The "Two Steps" Skeleton – A New Morphological Algorithm for Image Representation

N. D. Vizireanu, C. Pirnog, V. Lazarescu, and A. Vizirenu
University "Politehnica" of Bucharest, Romania

Abstract

This paper addresses the representation of binary images by means of mathematical morphology, a relatively new nonlinear theory for image processing, based on set theory. It considers images as sets (instead of vectors, as in the classical linear image processing), which permits geometry-oriented transformations of the images. The new image representation described in this work, called 'two steps" skeleton representation, is a natural extension of the morphological skeleton. This article will present the theoretical background of the morphological image representation, deduce the new representation and show some application examples. In the end, the possibility of generalizing the 'two steps" skeleton representation for multi-dimensional images will be analyzed, to extend the scope of its algebraic characteristics as much as possible. The applications of the 'two steps" skeleton representation that are presented will be illustrated by computer simulations.

Introduction – Image Representation via Mathematical Morphology

Image Representation is a key component in many tasks in computer vision and image processing. It consists generally of presenting an image in a form, different from the original one, in which desired characteristics of the image are emphasized and can be easily accessed. In the following pages we will consider some morphological methods for binary and grayscale image representation.

These methods are based on mathematical morphology, which is a relatively new, and rapidly growing, nonlinear theory for image processing.

Mathematical morphology is part of set theory, and it has a strong geometric orientation. For binary images, mathematical morphology provides a well-founded theory for analysis and processing. Therefore, binary morphological representations can be developed and analyzed. Mathematical Morphology involves the study of the different ways in which a structuring element interacts with a given set, modifies its shape, and extracts the resultant set.

Mathematical Morphology is a general theory that studies decompositions of operators between sets in terms of some families of simple operators: dilations, erosions, anti-dilations and anti-erosions. Nowadays, this theory is largely used in Computer Vision to extract information from images.

The basic operations are erosion and dilation. Based on these operations, opening and closing operations are defined. The morphological operations have been successfully used in many applications including object recognition, image enhancement, texture analysis, and industrial inspection.

Mathematical Morphology can be used in various areas of image processing, such as image compression, pattern recognition, object recognition, image enhancing, etc. For binary images, Mathematical Morphology provides a well-founded theory for analysis and processing. Therefore, Binary Morphological Representations can be developed and analyzed.

The new morphological image representation presented in this article is the 'two steps" skeleton. The 'two steps" skeleton (which will be denoted: '2SS'), is a natural extension of the morphological skeleton. It consists of first calculating the morphological skeleton using an Euclidean disk as structuring element, and then reiterating the above procedure on the residual image using lines for structuring elements. The original shape can be perfectly reconstructed.

For a better understanding of the way that 2SS works we must first present the theoretical background of the morphological skeleton.

Morphological skeleton – An overview

The skeleton is one of the main operators in mathematical morphology and it can be calculated entirely using the basic morphological operators.

The Fundamental Morphological Operators

Dilation and erosion are the fundamental operators of the Mathematical Morphology. The key process in the dilation and erosion operators is the local comparison of a shape, called structuring element, with the object to be transformed.

The structuring element is a predefined shape, which is used for morphological processing of the images. The most common shapes used as structuring elements are horizontal and vertical lines, squares, digital discs, crosses, etc.

The fundamental morphological operators are based on the operation of translation. Let *B* be a set contained in the Euclidean space *E*, and let *x* be a point in *E*. The translation of the set *B* by the point x, denoted B_x, is defined as follows:

$$B_x \triangleq \{b + x \mid b \in B\} \tag{1}$$

The dilation of the image *X* by the structuring element *B*, denoted $X \oplus B$, is defined by:

$$X \oplus B \triangleq \bigcup_{x \in X} B_x \tag{2}$$

For dilation: when the structuring element is positioned at a given point and it touches the object, then this point will appear in the result of the transformation, otherwise it will not.

The erosion of *X* by the structuring element *B*, denoted $X \ominus B$, is defined in the following way:

$$X \ominus B = \bigcap_{b \in B} X_{-b} \tag{3}$$

For erosion: when positioned at a given point, if the structuring element is included in the object, then this point will appear in the result of the transformation, otherwise not.

Advanced Morphological Operators

Based on the fundamental operators, two morphological operators are developed. These are the opening and closing operators. They are dual operators.

The opening operator, denoted "$\circ$", can be expressed as a composition of erosion followed by dilation, both by the same input structuring element:

$$X \circ B \triangleq (X \ominus B) \oplus B \tag{4}$$

The closing operator, denoted "$\bullet$", can be expressed as composition of dilation followed by erosion by the same input structuring element:

$$X \bullet B \triangleq (X \oplus B) \ominus B \tag{5}$$

The Morphological Skeleton

In 1978, Lantuejoul proved that the skeleton *S*(*X*) of a topologically open shape *X* in Z^2 can be calculated by means of binary morphological operations, in the following way:

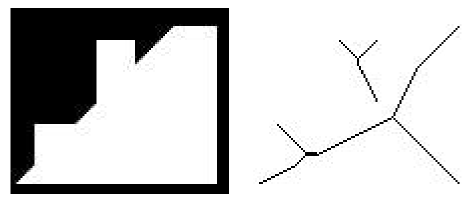
$$\begin{aligned} S(X) &= \bigcup_{n>0} S_n(X) \\ &= \bigcup_{n>0} \left\{ X \ominus nB - \bigcup_{\Delta n>0} \left[(X \ominus nB) \circ \Delta nB \right] \right\} \end{aligned} \tag{6}$$

where nB, ΔnB and B are respectively, the discrete topologically open discs with radii *n*, Δn and 1. $S_i(X)$ represents the reunion of the centers of the structuring elements with radius *i*.

The following step was a generalization of the skeleton, by defining structuring elements other than open disks. The generalized skeleton can be computed using as structuring element any kind of geometrical figure.

In Figure 1 is shown a binary image its morphological skeleton. The skeleton is obtained using a rhomb as structuring element.

The compression rate for this example is about 4%. This means that, for the skeleton, we need 25 times less information in order to reconstruct the original image.



a) *b)*

Figure 1. The original image (a) and its skeleton (b)

However, the reconstruction process needs additional information about the size of the structuring element for each point of the skeleton. This will reduce the compression rate to about 5.6%. By adding the information about the structuring element to the skeleton, the resulting image can be considered as a grayscale image. In this case, the resulting image is shown in Figure 2.

Figure 2. The skeleton completed with structuring element information

In this case, the value of each pixel represents the information about the radius of the corresponding structuring element. More to the point, the value of the pixel represents the value *n* of the radius.

The biggest problem with the skeleton representation is the fact that it contains many redundant points. These points are not needed for reconstruction, but appear in the skeleton. Several methods were proposed for reducing skeleton's redundancy, and the use of these methods reduces the number of redundant points in various degrees.

The image representations obtained from these methods are called reduced skeletons: *RS*.

Reconstruction of the Image from its Skeleton Representation

From the collection of subsets $\{S_n(X)\}_{n>0}$ and knowing the radius n for each pixel, the original shape X can be perfectly reconstructed in the following way:

$$X = \bigcup_{n\geq 0} S_n(X) \oplus nB \tag{7}$$

The Morphological Skeleton representation permits also partial reconstruction, yielding simplified versions of the original shape. This is obtained by eliminating from the skeleton the pixels with values smaller and equal to a given value k:

$$X \circ kB = \bigcup_{n\geq k} S_n(X) \oplus nB \tag{8}$$

The same results are obtained from the use of the reduced skeleton:

$$X = \bigcup_{n\geq 0} RS_n(X) \oplus nB \tag{9}$$

The "Two Steps" Skeleton

The method proposed in this paper is entirely based on the skeleton representation and it represents a generalisation of this method.

The main idea behind this method is to compute the skeleton twice. First, the skeleton of the binary image is computed using Lantuejoul's formula. The structural element used for this operation must be an open disk, a square or an elementary cross. The computation of the skeleton eliminates some of the redundant points from the original image.

Still, the skeleton has a large number of redundant points that reduce the compression rate. Because of these redundant points, the elements of the skeleton are highly connected. For example, the skeleton in Figure 1 (b) is made from ten straight lines. These lines are divided into two categories:

- The first category contains the lines that make with the horizontal angles that are multiple of 45 degrees;
- The second category contains all the other lines. If we take a closer look at these lines we notice that they are actually made of small horizontal and vertical lines (Figure 3).

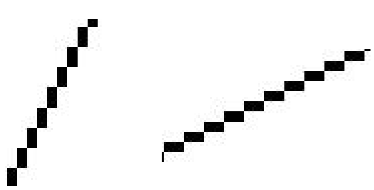


Figure 3. Details of the second category lines

We will eliminate the redundant points that exist in the skeleton in the same manner, by computing the skeleton of the skeleton.

$$2SS(X) = \bigcup_{n>0} SS_n(S_n(X))$$

$$= \bigcup_{n>0} \left\{ S(X) \,!\, nL - \bigcup_{\Delta n>0} \left[(S(X) \,!\, nL) \circ \Delta nL \right] \right\} \tag{10}$$

Since the skeleton is mainly composed of straight lines, the structuring elements used for the second skeleton computation will be:

- Horizontal line;
- Vertical line;
- 45 degree inclined line;
- 135 degree inclined line.

In equation (10), L represents these structural elements used for 2SS computation.

In Figure 4 is shown the 2SS of the image from Figure 1 (a). Is very easy to see that the number of points in 2SS is a lot smaller than the number of points in the skeleton.

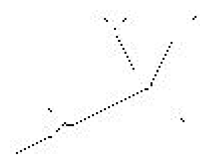


Figure 4. The "Two Steps" Skeleton

Figure 5 shows the redundant points that have been eliminated from the skeleton. For this example, these points represent about 2/3 of the number of points of the skeleton. This means that for the "two steps" skeleton we have increased the compression rate from 5.6% to 1.9%.

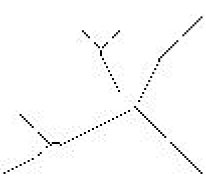
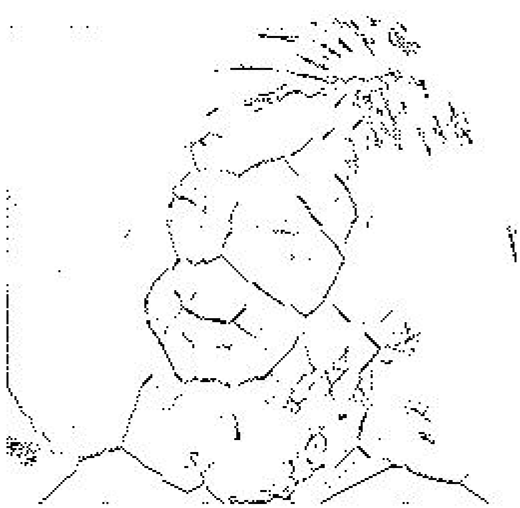


Figure 5. The difference between the skeleton and the 2SS

The last step of the 2SS method is to attach the information about the structuring elements to the points of the 2SS. As you have noticed, the "two steps" skeleton was computed from the skeleton without the structuring element information. However, this information must be supplied if we want to reconstruct the original image from its 2SS. This information can be added in a similar way to the morphological skeleton method, and the resulting image will be a grayscale image.

Conclusions

a)



b)

Figure 6. Original image (a) and its skeleton (b)

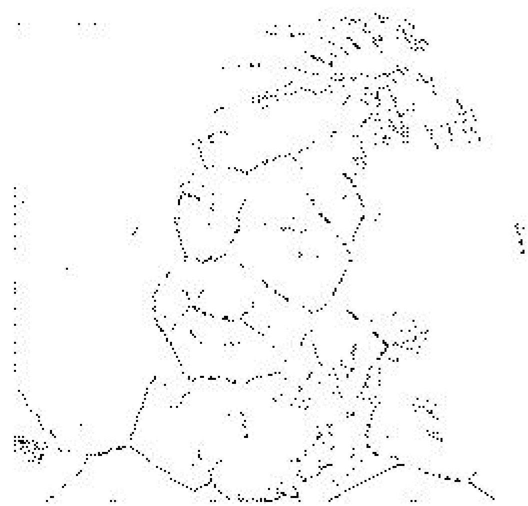
The "two steps" skeleton is a new and improved method for removing the redundant points from the morphological skeleton of a binary image.

For the binary images used in our experiments we have obtained compression rates up to 1.5% (the size of the compressed image is 1.5% from the original image) with perfect reconstruction.

Because of these properties, the "two steps" skeleton is recommended for image compression in areas where image quality is essential, where a degradation of the image due to the compression process is not accepted.

In Figure 6 (b) and Figure 7 (a) is shown the morphological skeleton and the 2SS of a binary image that is closer to the images currently used in image processing (Figure 6 (a)).

Figure 7 (b) shows the redundant points that have been eliminated from the skeleton. In this case, the compression rate is about 2.2%.



a)

b)

Figure 7. The 2SS (a) and the redundant points (b)

Given the properties of the 2SS, the next step would be the generalization of the 2SS for grayscale and color images. However, this is much more difficult, because in these cases we must work with spaces that have more than two dimensions.

References

1. C. Lantuejoul, “La Squelettisation et son application aux messures topologuiques de mosaiques polycristallines”, thesis de Docteur-Ingineur, School of Mines, Paris, France, 1978.
2 .J. Serra, “Image Analisis and Mathematical Morphology”, London Academic Press, 1982
3. J. Serra, “Skeleton Decompositions”, SPIE Vol. 1769, Image Algebra and Morphological Image Processing III, pp. 376-386, 1992.
4. Ph. Salembier, “Morphological Multiscale Segmentation for Image Coding”, Signal Processing, Vol. 38, pp.359-386, 1994.

Biography

Dragos Nicolae Vizireanu received his MS in Electrical Engineering from Georgia Institute of Technology in 1995, and his Ph.D in Electronics from the university “Politehnica” of Bucharest.. Since 2000, Associate Professor at the "Politehnica" University of Bucharest, Romania, Telecommunications Department. His work has primarily focused on: Real Time Digital Signal Processing, Digital Signal and Image Processing with focus on communications and multimedia systems, Hardware-Software DSP processor systems.