Fabrication Tolerance and Optimal Design of Spectral Sensitivities for Color Imaging Devices

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Abstract

A simplified hypothetical spectral sensitivity model was used to compare a variety of colorimetric quality factors proposed for the evaluation and design of spectral sensitivities for color imaging devices. Each spectral sensitivity function was parameterized with two or three variables. The range of parameters associated with best response from each quality factor was determined. Comparison showed that the ranges varied widely among the different quality factors. Selection criterion for appropriate quality factors for future practical application was discussed.

Introduction

The optimal design of color filters is essential to color imaging devices in the reproduction of object colors. Camera designers pay attention to colorimetric accuracy, noise sensitivity and illuminant invariance. Filters are often specified through minimizing color differences between the measurements and estimations for an ensemble of object colors. For instance, color differences might be derived through the calculation of mean squared color differences in uniform color spaces like CIE $L^*a^*b^*$ [4,8]. Sometimes color quality factors such as μ-factor proposed by Vora and Trussel [9,11] are used in color filter specification approaches.

However, the manufacturing process will finally determine whether the optimal filter is really feasible. If a filter design can be built but slight deviations cause its performance to deteriorate drastically, then such a filter could be a poor choice for putting it in a mass-produced product. Better smoothness of the ideal filtration function will increase the possibility of closeness between the ideally designed and practically fabricated filters. However small deviations from the desired curve may still cause loss of image quality. This paper proposes a new criterion for the optimal design of color filters based on a fabrication tolerance. A range of sensitivity function parameters which describe comparatively optimal filters are sought instead of a unique "absolutely" optimal filter. The larger the parameter range, the more tolerant the filter is to slight manufacturing errors.

A variety of color quality factors have been proposed and developed for various goals. The initial and perhaps the most widely known is Neugebauer's q-factor, [1] followed by Vora and Trussell's μ-factor, [2] Tajima's quality indices, [3] Shimano's Q_{st} and Q_{sf}, [12][13] Hung's camera rendering indices (CRI), [14] and Sharma and Trussell's Figure of Merit and its extension. [4] [11] These metrics are based on a number of different statistics. Q- and μ–factors calculate the geometrical difference between channel sensitivities and the color matching functions, Shimano's metrics measure the minimization of the mean square error of tristimulus, Sharma and Trussell's Figure of Merit calculates the minimization of the mean square $L^*a^*b^*$ color difference and Tajima looks at the characteristics of principal components of object reflectance spectra. No comparison has been done yet in the literature on the optimal ranges obtained from these metrics at the same level.

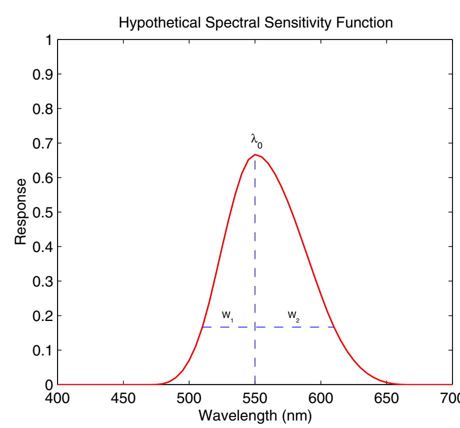

Figure 1. Hypothetical spectral sensitivity function model: Peak position locates at λ_0, left width is w_1 and right width is w_2.

The width and peak wavelength of sensitivity functions are important variables in color reproduction. General sensitivity function is modeled as asymmetric cubic spline function (Eq. (1)), as shown in Fg. 1. The geometric peak wavelength λ_0, left half-width w_1 and right half-width w_2 are the three primary parameters.

$$C(\lambda) = \begin{cases} \dfrac{w_2^3 + 3w_2^2(w_2 - |\lambda - \lambda_0|) + 3w_2(w_2 - |\lambda - \lambda_0|)^2 - 3(w_2 - |\lambda - \lambda_0|)^3}{6w_2^3} \\ \qquad 0 \le \lambda - \lambda_0 \le w_2 \\ \dfrac{w_1^3 + 3w_1^2(w_1 - |\lambda - \lambda_0|) + 3w_1(w_1 - |\lambda - \lambda_0|)^2 - 3(w_1 - |\lambda - \lambda_0|)^3}{6w^3} \\ \qquad -w_1 \le \lambda - \lambda_0 \le 0 \\ \dfrac{(2w_2 - |\lambda - \lambda_0|)^3}{6w_2^3} \qquad w_2 \le \lambda - \lambda_0 \le 2w_2 \\ \dfrac{(2w_1 - |\lambda - \lambda_0|)^3}{6w_1^3} \qquad -2w_1 \le \lambda - \lambda_0 \le -w_1 \\ 0 \qquad otherwise \end{cases} \tag{1}$$

Alternatively we can represent the function with these three parameters: peak wavelength λ_0 (as above), width parameter w and skewness parameter Δw, where

$$\begin{aligned} w_1 + w_2 &= 2w \\ w_2 - w_1 &= 2\Delta w \end{aligned} \tag{2}$$

It follows that

$$w = \frac{w_1 + w_2}{2} \tag{3}$$

$$\Delta w = \frac{w_2 - w_1}{2} \tag{4}$$

Δw describes the degree of skewness in the spectral sensitivity function. For symmetrical functions, $\Delta w = 0$. For more complicated spectral sensitivity shapes, lower and upper bound of spectral sensitivity functions may be introduced. Many real sensitivity functions have secondary peaks, so it may prove important to include those. Other real-world structure may be modeled in some future study as well. The simple curves have shown, so far, to be reasonable representations of filters believed to be easily fabricated.

By shifting peak wavelength of the cubic spline function from 400nm to 700nm, channel sensitivities from blue to red are simulated. By modifying the width parameters, the bandwidth of sensitivity functions is modified.

For the analysis, the possible geometrical peak wavelength for blue channel was varied from 400nm to 500nm, green channel from 500nm to 600nm and red channel from 550nm to 650nm, all in intervals of 10nm. The width parameter, w, was tested between 10nm and 110nm in increments of 10nm. The skewness parameter, Δw, was varied between –5nm and 5nm by increments of 5nm. By calculating all of these combinations, we can choose the combinations whose quality factors satisfy some pre-defined minimum conditions, such as $\mu \ge 0.98$. In order to compare the optimal ranges from different quality factors, these parameters were held constant:

$$\Delta w_R = \Delta w_G = \Delta w_B = 0nm$$

$$\begin{aligned} w_R &= 50nm \\ w_G &= 70nm \\ w_B &= 40nm \end{aligned}$$

For all of the evaluated quality factors, the perfect score is that of 1.

Optimal Range Obtained with *q*-Factor

Neugebauer's *q*-factor was proposed forty years ago and was the first the so-called quality factors. [1-4,9-14] Its physical meaning is to approximate a spectral sensitivity function of a color input device with the linear combination of one set of color matching functions. The set of color matching functions chosen does not affect the quality factor, since the orthornormal color matching functions are always equivalently used instead. *Q*-factor can be represented with matrix notation as:

$$\begin{aligned} q(m) &= \frac{\|P_V(m)\|^2}{\|m\|^2} = \frac{\|A \cdot (A^T A)^{-1} A^T \cdot m\|^2}{\|m\|^2} \\ &= \frac{m^T A (A^T A)^{-1} A^T m}{m^T m} \end{aligned} \tag{5}$$

where A is a color-matching function matrix (n×3), m is the spectral sensitivity function (n×3), and the superscript, ^{T}, is the matrix transpose operator, assuming n-wavelength sampling is used in the visible range.

Figure 2 shows a contour plot of Neugebauer's *q*-factor relative to varying peak wavelength of our hypothetical channel sensitivities. The optimal range of q>0.95 is found to be the contour as shown in Figure 2. For results where q is high, e.g. q>0.95, two separate width-peak continuous areas are found. These show blue region to be limited, while the red-green region to be quite large and connected. Neugebauer's *q*-factor does not treat the three filters as a system, so for this factor the simultaneous selection of optimal three sensitivity functions cannot be obtained.

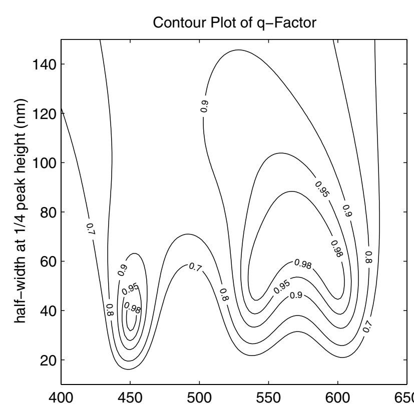

Figure 2. Optimal range defined by q-factor

Optimal Range Obtained with μ-Factor

Vora and Trussell's μ-factor has been discussed extensively.[2,3,4,9,10,11,12,13] In general, there always exists some difference between the color-matching functions and the camera spectral sensitivities by linear transformation. For example, Equation (6) describes this difference: [11]

$$\mu_A(M) = \frac{\mathrm{Trace}\{AA^T M (M^T M)^{-1} M^T\}}{\mathrm{Trace}\{AA^T\}} = \frac{\mathrm{Trace}\{AA^T OO^T\}}{\mathrm{Trace}\{AA^T\}} \tag{6}$$

where M is the camera spectral sensitivities matrix ($n\times 3$ if three channels are used), and A typically is the CIE 1931 2° $[\bar{x}, \bar{y}, \bar{z}]$ color matching functions, but it could be any set of color matching functions in general. Sometimes the viewing and taking illuminants are incorporated into the sensitivity functions and color-matching functions. This paper will assume such incorporation. If the equi-energy illuminant is used, A and M will return to its usual definition. The quality factor of a set of given spectral sensitivities will change if the reference color-matching function set is changed; so does the illuminants (We let the viewing illuminant and the taking illuminant be the same for consistent comparison). Vora and Trussell's μ-factor is defined when the orthonormal color matching functions U derived from A are used, which makes the value unique for a set of given spectral sensitivities:

$$\mu_U(M) = \frac{\mathrm{Trace}\{UU^T M (M^T M)^{-1} M^T\}}{\mathrm{Trace}\{UU^T\} \to 3} = \frac{\mathrm{Trace}\{UU^T OO^T\}}{3} \tag{7}$$

The optimal ranges defined by $\mu_A(M)$ and $\mu_U(M)$ are different, since different weights are used in the functions. Figure 3 demonstrates the use of $\mu_A(M)$. It shows the optimal range of peak positions when $\mu_A(M) \geq 0.95$, standard CIE 2° $[\bar{x}, \bar{y}, \bar{z}]$ color matching functions and illuminant D65 is used as in Eq. (6). $\mu_U(M)$ is used in making Figure 4-6. These figures show the optimal range of peak positions when $\mu_U(M) \geq 0.95$ and illuminants equi-energy, D65 and A are used. Within the $\mu_U(M)$ figures, it can be plainly seen that the regions have only minor difference between the illuminant changes. Much larger differences are noted between the use of $\mu_A(M)$ and $\mu_U(M)$. Fg. (3) shows the peak of green could be less than 500nm if the peak wavelengths of blue and red are appropriate, and there exist conditions where the peak wavelength of red could be higher than 650nm. Both metrics limit the peak position of blue sensitivity to some interval, about 435nm-450nm, and μ_U also limits the optimal peak wavelength of red to 570nm-600nm. Detailed analysis shows the boundary of the region changes when the illuminant changes.

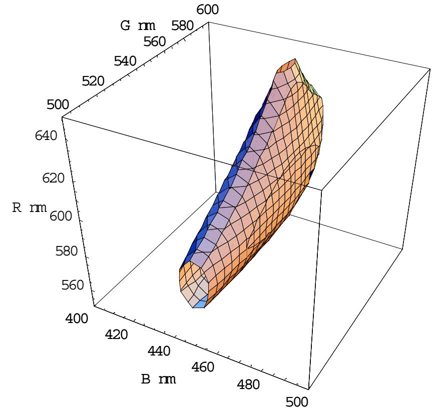

Figure 3. $\mu_A(M) \geq 0.95$ *when D65 is used*

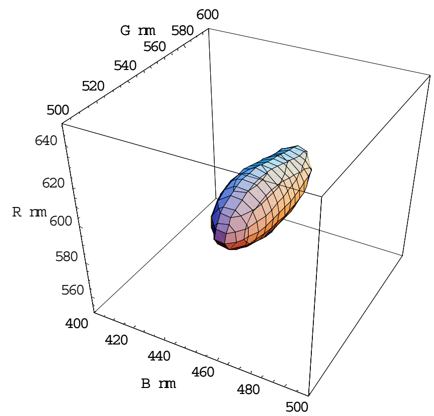

Figure 4. $\mu_U(M) \geq 0.95$ *when equi-energy illuminant is used*

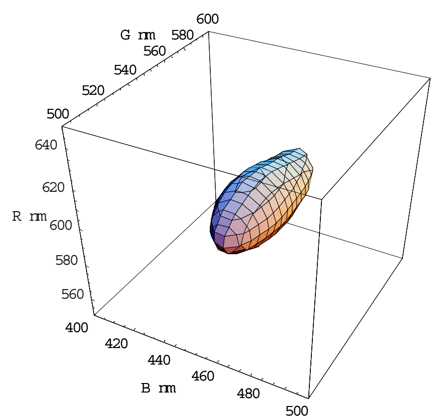

Figure 5. $\mu_U(M) \geq 0.95$ *when D65 is used*

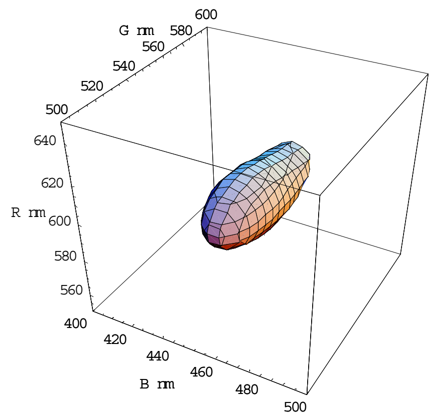

Figure 6. $\mu_U(M) \geq 0.95$ *when A is used*

Optimal Range obtained with Q_{st} and Q_{sf}

The real object reflectance would be very helpful in transforming camera signal into the colorimetric values. By minimizing the mean-squared error between the estimated and measured tristimulus values, Shimano's Q_{st} and Q_{sf} quality factors [12,13] can be equivalently defined as:

$$Q_{st} = \frac{\mathrm{trace}(A_L A_L^T K_r G(G^T K_r G)^{-1} G^T K_r)}{\mathrm{trace}(A_L A_L^T K_r)} \quad (8)$$

and

$$Q_{sf} = \frac{\mathrm{trace}(U_L U_L^T K_r G(G^T K_r G)^{-1} G^T K_r)}{\mathrm{trace}(U_L U_L^T K_r)} \quad (9)$$

where

$$K_r = E\{rr^T\} = \frac{1}{n_{samples}} \sum_{i=1}^{n_{samples}} r_i r_i^T \quad (10)$$

is the correlation matrix for the ensemble of object reflectance spectra. $A_L (= L_{view}\,\mathrm{A})$ and $G (= L_{take} M)$ already include the illuminant factor inside, [4] and U_L is the orthonormal fundamental vectors derived from A_L. A is the color-matching functions matrix, typically the CIE 1931 2° $[\bar{x}, \bar{y}, \bar{z}]$, and M is the spectral sensitivity functions matrix. Both Q_{st} and Q_{sf} are the data-dependent metrics for spectral sensitivity functions. In the experiment, Vrhel and Trussell's data set, which contains 354 object colors, is used when the statistics of reflectance spectra is necessary. [7]

The optimal range obtained with Shimano's Q_{st} and Q_{sf} can be shown in Fg. (7) and Fg. (8). While Vora and Trussell's μ-factor misses some good sensitivity functions, [3][13] both two measures here try to "beautify" all spectral sensitivity sets. If the threshold is chosen as 0.95 or 0.98, every combination in the peak position cube will qualify "optimal". Instead, here a threshold of 0.9995, which is very close to 1, is chosen to define the optimal space. Shimano's papers [12][13] also demonstrate that most quality factor values he calculated are very close to 1 even when different object reflectance set is used. Comparing Figure 7 and 8, they are obviously different, which is similar to the difference to that between Figures 3 and 5. Since a threshold of 0.9995 is used, it will be difficult to discriminate which sensitivity set is better if quality factors of both sets are larger than 0.9995. Compare Figures 5 and 8, the optimal range of blue is wider than that in Figure 5, while the optimal range of red is narrower. Similar feature can be found between Figures 3 and 7.

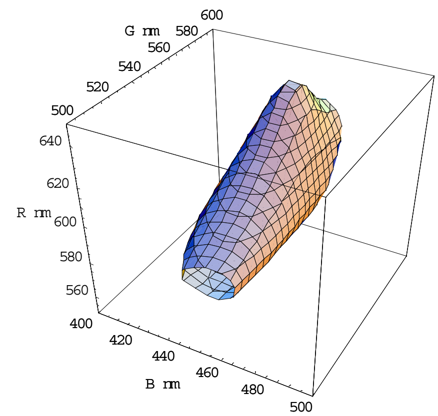

Figure 7. $Q_{st} \geq 0.9995$ *when D65 is used*

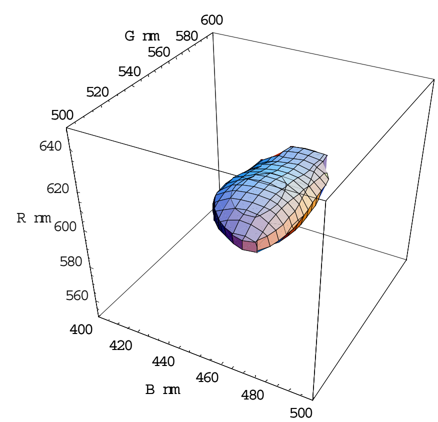

Figure 8. $Q_{sf} \geq 0.9995$ *when D65 is used*

Optimal Range obtained with *FOM*/*MG*

Sharma and Trussell's Figure of Merit (*FOM*) [4] is based on the optimization within CIE$L^*a^*b^*$ color space, which is considered as a more perceptually uniform color space than CIEXYZ. It also takes the signal-independent random noise into account. This quality factor should be more coincident with the real world. Like Shamano's metrics, it also depends on the selection of recording and targeting illuminants, as well as the statistical characteristics of the ensemble of object reflectance used. Figure of Merit can be defined as

$$q_{FOM}(A_v, G) = \frac{\tau(A_v, G, K_r, K_\eta)}{\alpha(A_v, K_r)} \tag{11}$$

where

$$0 \le \tau(A_v, G, K_r, K_\eta) \le \alpha(A_v, K_r) \tag{12}$$

$\alpha(\cdot)$ and $\tau(\cdot)$ are two functions, [4] K_r and K_η are the correlations of the reflectance spectra set and the random noise. We have defined our own simple modification to *FOM*. We name it Measure of Goodness (*MG*):

$$q_{MG} = 1 - \sqrt{1 - \frac{\tau(A_v, G, K_r, K_\eta)}{\alpha(A_v, K_r)}} = 1 - \sqrt{1 - q_{FOM}} \tag{13}$$

Here the average color difference of an ensemble of spectra varies linearly against quality factor. [11]

If signal-independent noise is ignored, which means the noise correlation matrix $K_\eta = 0$, we can obtain the optimal range. For comparison, illuminants D65 and A will be used in the calculation of optimal range. Figures 9 and 10 demonstrate the results when illuminant D65 and A are used with *FOM*, and Figures 11 and 12 demonstrate the results when illuminant D65 and A are used with *MG*. At a level of 0.98, the optimal ranges of peak wavelengths tend to be quite large for both D65 and A. The optimal peak wavelength of blue is extended from 420nm to 480nm, but that of green could be from less than 500nm to 600nm, and that of red could be from 560nm to more than 650nm. The difference between Figures 9 and 10 shows that the peak wavelength of red could be even higher for A, since A has higher power spectral distribution in red wavelength than D65. Figures 11 and 12 show that $MG \ge 0.95$ is a more strict condition than $FOM \ge 0.98$. Since *MG* has a linear relationship to $L^*a^*b^*$, it would be a more effective choice for selecting the peak wavelengths of sensitivity functions. Probably $MG \ge 0.90$ is a good condition to determine the region in practice.

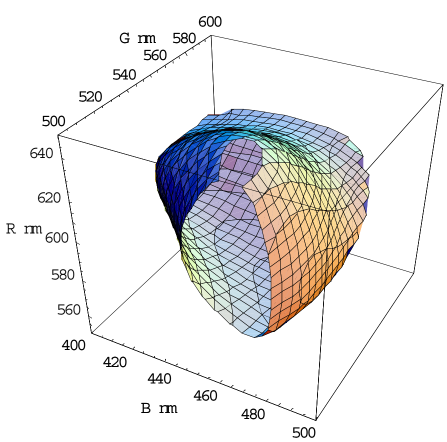

Figure 9. FOM ≥ 0.98 *when D65 is used*

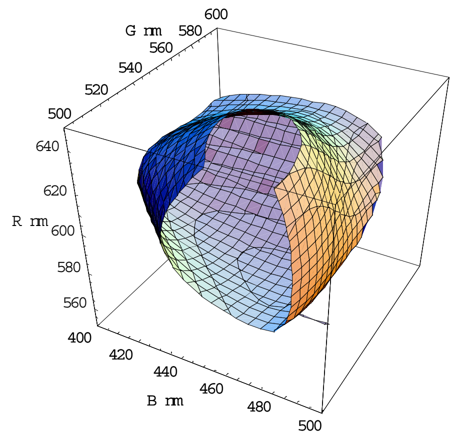

Figure 10. FOM ≥ 0.98 *when A is used*

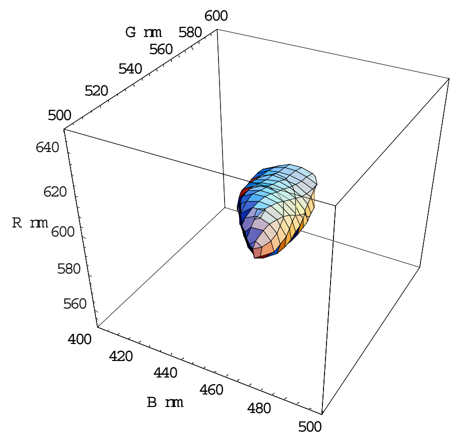

Figure 11. MG ≥ 0.95 *when D65 is used*

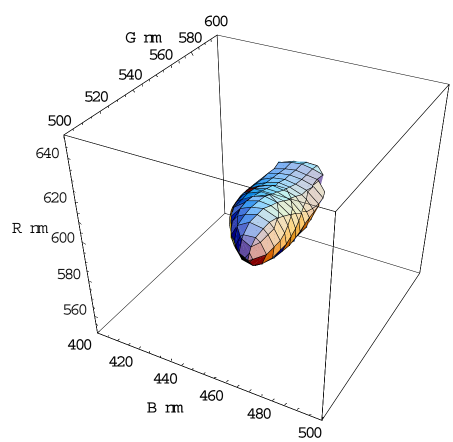

Figure 12. MG ≥ 0.95 *when A is used*

Conclusions

In this paper, several color quality factors have been explored to obtain the optimal peak positions of spectral sensitivities when their widths and skewness have been given. The optimal regions are different from each other, which demonstrates that one should exercise caution in the use of quality factors. The following has been concluded:

(1) The region of optimal peak positions is continuous. For sensitivity functions with peak parameters which sit within a large region of high factors, the fabrication tolerance would be relatively high. For other sensitivity curve parameters the same can be said.

(2) The region of optimal peak positions becomes larger when the quality factor level is lowered. The region with lower quality factor number will include that with higher quality factor value.

(3) We haven't demonstrated here, but in fact the shape of the region depends on the choice of the width parameter and skewness. This may be reported in the future

(4) μ-factor tends to be overly discriminating, while Q_{st} and Q_{sf} are likely to include everything. If the color difference is the final judge, then the region obtained with *FOM*/*MG* shows more promise.

(5) The value of most quality factors is more or less affected by the statistics of the data set and the characteristics of illuminants. In the future, these factors and noise amplification will be considered for real-world sensitivity optimization.

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Biography

Shuxue Quan received his B.S. and M.S. degree in Optical Engineering from Beijing Institute of Technology in 1994 and 1997. Since 1997 he has been a Ph.D. candidate in Imaging Science with Rochester Institute of Technology (RIT). His work primarily focuses on the optimal design of spectral sensitivity functions for color imaging systems. He is a student member of IS&T.

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