Penetrating Ink and Halftone Reflectance

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Abstract

A simple model of a halftone print consists of an ink layer on top of the paper. Light is transmitted through the ink layer on incidence, is reflected by the paper and is transmitted again through the ink layer on reflection. The transmittance is given by Beer's Law. For many types of paper, however, a more realistic model of the halftone process must include the effects of ink penetration into the paper. In this case, Beer's Law does not adequately describe the interaction of light with the ink because of scatter by the paper fibers. In the current work, the reflection and absorption of light is determined theoretically by solving the diffusion approximation to the radiative transfer equation for both the case of ink layer on top of the paper and the case of ink penetration into the paper.

1. Introduction

There has been some effort in recent years to develop "first principle" models of the halftone process[1, 2, 3, 4, 5, 6]. Such models use the physics of light interacting with paper and ink to relate halftone color to various physical quantities that characterize the halftone print. The objective is not so much to develop new algorithms for printing technology, but to better understand the fundamental processes that are involved. One of these processes that must be accounted for is the diffusion of photons within the paper substrate, an effect that leads to optical dot gain. Another fundamental process is ink penetration into the paper substrate. In this article, the effect that ink penetration has on halftone color is compared to the case in which there is no penetration. In both cases, there is photon diffusion within the paper substrate.

Many models have treated the halftone print as consisting of an ink layer lying on top of a paper substrate.[3, 4, 5, 6] In such a model, incident light is transmitted through the transparent ink layer, is scattered by the paper fibers and diffuses within the substrate, and is again transmitted through the ink layer on reflection. Selective absorption occurs only on transmission through the ink layer, and the transmittance spectrum is given by the Beer's law, as the ink layer is transparent. A more realistic model for many types of paper – particularly uncoated papers – is one in which the ink penetrates into the paper[10, 11]. The ink layer is part of the substrate, and so the substrate does selectively absorb. Light enters the paper, is scattered by the paper fibers and absorbed by the ink. In this case, Beer's law does not correctly describe the interaction of light with the ink layer because of scatter by the paper fiber.

In this paper we compare theoretical models for the case of non-penetrating ink (NPI) and the case of ink penetration (PI) into the paper. In the NPI model, the ink lies in a distinct layer on top of the paper substrate, and in the NPI model the ink concentration decrease exponentially with depth. In order to compare the two models, the total number of ink dye molecules is the same in both cases.

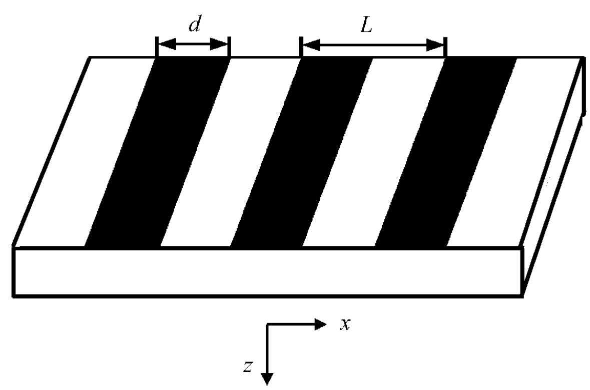

Figure 1: Line halftone with line-width d and line frequency $1/L$.

To simplify the calculations, we treat the case of a line halftone – the inked regions consist of parallel lines separated by the distance L and having width d, as shown in Figure 1. It has been shown both theoretically and experimentally that optical dot gain only slighty depends on dot shape,[9] so one would expect that the results presented here would largely hold for circular dot halftones as well. The fractional ink coverage is given by d/L. The x-axis is perpendicular to the lines, and the positive z-axis is pointing downwards into the paper. The incident light travels in the positive z direction. The paper top surface is at $z = 0$ and the bottom surface is at $z = t$,with t the paper thickness. All lengths are in units of the paper thickness t. It is assumed the backing is black, so none of the photons transmitted through the paper are reflected back. We treat the case of an AM halftone with line frequency $(2t)^{-1}$.

The directional incident photons enter the paper, where they are scattered and absorbed. The diffusion approximation assumes that the scattered photons become (nearly) diffuse[5, 7, 8]. Within the paper, there are directional incident photons and diffuse scattered photons. The reflected light consists of the diffuse photons that exit the paper top surface.

2. Diffusion Model

Light entering the paper substrate is scattered and becomes approximately diffuse. The density of these nearly diffuse photons within the substrate can be calculated from the diffusion approximation to the radiative transfer equation[5, 7]:

$$-D\nabla^2 u(x,z) + \gamma_a u(x,z) = \gamma_s F(x,z) \tag{1}$$

where $u(x,z)$ is the photon density and D is the diffusion coefficient, given by:

$$D = \frac{1}{3\gamma_t},$$

and γ_a, γ_s, and γ_t are the absorption, the scattering, and the transport coefficients of the substrate. The transport mean free path is: $l^* = 1/\gamma_t$, and is the average distance a photon travels before being either scattered or absorbed. γ_a and γ_s are the probabilities per unit length that a photon is absorbed or scattered respectively. The transport coefficient is given by:

$$\gamma_t = \gamma_a + \gamma_s.$$

It is assumed that the ink does not scatter, that all scatter is due to the paper fibers. Therefore, the scattering coefficient for the ink-penetrated paper is the same as that for bare paper. For NPI, $\gamma_a = 0$, as the paper has very little absorption. For PI, the absorption coefficient depends on the ink concentration, which varies with position, and on the ink spectral characteristics and can be written as:

$$\gamma_a(x,z;\lambda) = \sigma_a(\lambda)\rho(x,z),$$

where σ_a is the absorption cross-section of the ink and $\rho(x,z)$ is the ink concentration. The concentration decreases with depth and is given by:

$$\rho(x,z) = \rho_0 h(z/\alpha)\, \mathcal{C}_i(x),$$

with ρ_0 the average concentration in inked regions, $h(z/\alpha)$ the ink penetration function, and α the penetration depth. Ink penetration is given by:

$$h(z/\alpha) = \frac{2}{\sqrt{\pi}} \exp\Big[-(z/\alpha)^2\Big].$$

$\mathcal{C}_i(x)$ is the ink distribution function and is zero between the lines of ink and equal to one at points x within the lines:

$$\mathcal{C}_i(x) = \begin{cases} 1, & \text{if a line at point } x \\ 0, & \text{if no line at point } x \end{cases} \tag{2}$$

The quantity $F(x,y)$ is the reduced intensity which is the directed incident light that has entered the paper. This directed incident flux is reduced as it travels into the paper due to scatter and absorption and is the source of the scattered, nearly diffuse photons. The reduced intensity satisfies the following equation:[5]

$$\frac{d}{dz}F(x,z) = -\gamma_t(x,z)F(x,z).$$

As indicated above, for the PI case γ_t is a function of x and z. One finds:

$$F(x,z) = F_0 \exp\Big[\gamma_s z + \gamma_a' \,\alpha\, \mathrm{erf}(z/\alpha)\, \mathcal{C}_i(x)\Big]$$

where F_0 is the incident flux, $\gamma_a' = \sigma_a(\lambda)\rho_0$ is the average absorption coefficient, and $\mathrm{erf}(x)$ is an error function.

For NPI, the light passes through the ink-layer before entering the paper, so that

$$F(x,z) = F_0\, \tau(x) \exp(-\gamma_t\, z)$$

where τ is the transmittance of the ink layer given by $\tau(x) = \tau_0\, \mathcal{C}_i(x)$ with τ_0 the ink transmittance. In this case γ_t is constant.

In comparing the two cases, the number of ink dye molecules is kept constant. Therefore if the ink layer has transmittance τ_0, then

$$\gamma_a' = \frac{-\ln(\tau_0)}{\alpha}.$$

The photon density $u(x,z)$ does not include the photons of the reduced intensity $F(x,z)$, but only the scattered nearly diffuse photons. The photon density $u(x,z)$ is the solution to Eq. (1) subject to the following boundary conditions: there are no diffuse photons entering the paper from top or bottom. This can be expressed as:[5]

$$u(x,0) - \frac{D}{2}\frac{\partial}{\partial z}u(x,z)|_{z=0} = 0$$

for the top surface, and

$$u(x,t) + \frac{D}{2}\frac{\partial}{\partial z}u(x,z)|_{z=t} = 0$$

for the bottom surface. Figures 2 and 3 show contour plots of the photon density, $u(x,z)$, as the solution to Eq. (1) for NPI (Figure 2) and PI (Figure 3). The solutions were obtained numerically using the Matlab Partial Differential Equation Toolbox. The light is incident from the top, and regions with ink (0.5 fractional coverage) are indicated by the thick lines on top. As one would expect, the photon density is maximum between the regions of ink, as indicated by the oval shaped maxima. For NPI, the regions *covered* by ink have significant photon density due to diffusion from the regions of higher density to these regions of lower density. For the PI case, the density in inked regions is nearly zero. This is because photons that diffuse into those regions are absorbed by the ink.

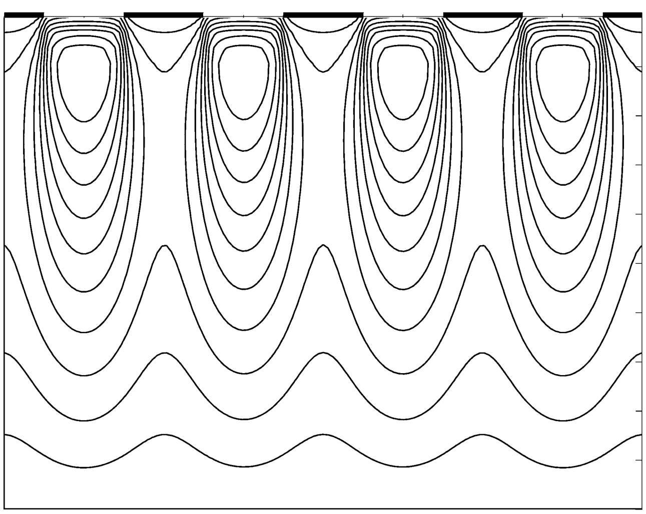

Figure 2: NPI. Contour plot of photon density within the paper. Light is incident from the top, and $d/L = 0.5$, $\gamma_s = 15$. The z-axis is vertical, x-axis is horizontal, and inked regions are indicated by the short thick lines on top.

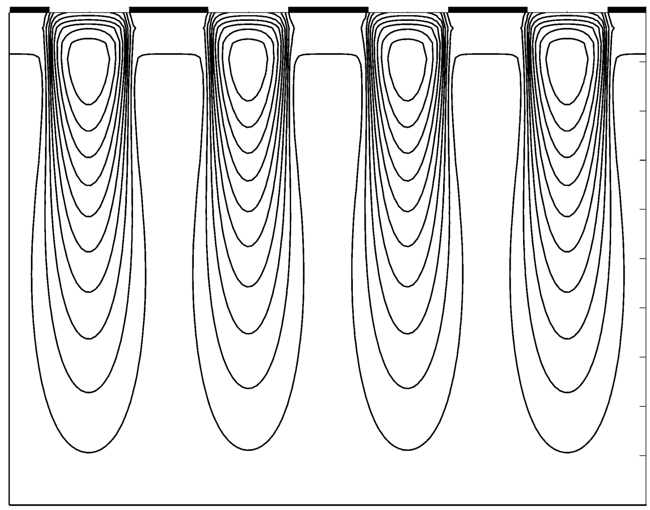

Figure 3: PI. Contour plot of photon density within the paper. Light is incident from the top, and $d/L = 0.5$, $\gamma_s = 15$, $\alpha = 0.2$. The z-axis is vertical, x-axis is horizontal, and inked regions are indicated by the short thick lines on top.

3. Reflection and Absorption

The reflected light is the flux passing through the paper top surface in the $-z$ direction. For NPI the light passes through the ink-layer on reflection, and is given by:[5]

$$F_-(x) = \frac{1}{2}\tau(x)\,u(x,0).$$

For PI the reflected light is:[5]

$$F_-(x) = \frac{1}{2}u(x,0).$$

The reflectance as a function of position for both PI and NPI is shown in Figure 4. One notes that the reflectance *between* dots (the peaks in Figure 6) is less for PI than for NPI. The photon density between dots is less for PI due to the absorption of photons that diffuse into inked regions – This is due to the fact that the photon density between the dots is less in PI (compare Figures 2 and 3), and since the density is less, the number of photons that diffuse through the top surface between dots is less as well.

One also notes that the reflectance *from* dots is greater for PI than it is for NPI. The amount of absorption a given ray undergoes as it travels through an absorbing medium depends on the path length of the ray through the medium. If the medium scatters, then different rays exiting the medium have different path lengths, and therefore undergo different absorptions. The greater the scatter power of the medium,

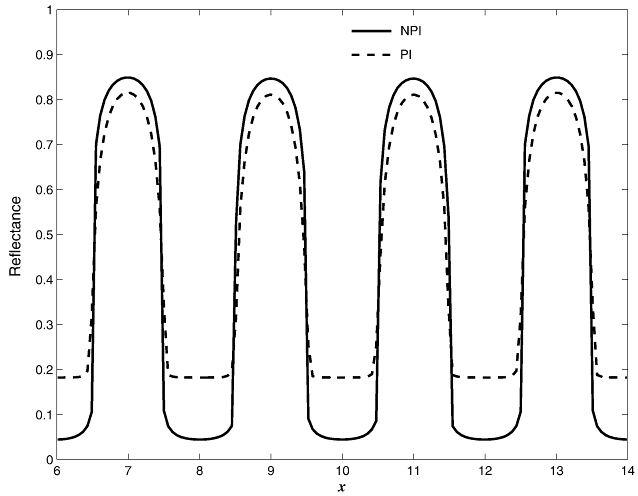

Figure 4: The reflectance as a function of position x for both PI and NPI. With $d/L = 0.5$, α = 0.2, and $\gamma_s = 15$.

the shorter the average path length, and therefore the less is the absorption of the incident light.

With NPI, in contrast, nearly all the rays exiting through the dots (except those that have diffused from between the dots) will have passed completely through the ink twice, and so have undergone maximum absorption. As a result, reflectance from dots is much less than in PI.

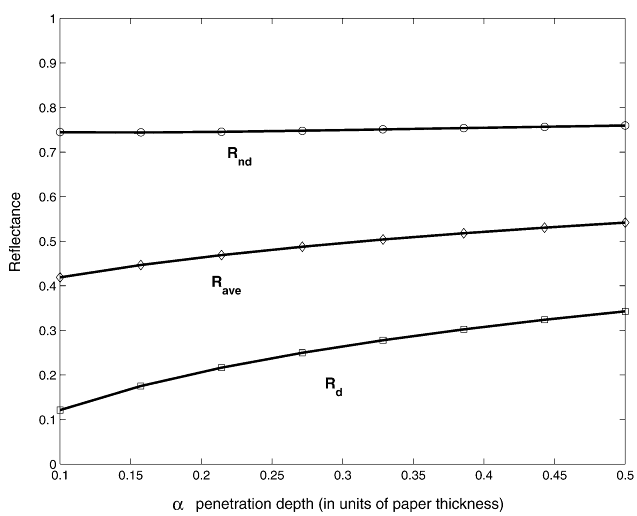

Figure 5: The reflectance as a function of penetration depth α. R_d is the average reflectance from dots, R_{nd} is the average reflectance from between dots, and R_{ave} is the total average relectance. With $d/L = 0.5$, and $\gamma_s = 15$.

Figure 5 shows the average reflectance, the average reflectance from dots, and the average reflectance between dots for penetration depth varying between 0.1 to 0.5 the thickness of the paper. As the ink penetrates further into the paper, the reflectance increases, and virtually all this increase comes from reflectance off the dots. This is due to the fact that while the distance the photons diffuse into the paper remains the same (it depends on γ_s), the ink concentration at distances less than α decreases with increasing penetration depth. With path lengths constant and ink concentration decreasing, the amount of absorption decreases as well.

The reflectance from between the dots does not vary much with penetration depth. With scattering coefficient $\gamma_s = 15$, very few photons diffuse into the paper as far as $\alpha = 0.1$, so diffusion from regions of high density to regions of low density does not change much as α inceases greater than 0.1. Figure 6 compares the absorption, $A = 1 - R - T$, as a function of scattering coefficient γ_s for several different α's with the absorption for the NPI case. A significant difference between PI and NPI is the fact that with PI, for $\gamma_s > 3$, the absorption *decreases* as scattering power increases.

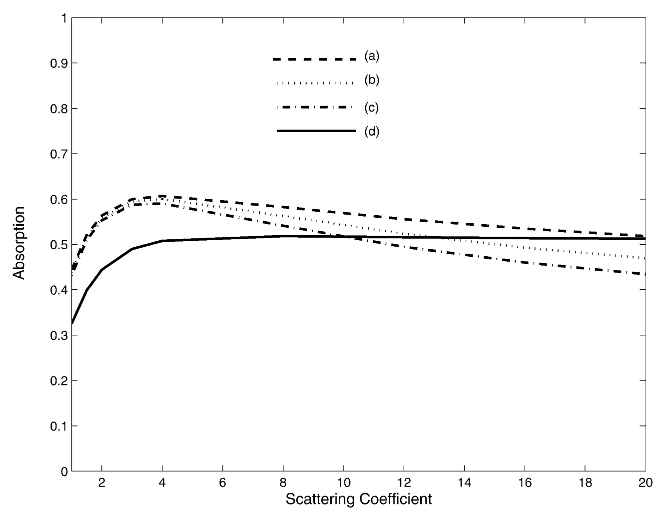

Figure 6: Absorption as a function of scattering coefficient γ_s, with $d/L = 0.5$ for several different α. (a) PI, α = 0.4; (b) PI, α = 0.25; (c) PI, α = 0.1; (d) NPI

For NPI, the greater the scattering power, the more photons diffuse to ink covered regions where they are absorbed as they exit the paper. Thus absorption increases as scattering power increases up to a maximum absorption, at which point the photons are "completely scattered."[5] This corresponds to the Yule-Nielsen n-factor equal to 2. Further increase in scattering power has no effect on absorption.

Similarly for PI, at small γ_s absorption increases with increasing scatter power since more photons diffuse to ink penetrated regions where they are absorbed. The absorption is greater, however, than for NPI because of absorption of these diffused photons.

Past the point of "complete scatter" instead of constant absorption as for NPI, the absorption declines for PI. This decrease in absorption is for the reasons indicated above: As the scattering power increases, the distance the photons diffuse into the paper decreases, the average path length decreases, and the so the absorption decreases. This is clearly indicated in Figure 6.

4. Conclusion

The theoretical model constructed here to account for ink penetration into paper shows some significant differences from that for which there is no ink penetration.

Most of the difference comes from two factors: (a) for PI, photons that diffuse from high density regions to low density region are absorbed by the ink in those low density regions; and (b) light entering the paper through dots has significantly less absorption in PI because of scatter by the paper fibers. For scattering power less than "complete scatter" (a) predominates, and the reflectance for PI is less than it is for NPI; for scattering power something greater than "complete scatter" (b) predominates and reflectance for PI is greater than it is for NPI.

It would seem that for papers in which ink penetration is a factor, one needs the more complicated model to adequately describe the results. There is a need for some experimental work to see whether in fact (a) and (b) above are significant, and how these processes can be accounted for phenomenologically.

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