Influence of Resolution on Scanner Noise Perceptibility

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Abstract

The perceptibility of scanner noise as a function of resolution is studied using a model for the human visual system and for the scanner noise. The visual system is modeled using a point-wise nonlinearity followed by a lightness contrast-sensitivity-function. The noise model incorporates a signal-dependent noise component and a signal-independent noise component. The system is analyzed to determine the perceived signal-to-noise ratio (SNR) as a function of the measured SNR. The findings support the intuition that as resolution is increased a lower measured SNR is acceptable because the eye effectively averages over the pixels at the higher resolution. Roughly speaking, the acceptable levels of measured SNR are inversely proportional to the resolution of the scanner. The overall impact of increasing resolution in a scanner by changing the sensor while keeping the lamp and the optics fixed is also analyzed in the same framework. The analysis indicates that if the signal-dependent component of the noise dominates, the perceived SNR does not degrade with increased resolution, but if signal-independent noise is also significant, the perceived SNR degrades with an increase in resolution.

Introduction

When calculating a signal-to-noise (SNR) ratio, the standard deviation of the noise signal is commonly used as a noise metric due to its ease of computation. Its numerous shortcomings are well known. The biggest one being that it does not take into account the perceptibility of the noise. In scanners, as resolution is increased the light gathering area per pixel is reduced – resulting in a reduced signal level (assuming the lamp output is not increased to compensate) and consequently a decrease in SNR. Thus if an SNR specification is set independent of scanner resolution (and viewing parameters for the scanned images), it is harder to meet the specification as the resolution increases. As the resolution is increased (and viewing parameters remain unchanged), a greater fraction of the noise energy is distributed in the higher frequencies, which are not as visible. This would indicate that a lower SNR would be tolerable at higher resolutions. Several researchers have used modified noise metrics that take into account perceptibility. The most common of these use filtered noise energy as a correlate of perceived noise instead of simply using the complete (unfiltered) noise energy. Typically, this is performed by using a simple linear shift invariant model of the human constrast sensitivity which defines the "filter" used for filtering the noise. A more detailed description of the motivation and use of such an approach in image coding can be found in Mannos and Sakrison[1]. In this paper, we apply a similar methodology for evaluating a more perceptual noise metric for scanners and use that metric to determine how much degradation in measured SNR can be tolerated with increase in resolution.

Simplified Vision Model

For the analysis in this paper, in order to evaluate the perceived impact of noise added to an image the simple vision model shown in Figure 1 is used. This model is adapted from Mannos and Sakrison.[1] The model consists of a point wise non-linearity representing the conversion from measured luminance to perceived lightness and a band-pass lightness contrast sensitivity function. The point wise nonlinearity is represented as the CIE lightness function (which is the common approximation to the transformation from a luminance input space into a perceived lightness space):

$$l(u) = 116t(u) - 16 \tag{1}$$

where,

$$t(u) = \begin{cases} u^{1/3} & u > 0.00856 \\ 7.787u + \dfrac{16}{116} & u \le 0.00856 \end{cases}$$

The second stage representing the lightness contrast function is expressed as a linear shift invariant filter with separable 2-D frequency response given by:

$$H(f_x, f_y) = H_r(f_x) H_r(f_y) \tag{2}$$

where f_x and f_y are the spatial frequencies along the x and the y spatial dimensions, respectively, and

$$H_r(f) = 2.6(0.0192 + 0.114|f|)\exp(-0.114|f|) \tag{3}$$

Note the model differs from the one presented in Mannos and Sakrison[1] in two respects: firstly the model in the original paper assumed a radially symmetric contrast sensitivity function and secondly the argument of the exponent had a power of 1.1. Both these simplifying assumptions, do not change the contrast sensitivity function (CSF) sufficiently to have an impact on the results and conclusions of this paper. However, they allow analytic integration and evaluation of perceived noise power and SNR.

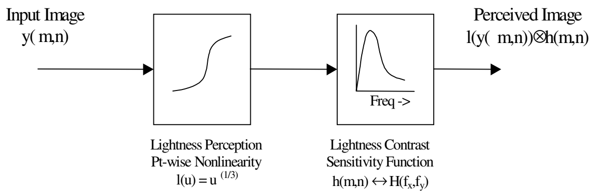

Figure 1. Simple Vision Model for obtaining an approximate perceived image.

While the model is simplistic, it captures the major characteristics of the human visual system and therefore allows us to capture first order effects. A more complete model (based on the several visual difference predictors in the literature) could alternately be used but the required simulations would be time consuming and harder to interpret.

System Model

Figure 2 shows the system model used for the analysis in this paper. The model computes a perceived difference-noise image between a noise-less input image *i(m,n)* and an image with additive scanner noise. The perceived image of the original image is obtained by propagating it through the visual model. The perceived image of the image with scanner noise is calculated by adding the scanner noise to the original noiseless image and then propagating the sum through the visual model. The difference between these two perceived images represents the perceived noise. The notation used is as follows:

$i(m,n)$ - original noise-less image
$v(m,n)$ - scanner noise
$d(m,n) = i(m,n) + v(m,n)$ - noisy "scanned" image
$v_p(m,n)$ - perceived noise image
$l_i(m,n) = l(i(m,n))$ - input image after the point-wise lightness nonlinearity

If the visual model of the last section is used, the perceived noise image can be written as

$$\begin{aligned} v_p(m,n) &= h(m,n) \otimes l(\, i(m,n)+v(m,n)) - h(m,n) \otimes l(\, i(m,n)\,) \\ &= h(m,n) \otimes [\, l(i(m,n) + v(m,n)) - l(i(m,n)\,)\,] \end{aligned} \quad (4)$$

where ⊗ represents the convolution operation.

The energy of $v_p(m,.n)$, defined as, $\Sigma\, (v_p(m,n)\,)^\wedge 2$ can then be used as an indicator of the perceived noise energy in the image. This expression assumes that the bandwidth of $H(f_x,f_y)$, the Fourier Transform of $h(m,n)$, is less than half the sampling frequency, f_s (which is a good approximation for typical scanner resolutions).

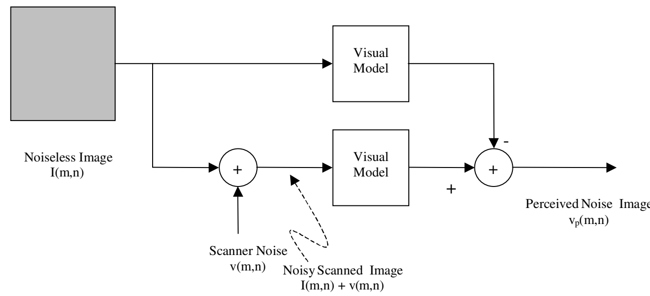

Figure 2. System Model for obtaining Perceived Approximation to an Image.

Perceptual Signal to Noise Ratio

Using the system model presented in the last section, a perceptual or visual SNR (VSNR) based on a visual model can be defined in much the same way as SNR is defined, VSNR=√(perceived signal energy / perceived noise energy)

$$= \sqrt{\frac{\int_{f_y=-f_s/2}^{f_y=f_s/2} \int_{f_x=-f_s/2}^{f_x=f_s/2} \left| H(f_x,f_y) L_i(f_x,f_y) \right|^2 df_x df_y}{\int_{f_y=-f_s/2}^{f_y=f_s/2} \int_{f_x=-f_s/2}^{f_x=f_s/2} P_{v_p}(f_x,f_y) df_x df_y}} \quad (5)$$

where f_x , f_y denote the spatial frequencies along x and y spatial directions, respectively; f_s represents the sampling frequency, $P_{v_p}(f_x, f_y)$ denotes the power spectral density of $v_p(m,n)$ the perceived noise image (assumed to be at least wide-sense stationary); and the remaining functions indicated by upper case letters are the (spatial) Fourier transforms corresponding to their lower case counterparts.

Scanner Noise Model

Scanner noise arises from two sources: a signal dependent shot noise component and a signal independent random/dark noise component. The standard deviation of the shot noise is proportional to the square root of the signal level (due to poisson statistics for photon arrival). Accordingly, it is assumed that the expression for the noise, v(m,n), at any pixel is given by:

$$v(m,n) = \sigma_1 \sqrt{i(m,n)}\, v_1(m,n) + \sigma_2\, v_2(m,n) \quad (6)$$

The first term part represents the image-dependent noise which has a standard-deviation proportional to the square-root of the signal level, with σ_1 as the proportionality factor (throughout this paper we will assume that the input image is normalized to lie between 0-black and 1-white). The second term represents the signal independent noise. The noise terms, $v_1(m,n)$ and $v_2(m,n)$ are assumed to be uncorrelated spatially and with each other, with zero-mean, and unit variance.

Perception of Noise in Uniform Patches

While the methodology described above could conceptually be used to determine the perceptibility of noise in any image or any class of images, in most cases simulations or simplifications would be required for evaluation of the VSNR. In this paper, we consider the case for uniform input images *i(m,n) = i*, for which the analysis can be done in closed form using simplifying assumptions. Using the scanner noise model presented in the last section, for a uniform image with stationary, white noise, the power spectral density of the noise (power/unit area) is given by

$$P_v(f_x, f_y) = (i\sigma_1^{\ 2} + \sigma_2^{\ 2}) / f_s^{\ 2} \tag{7}$$

Conventionally, the "measured" scanner signal to noise ratio at a given signal level is defined as

$$SNR_i = \frac{signal\_level}{noise\_std\_deviation} = \frac{i}{\sqrt{i\sigma_1^2 + \sigma_2^2}} \tag{8}$$

The expression for the perceived noise in equation (4) can be simplified by assuming the added noise v(m,n) is small. In this case the Taylor series can be used to approximate

$$v_p(m,n) \approx h(m,n) \otimes [l^{'}(i(m,n))v(m,n)] \tag{9}$$

Using this approximation perceived SNR reduces to

$$VSNR = |H(0,0)| K(i) SNR_i W_H(f_s)\,, \tag{10}$$

where

$$K(i) = l(i) / (|l^{'}(i)| i)$$

is a signal level dependent term that does not depend on resolution (f_s), and the function

$$W_H(f_s) = \frac{f_s}{\sqrt{\int_{f_y=-f_s/2}^{f_y=f_s/2} \int_{f_x=-f_s/2}^{f_x=f_s/2} |H(f_x, f_y)|^2 \, df_x df_y}} = \frac{f_s}{\left(30.8 - \left(30.8 + 3.5 f_s + 0.192 f_s^2\right) \exp(-0.114 f_s)\right)} \tag{11}$$

captures dependence of perceptual SNR on resolution f_s .

A plot of the function $W_H(f_s)$ which relates perceptual SNR to measured SNR as a function of scan resolution is shown in Figure 5 for a 25 cm viewing distance. The plot of the function shows a minimum at approximately 175 dpi indicating that the scanner noise is most visible at a scanner resolution (sampling frequency) of 175 dpi. This minimum corresponds to the scanner sampling frequency at which the scanner noise is "most visible" on either side the visibility of scanner noise is reduced. The minimum arises due to the band-pass nature of the lightness contrast function chosen. If the lightness contrast function is chosen to be low-pass, the function will be a monotonically increasing function of f_s. Note also that the decrease in noise perceptibility for low sampling frequencies is not really useful because use of these lower sampling frequencies would introduce aliasing and other undesirable artifacts in images other than uniform patches.

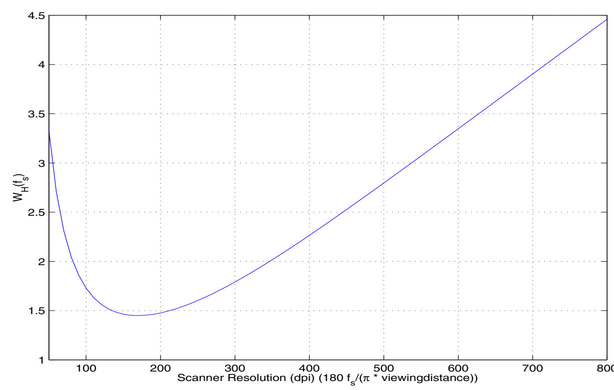

Figure 3. Function W_H(f_s) relating perceptual SNR to measured SNR as a function of resolution.

Note that for scanner resolutions over 250dpi, perceptual SNR is related to the "measured" SNR in almost direct proportion to the resolution. This can also be intuitively inferred by looking at (11) and the lightness contrast sensitivity function in (3). The lightness contrast sensitivity function falls to a level very close to zero at high spatial frequencies. Therefore, as the scanner resolution is increased beyond 250dpi, the denominator term in equation (11) remains unchanged because the increased interval of integration for the denominator term corresponds to a region over which the CSF is close to zero and therefore does not contribute anything. This clearly indicates that at higher resolutions a much lower "measured" SNR should be acceptable because it is equivalent to a lower perceptual SNR. Note however, that this assumes that the image is not magnified/scaled up after scanning and that the viewing distance is fixed.

Measured SNR Variation with Resolution

The analysis of the previous sections provided a means for relating measured SNR to perceptual SNR. For scanner design, it is also useful to understand how change in resolution impacts measured SNR. Typically, the resolution of a scanner is increased by increasing the number of pixels in the sensor array used for image capture. Thus in order to double the scanner resolution along each dimension a single "pixel site" is split into 4 pixel sites. If it is assumed that the scanner lamp and optics are left unchanged, the impact of increasing the resolution by increasing the number of elements in the sensor array can be analyzed using the assumption that the light incident on the sensor array is unchanged. The signal level for each pixel is then dependent on the area of the pixel site. Consider the case when the sampling frequency is changed from an original value f_0 to a new value f_1 , with $t = f_1 / f_0$ representing the factor by which the sampling frequency is changed (in each direction). If the original signal strength per pixel is S_0, the signal strength when the resolution is changed by a factor t in both x and y directions is given by S_0/t^2 (since the area per pixel is scaled by t^2). The signal dependent portion of the noise is due to the Poisson statistics of photons and its variance will therefore also be scaled by the same factor. If we assume that the signal independent noise is unchanged with change in resolution, and the original SNR is given by the expression of equation (8), the measured SNR at the new resolution at a signal level i is given by

$$SNR_i(t) = \frac{\left(i/t^2\right)}{\sqrt{\left(i/t^2\right)\sigma_1^2 + \sigma_2^2}} = \frac{f_0 i}{f_1\sqrt{i\sigma_1^2 + f_1^2\sigma_2^2 / f_0^2}} \qquad (12)$$

Note that if the signal independent component of the noise is small, the above equation indicates that the measured SNR is in inverse proportion to the sampling frequency.

Overall Impact of Change in Resolution on Perceptual SNR

Equation (12) can be substituted in equation (11) to obtain a single expression for the perceived SNR that comprehends the overall impact of change in resolution: incorporating both the effect of perception and the change in noise statistics due to change in sensor pixel area. For the rest of this section, it is assumed that the noise statistics σ_1 and σ_2, for the noise model in equation (6) are computed at a certain reference sampling frequency (resolution) f_0 the noise statistics for a different resolution are computed from these statistics using the model of the last section. The perceived SNR corresponding to a sampling rate (resolution) of f_1 is then given by

$$\begin{aligned} VSNR(f_1) &= \frac{\left|H(0,0)\right| K(i) W_H(f_1) f_0 i}{f_1\sqrt{i\sigma_1^2 + f_1^2\sigma_2^2 / f_0^2}} \\ &\approx \frac{\left|H(0,0)\right| K(i) f_0 i}{(30.8095)\sigma_1\sqrt{i + (f_1^2\sigma_2^2)/(f_0^2\sigma_1^2)}} \end{aligned} \qquad (13)$$

where the approximation is valid for resolutions over 250 dpi. If

$$\sigma_1^2 << f_1^2\sigma_2^2 / f_0^2,$$

i.e., the signal independent component of the noise is negligible as compared to the signal dependent noise (13) reduces to a constant value independent of the resolution. If alternately, it is assumed that the signal dependent noise is negligible compared to the signal independent noise, the visual SNR in (13) varies inversely with the resolution. The assumption of negligible signal dependent noise is however, unrealistic due to the inherent Poisson statistics of photon arrival. Typically, both signal dependent and signal dependent noise are present. For the rest of this analysis, we consider specific numbers for evaluating the perceptual SNR: a reference sampling frequency of f_0=*400dpi* and noise std. deviations $\sigma_1=0.0024$, $\sigma_2=0.0025$. These numbers correspond to actually estimated noise variances from a scanner. For these chosen values, relative overall VSNR (normalized wrt the VSNR value at 400dpi) is shown in Figure 4 (for a signal level $i=1$). Note that due to the signal independent noise component, the overall perceptual impact of changing resolution while keeping other scanner components (lamp, sensor) fixed is a decrease in perceptual SNR. The decrease however is at a much smaller rate than what would be expected based purely on the "faster-than-linear" rate of decrease in measured SNR indicated by equation (12).

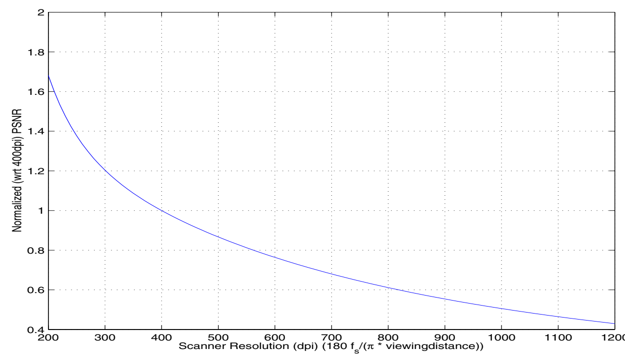

Figure 4. Overall relative VSNR (wrt 400dpi) as a function of scanner resolution.

Conclusions

Scanner specifications defined in terms of measured SNR should be dependent on scanner resolution. A good rule of thumb applicable for typical scanner resolutions of interest is that the scanner SNR specified at a higher resolution can be degraded (scaled down) by the same factor by which the scanner resolution is scaled up. If the scanner resolution of a scanner is increased by simply splitting the pixel area of a sensor into multiple sensors while keeping the light level and the scanner optics identical, the perceived SNR degrades with increasing resolution. The rate of degradation depends on the relative amounts of signal dependent and signal independent noise. If the signal dependent noise dominates, there is little change in perceptual SNR with increase in resolution (for typical scanner resolutions). If the signal independent noise dominates, the perceptual SNR falls by a factor proportional to the increase in resolution.

References

1. J. L. Mannos and D. J. Sakrison, “The Effects of a Visual Fidelity Criterion on the Encoding of Images,” *IEEE Trans. Info. Theory*, vol. **IT-20**, No. 4, July 1974.

Biography

Gaurav Sharma received the PhD degree in electrical engineering from North Carolina State University, Raleigh in 1996. Since Aug. 1996 he has been employed as a member of the research and technical staff at Xerox Corporation's Digital Imaging Technology Center in Webster, NY. He is also involved in teaching in an adjunct capacity at the Rochester Institute of Technology, Rochester, NY. His current research interests include color science and imaging, signal restoration, image halftoning, and error correction coding. Dr. Sharma is a senior member of the IEEE, and a member of Sigma Xi, Phi Kappa Phi, the Society for Imaging Science and Technology, and the IEEE Signal Processing Society.