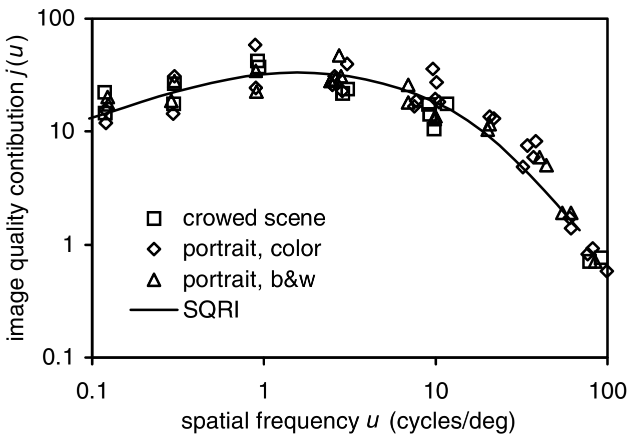

Figure 6. Image quality contribution of different spatial frequency areas calculated with Eq. (4) for experimental data measured by Carlson & Cohen[7] (1980). The solid line has been calculated with Eq. (4), valid for the SQRI.

For a better insight in the image quality distribution over the different spatial frequencies, the average values of the data points of Figure 6 are plotted in Figure 7 with a linear vertical scale The surface area under the curve now corresponds with the total image quality.

In the experiment by Carlson and Cohen, only the spatial frequency where the MTF drops to 50% was changed. The question rises, how the image quality varies, if the spatial frequency is constant and the modulation is only changed. In the SQRI, a square-root relation with the modulation is assumed. Watt and Morgan[8] (1983) made an experiment where they changed the contrast of a single step function and measured the just-noticeable change of contrast as a function as a function of contrast. The effective spatial frequency in this experiment was determined by the sigma of the blur and was constant. It appeared to be 4.5 cycles/deg. The values of $j(u)$ calculated from the data of this experiment are shown in Figure 8 as a function of contrast. They show a good agreement with the solid line that was calculated with the SQRI and indicates a square-root relation with contrast.

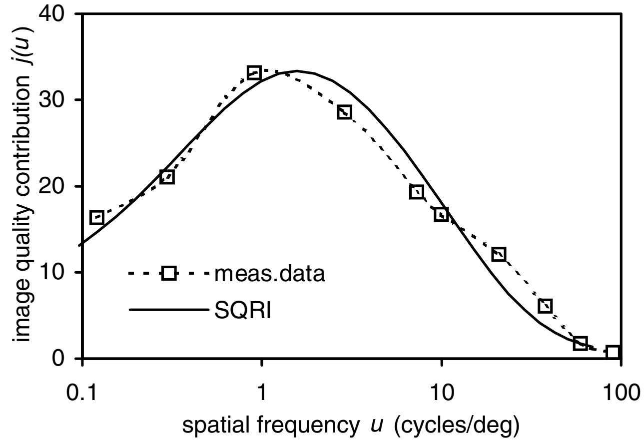

Figure 7. Average of the measurement data of Figure 6 plotted with a linear vertical scale The solid line is the SQRI prediction. The surface area under the curves now represents the total image quality.

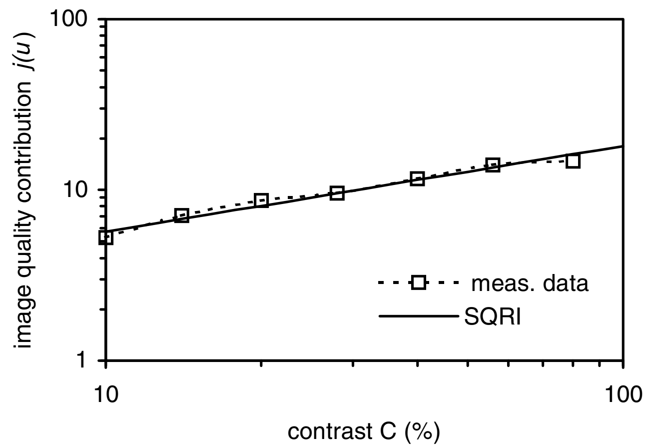

Figure 8. Image quality contribution as a function of contrast for measurements by Watt & Morgan[8] (1983). The solid line has been calculated with the SQRI and indicates a square-root relation with contrast.

Effect of MTF on Image Quality

The effectiveness of the SQRI for the description of image quality may be shown from a test with images produced with different MTFs. Higgins[9] (1977) made an investigation of the perceived quality of photographic images reproduced with 22 different MTFs. Some of these MTFs had a quite irregular shape, like the examples shown in Figure 9.

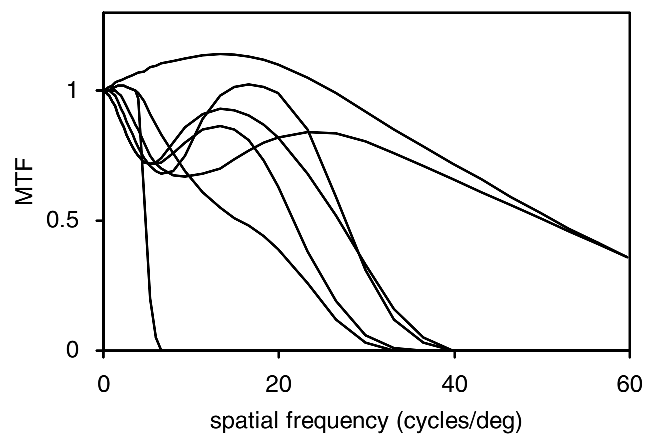

Figure 9. Seven of the 22 different MTFs used in the experiment by Higgins[9] (1977)

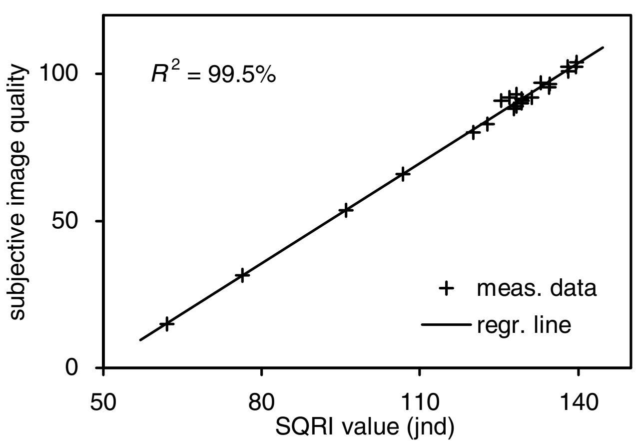

Figure 10. Linear regression between subjective image quality and SQRI value for measurements by Higgins[9] (1977) of photographic pictures reproduced with 22 different MTFs. The correlation between measurements and calculations is 99.5%.

The measurement results are shown in Figure 10 as a function of the calculated SQRI value. They are the average of the judgments by 20 observers and extend over a large range of image quality. The figure shows that there is a very good agreement between measurements and calculations. The straight line through the data has been calculated with a linear regression The correlation between measurements and calculations is 99.5%.

Effect of Simultaneously Varied Parameters on Image Quality

The subjectively perceived quality of an image is not only determined by the MTF of an imaging system, but also by other parameters that influence the perceived image quality, like luminance, contrast, image size, gamma, noise, etc. A good image quality metric should, therefore, also give a good correlation with actually perceived image quality, if these parameters are varied simultaneously. We will give here a few examples of the behavior of the SQRI for measurements of such simultaneous variations.

In the investigation by Higgins mentioned in the previous section, he also made an experiment with different MTFs in combination with different amounts of noise. Figure 11 shows the linear regression between measured data and calculated SQRI value for this experiment. For the calculation of the SQRI, it is assumed that the noise causes an increase of the modulation threshold.[2] The correlation between measurements and calculations is 99.7%.

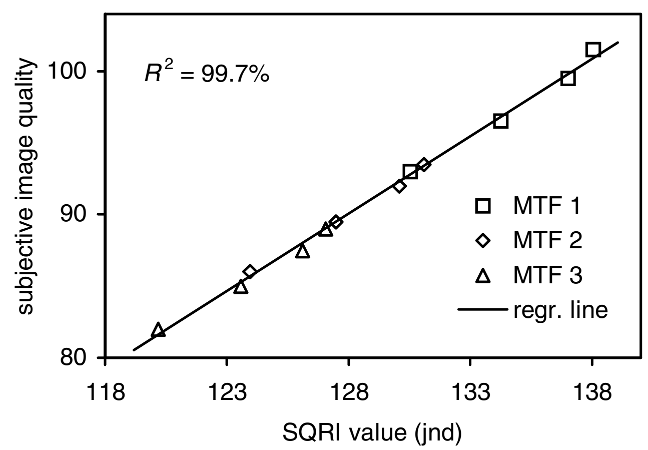

Figure 11. Linear regression between subjective image quality and SQRI value for measurements by Higgins[9] (1977) of photographic pictures with four different amounts of noise reproduced with three different MTFs. The correlation between measurements and calculations is 99.7%.

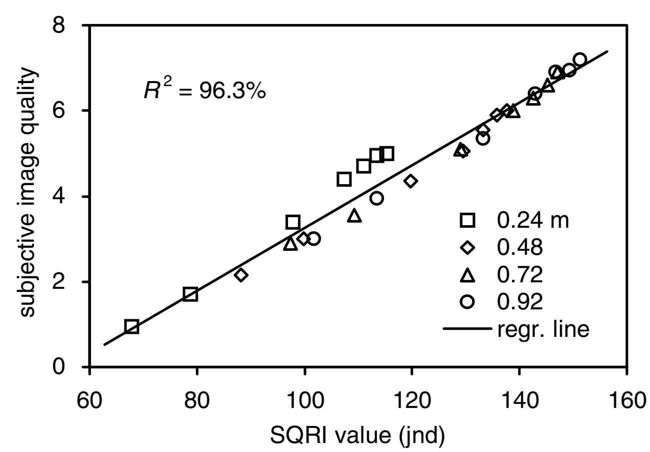

Figure 12. Linear regression between subjective image quality and SQRI value for measurements by Westerink & Roufs[10] (1989) with color slides projected with different resolutions and sizes. The correlation between measurements and calculations is 96.3%.

Figure 12 shows the linear regression between measured data and calculated SQRI value for an investigation by Westerink & Roufs[10] (1989), where color slides were projected with different resolutions and sizes at a constant viewing distance of 2.9 m. In the SQRI, the effect of image size is taken into account by its effect on the modulation threshold[2]. The correlation between measurements and calculations is 96.3%.

Figure 13 shows the linear regression between measured data and calculated SQRI value for an investigation by van der Zee & Boesten[11] (1980), where the same color slides were used, but projected with different luminance and sizes. The viewing distance was also 2.9 m. In the SQRI, the effects of luminance and mage size are both taken into account by their effect on the modulation threshold.[2] The correlation between measurements and calculations is 97.7%.

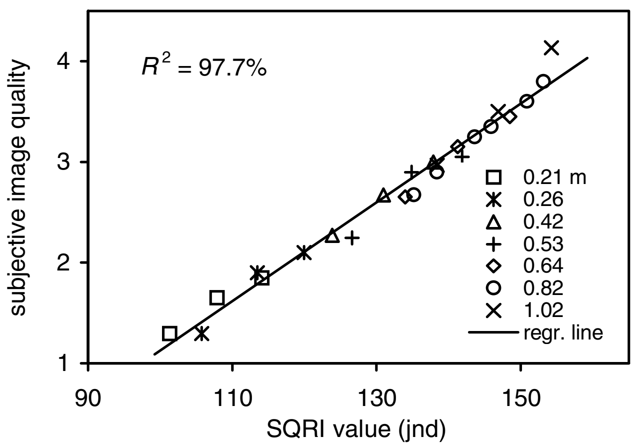

Figure 13. Linear regression between subjective image quality and SQRI value for measurements by van der Zee & Boesten[11] (1980) with color slides projected with different luminance and sizes. The correlation between measurements and calculations is 97.7%.

Conclusion

The perceived quality of an image increases approximately proportional with the square root of the modulation of the spatial frequency components. The coefficient in this relation is independent of spatial frequency, if the modulation is divided by the threshold modulation of the concerning spatial frequency. A good metric for image quality is the SQRI, that is based on this principle. This appears from tests of this metric with various image quality measurements.

References

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Biography

Peter Barten was born in Amsterdam, the Netherlands. He graduated in physics at the Technical University in Delft and received his Ph.D. from the Technical University in Eindhoven. He worked many years at Philips in Eindhoven, where he was in charge of the development of color CRTs. After his retirement, he specialized on human vision and image quality. He wrote many technical papers and recently published a book on this subject. He is a fellow of the SID and a member of SPIE.