Multispectral Imaging of the Archimedes Palimpsest

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Abstract

The Archimedes Palimpsest is a thousand-year-old manuscript. It contains seven treatises of the mathematician, Archimedes, who died in 212 BC. Eight hundred years ago, this manuscript was erased and overwritten with a Christian prayer book. Today, scholars working with the Walters Art Museum of Baltimore, Maryland, want to compare the underlying text of Archimedes with present-day copies of his writings. Two teams of imaging scientists, one from Johns Hopkins University and our team from the Rochester Institute of Technology and Xerox Corporation, have completed a preliminary study of the efficacy of using multispectral imaging and related techniques to recover the faded Archimedes text.

In this talk, we report on an image processing method, commonly used in remote sensing, that was used to extract the Archimedes text and suppress the prayer-book writings. The results show that this linear mixing model, using a least squares fit with operator-specified spectral signatures, can separate the underlying Archimedes text from the writings of the prayer book. Although these results are preliminary, they have already begun to reveal new information about the writings of Archimedes.

Introduction

The significance of the Archimedes Palimpsest has been described in another talk in this conference (Noel, 2001). The manuscript is written on parchment, which is treated animal skin. It was last studied in detail in the early 1900s by Heiberg, using a magnifying glass and monochrome photography. Since that investigation, the condition of the manuscript has degenerated due to poor storage conditions which has led to the growth of mold.

Much of the original text of the Archimedes treatises is now very difficult to read. To the eye, the colors of the inks used for the two texts seem slightly different. This suggests that spectral reflectance measurements might be useful for segmenting the original text. In this study, we collected multispectral images of five leaves of the manuscript and applied imaging classification algorithms, borrowed from the field of remote sensing, to enhance the original writing while suppressing the text of the prayer book.

Multispectral Imaging

Images were collected using a Roper/Photometrics *SenSys*® monochrome digital camera. The sensor is the Kodak 1602E "Blue Plus" CCD, which exhibits enhanced sensitivity in the near-ultraviolet and blue regions of the spectrum. The CCD array contains 1536×1024 pixels and the gray values are quantized to 12 bits per pixel (4096 gray levels). We chose this scientific camera over an available "point-and-shoot" digital camera with a larger-format CCD (3072×2048) to eliminate problems with the RGB color-filter array, its associated antialiasing filter and color interpolation, and 8-bit quantization (256 gray levels per pixel per color.) The 12-bit quantization of the scientific camera allows recording of very subtle changes in reflectivity.

The passbands of the images were selected by 1"-diameter glass absorption filters carried in the camera's filter wheel. We used standard astronomical UBVRI filters with passbands approximately 100nm wide centered at 100-nm increments between 350nm and 750nm. Other experiments used a liquid-crystal tunable filter (LCTF) from CRI, Inc., which can select passbands that are 10nm wide over the range from about 420nm-720nm.

We used two illumination sources to image the manuscript: white light from tungsten lamps and "longwave" ultraviolet light ($\lambda = 365$nm). The parchment fluoresces under ultraviolet illumination, enhancing the contrast of the faded ink and improving the readability of the original text.

During imaging, the manuscript rested on a low-reflectance surface (a *Spectralon*® broadband 2% reflector from Labsphere, Inc.). This reduced the intensity of light from the backing and thus reduced the visibility of writing on the opposite side of the leaf.

Spectral Signatures

Although the two inks look very similar, there were made and written in different centuries. Because of this large gap in time, the two inks may respond differently to different wavelengths of illumination. The specific response of a material to wavelength is called its spectral signature. If the two inks have different spectral signatures, then it may be possible to separate them using multispectral imaging.

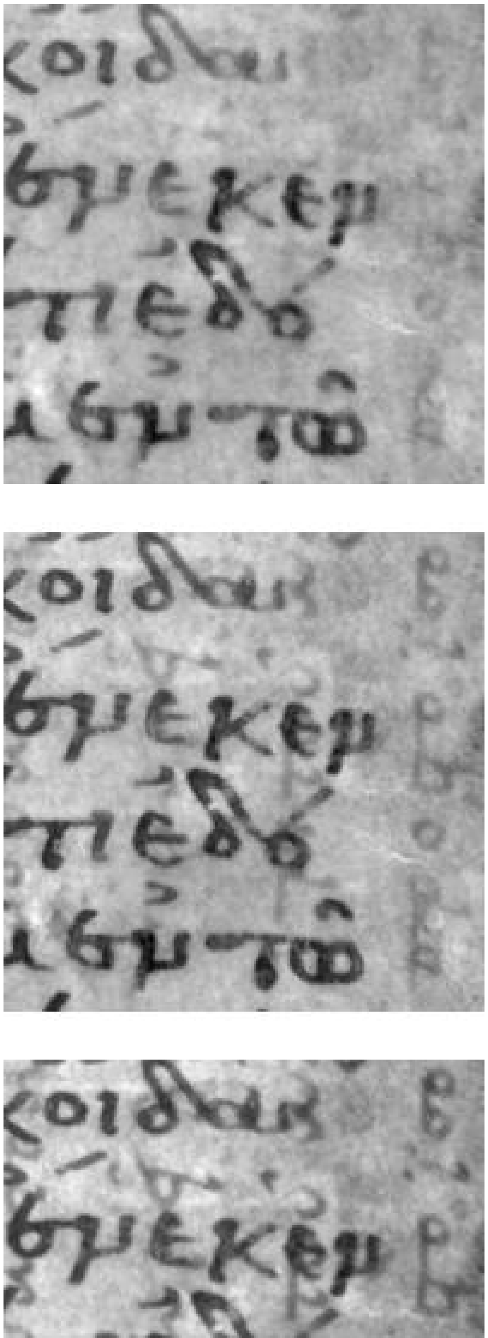

Figure 1. Three spectral bands of the Archimedes Palimpsest, under tungsten illumination. From the top, the bands are red, green and blue. The erased writing runs vertically in the images.

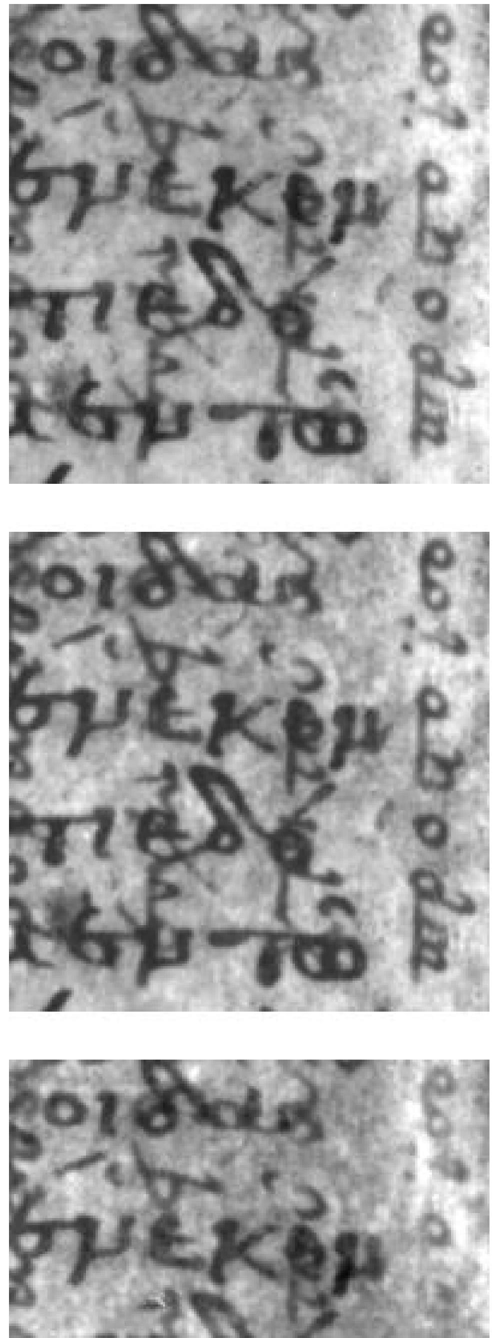

Figure 2. The same region under ultraviolet illumination, with corresponding red, green and blue bands. The figures show that the erased (vertical) writing varies significantly with wavelength.

Images of a small section of one leaf are shown in Figures 1 and 2. In both figures, the red, green and blue bands are shown separately, with the red band at the top. Tungsten illumination was used for the images in Figure 1, and ultraviolet illumination in Figure 2. The vertical writing in these images is the original writing that was erased, while the horizontal writing is the more recent prayer book.

With tungsten illumination, one line of the vertical writing is just visible. It is barely visible in the red separation, at the top of the figure, and becomes progressively easier to see in the green and blue separations. Other vertical lines of writing become visible in the green and blue separations in the regions that overlap the horizontal text. Clearly, the two inks respond differently with wavelength when illuminated with visible light.

With ultraviolet illumination, the vertical writing is now much clearer in all separations, even in the overlap regions. With this illumination, however, the maximum contrast is in the green separation, rather than the blue. Even here, the ink of the vertical writing varies in response, as a function of wavelength, much more than does the horizontal writing.

These images show that the two inks have different spectral signatures, which can be distinguished in visible and in ultraviolet illumination. The process of differenttiating between two materials with different spectral signatures has been studied extensively in the field of remote sensing. A least squares image processing algorithm, based on a linear mixing model, was used and is described in the next section.

Image Processing

Image Correction

The first image processing step is to correct the image data for known camera, sensor and illumination characteristics. Three corrections were made in this experiment, to correct for non-uniformity of illumination, for pixel defects in the sensor and for slight angular tilts of the filters.

The illumination over the parchment cannot be made completely uniform. To compensate for this non-uniformity, a flat, i.e. an image of a white target, is taken whenever the camera or illumination configuration is changed. These flats are blurred and divided into each data image to give a more uniform contrast across the image.

A second correction was made for defective pixels in the camera sensor. These pixels are always on, producing isolated white pixels in the recorded image data. Since it is the pixel elements themselves that malfunction, the defects always occur in the same locations in the recorded images. These locations can be detected from a dark image. The images are corrected by interpolating over the defective data points using the neighboring pixel values.

The last correction was made for slight shifts in the images between the different spectral bands. These shifts are caused by slight angular tilts of the filters in the filter wheel. The shifts in the images correspond to global lateral translations in two dimensions, which can be measured and digitally corrected with interpolation.

Linear Mixing Model

There are several methods in remote sensing, used to classify parts of an image based on individual spectral signatures. The linear mixing model[1] is one of those methods. It calculates the relative amount of each class contained in any given pixel. In remote sensing, the mixing of the classes is due to low resolution imaging that blurs two parts of an image together. For the palimpsest, the mixed classes arise from the superposition of the two inks.

The linear mixing model assumes that each pixel is composed of a linear combination of all of the classes. Any given pixel has a value, $v(\lambda)$, given by:

$$v(\lambda) = \sum a_i \, c_i(\lambda), \tag{1}$$

where $v(\lambda)$ is the response at a given pixel location, as a function of wavelength, a_i is a constant of proportionality, and $c_i(\lambda)$ is the value of a pixel from the i-th class.

The values, $c_i(\lambda)$, are determined by selecting specific pixels of a given class, that are pure examples of that class, and do not contain spectral content from any of the other classes. These pixels are also called endmembers, because they are at the extreme ends of their class.

The linear model can also be expressed in matrix form:

$$V = C\,A, \tag{2}$$

where V is a column vector with one spectral band per row, A is a column vector of proportionality constants, each row a different class, and C is a matrix, where each column is a endmember pixel and each row is a spectral band. The linear model represents each pixel as linear combination of the spectral signatures of the different classes.

If C is a square matrix, i.e. the number of columns (or classes) equals the number of rows (or spectral bands), then the proportionality vector, A, is found by multiplying V with the inverse of C. On the other hand, if the number of spectral bands is greater than the number of classes, then a least squares solution can be found. This solution is the standard pseudoinverse[2]:

$$A = (C^T C)^{-1} C^T\, V. \tag{3}$$

For the case of ink on parchment, a linear model does not apply. Ink superimposed on parchment or on another ink is not additive, but instead is multiplicative. It can be converted to an additive model by taking the logarithm of the pixel values, converting intensity values to densities:

$$V' = \log(V), \tag{4}$$

$$C' = \log(C). \tag{5}$$

The pixel densities, V', can now be represented as a linear combination of the spectral signature densities of the different classes, C'.

The least squares solution, shown in equation 3, using densities, was implemented to process the images taken of the five leaves. This solution is applied to each pixel, yielding a proportionality constant at each pixel location. The display of this constant in image form, constitutes a reconstruction of the corresponding class.

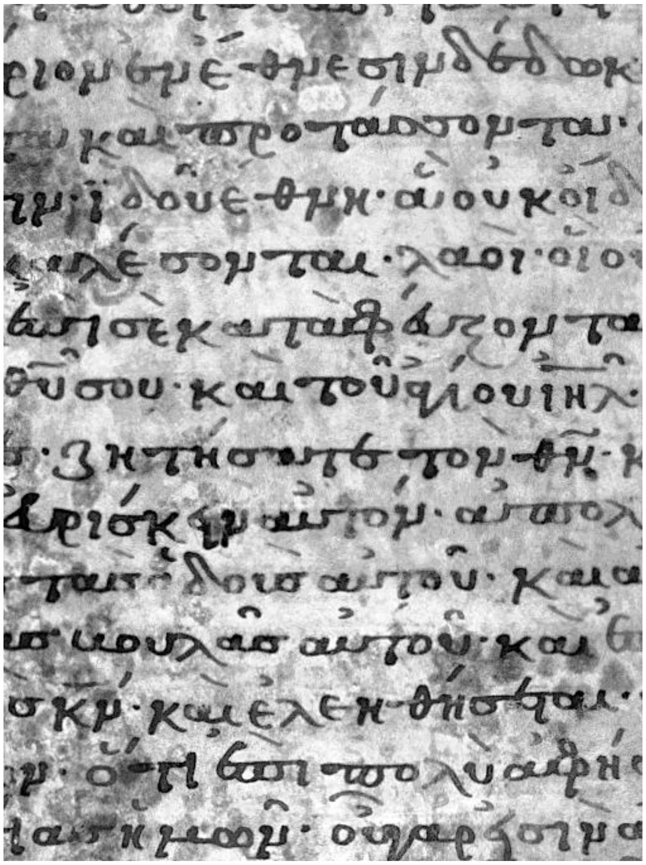

Figure 3. The luminance channel of part of one leaf is shown here. Very little underwriting is visible in this image.

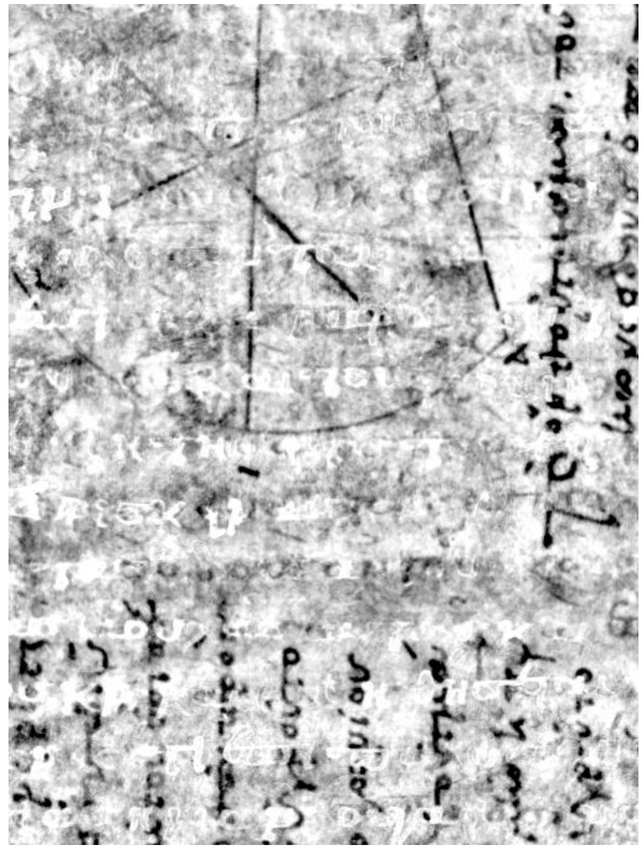

Figure 4. The vertical text is the reconstruction of the erased Archimedes writing using the modified linear mixing model.

Experimental Results

There were ten spectral bands taken, using the five astronomical filters and the two illuminations. The luminance channel of a merged red, green and blue image is shown in Fig. 3.

After the bands were registered, several individual pixels were identified as representative of the different classes. It was possible to identify locations where only underwriting or only overwriting is visible, for example, by comparing the images taken with ultraviolet and tungsten illumination.

There were six classes identified, one class for each ink and four different classes for the parchment. More than one class of parchment was necessary because of the variation due to mold etc. across a leaf.

Fig. 4 shows a reconstruction of the erased underwriting for the same region of the parchment that is shown in Fig. 3. The proportionality constant for the underwriting class is calculated for each pixel and displayed in an image format. This image has been enhanced to increase the contrast of the text. The image is noisy and includes impressions of the overwriting. The authors are continuing to develop methods to improve the image quality of these results.

Conclusions

The least squares estimation method, using a modified linear mixing model, can recover erased text on a parchment manuscript. The spectral differences between the two inks in the Archimedes Palimpsest are sufficient to enable the two writings to be separated by this method, thereby dramatically improving the legibility of the underwriting.

Acknowledgments

The authors wish to thank the Walters Art Museum of Baltimore, Maryland and the owner of the manuscript for this unique opportunity to increase society's knowledge of its cultural heritage. All the photographs in this paper were produced by the Rochester Institute of Technology, Johns Hopkins University and the Xerox Corporation. Copyright resides with the owner of the Archimedes Palimpsest.

References

1. Robert Schowengerdt, Remote Sensing, Academic Press, San Diego, CA, 1997, p. 447.
2. Robert Schowengerdt, p. 454.

Biography

Keith Knox is a Principal Scientist in the Xerox, Digital Imaging Technology Center. He joined Xerox in 1974, after completing a B.S. in Electrical Engineering and a Ph.D. in Optics from the University of Rochester. His research interests include image processing, digital halftoning, image enhancement and restoration. Keith is a Fellow of both IS&T and OSA.