

The Advantages and Disadvantages of Small Pixels

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Abstract

Digital cameras whose imagers are comprised of small pixels provide the following advantages when compared to cameras whose imagers are comprised of large pixels.

- Small pixel imagers use shorter focal length lenses than large pixel imagers to obtain the same magnification.
- Short focal length lenses are generally faster than long focal length lenses.
- Short focal length lenses yield greater depth-of-field/focus than long focal length lenses.
- Shorter lenses and smaller imagers greatly influence the compactness of a camera.
- A small pixel imager requires less silicon than a large pixel imager requires for a given resolution.

The one disadvantage of using an imager with small pixels is that the optics of such a short focal length camera requires very tight manufacturing tolerances. This factor may outweigh the benefits mentioned above.

There is an absolute lower limit, based on physical optics, for the size of a pixel used to capture an image in visible light. If one specifies two pixels per optical spot size and uses an f/0.5 lens to produce an optical spot size of 0.5 microns, that pixel size limit is 0.25 microns. (An aperture of f/0.5 is the smallest possible and probably not available at any price, however.)

A second limit is based on the noise caused by variation in the number of photons reaching each pixel. Imager noise caused by thermal effects and/or clocking is ignored.

A single expression for determining a “best” pixel size has been developed. It includes illumination level, signal-to-noise ratio, resolution, lens aperture, and exposure time. This expression can be used to optimize system performance and cost by evaluating the cost of focusing tolerances, other camera parameters, silicon costs, etc.

Traditional Method of Choosing Camera Spatial Resolution

First, taking the traditional approach for determining the required film resolution, consider print size and eye resolution. The spatial resolution of the eye is about 10 cycles per mm at about 10 inches. Assuming the print will be viewed at "normal" viewing distance and then as close as the eye can focus, the print material resolution should be at least as good as the resolution of the eye at 10 inches, 10 cycles/mm. The resolution required in the negative is greater than the print resolution by a factor of the magnification required to enlarge the negative size to the print size. As the format size changes it is clear that the required resolution at the camera image plane changes. A 4” x 5” format film will only be enlarged 2 times to produce an 8” x 10” image but a 35 mm format film (24 mm x 36 mm) will be enlarged 8 times to produce an 8” x 10” image. To have adequate resolution in the enlargement, the image on the 4” x 5” film must have twice the linear resolution of the eye (500 cycles/inch or 20 cycles/mm). The image on 35 mm film has to have 8 times this resolution (2000 cycles per inch or 80 cycles/mm). The small format camera has the disadvantage of requiring high-resolution film, but it has the advantage of small size. There are other advantages to small format size that are not immediately apparent, but these advantages cannot be realized if the required film or imager resolution cannot be obtained.

Perspective

The concept of “correct” perspective leads to an insightful view of the optics used to produce digital or traditional photographic images. It has always been difficult to trade off lens focal length, format size, and film or imager resolution. The tradeoffs are straightforward when they are based on a system that has correct perspective. This discussion also justifies the pressure that drives photographic systems to shorter focal lengths and smaller film sizes (formats).

When the conditions for correct perspective are met it will be clear that a photographic system must at least match the angular resolution of the eye to have “good” resolution. The angular resolution of the eye, about 1 minute of arc (equivalent to 0.0003 radian, or 0.3 milliradian), is the angle that the smallest resolvable feature subtends on the eye, this angle is roughly constant at all viewing distances. The advantage of specifying angular resolution is that all photographic systems with the same resolution quality have the same angular resolution regardless of format size.

In addition, the depth of field and ultimate resolution of photographic systems are strictly a function of aperture size when defined in terms of angular resolution. The angular resolution defines the smallest aperture size the system can have based on diffraction. The largest aperture size is limited by the required depth of field. Given a resolution and depth of field specification, the lens focal length and format size can be adjusted until the desired system sensitivity is reached.

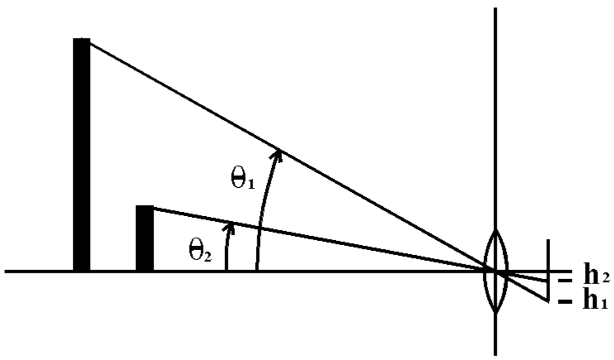

Figure 1. Unit angular magnification

To reproduce the perspective of a scene, the angles the objects in the scene subtend on the eye have to be reproduced.[1] Figure 2 shows this geometry. If a contact print of a negative is viewed from the lens location (one focal length from the print), the image of the objects with height h_1 and h_2, will subtend the same angle, θ_1 and θ_2, on your eye that they subtended in the original scene. Figure 2 also shows that the same geometry can be demonstrated by reflecting the image plane into object space through the plane of the lens. Figure 2 also shows that if the image is enlarged the scene angles will be reproduced when the scene is viewed from a distance equal to the print magnification times the lens focal length.

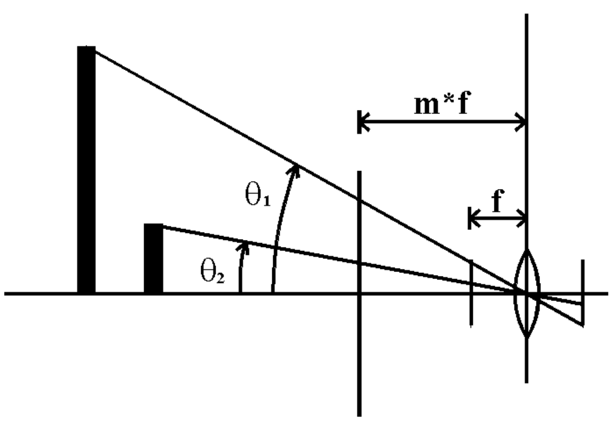

Figure 2. Print magnification

It is common for the "normal" focal length lens for a given format to equal the diagonal of the format. It is also common to view prints from a distance a little more than the diagonal of the print.[2] If a negative made with a normal lens is enlarged and not cropped, and the print is viewed from a distance about equal to its diagonal, then the print is being viewed from a distance equal to the print magnification times the lens focal length. This leads to "correct" perspective, sometimes called unit angular magnification, as defined above. These rules are not hard and fast, photography is an art. Technically correct perspective does not always produce the most effective print, but deviations can be compared to correct perspective.

Angular Resolution

Why is a discussion of perspective important to a discussion of system resolution? If a print is always viewed so that the perspective is correct then the system resolution, right to the resolution of the print, should have at least the same angular resolution as the eye. Figure 3 shows that the required angular resolution is the same regardless of format size. Given the angular resolution Δ, all of the format sizes, A to E, have the same required angular resolution, even though the spot size required for format E is much smaller than the spot size requirement for format A.

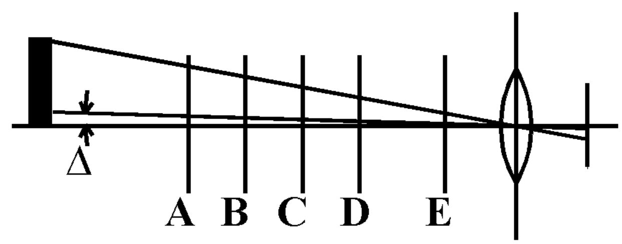

Figure 3. Angular magnification

Angular Resolution–Diffraction

If a point object is imaged by a perfectly corrected lens, the image is not the infinitely small point shown at the top of Fig. 4. Diffraction at the lens aperture produces a finite size spot, called an Airy disk, as shown at the bottom of Fig. 4. The Airy disk is a central spot surrounded by alternating dark and light rings. The central spot is small, about 5 microns for an f/4 lens. The Airy disk is the point-spread function for the lens. The central spot size (d) (diameter of the first dark ring) due to diffraction is a function of wavelength (λ) and effective f/# (Ne), f/# is the focal length/lens aperture.

$$d = 2.44 * \lambda * Ne \qquad (1)$$

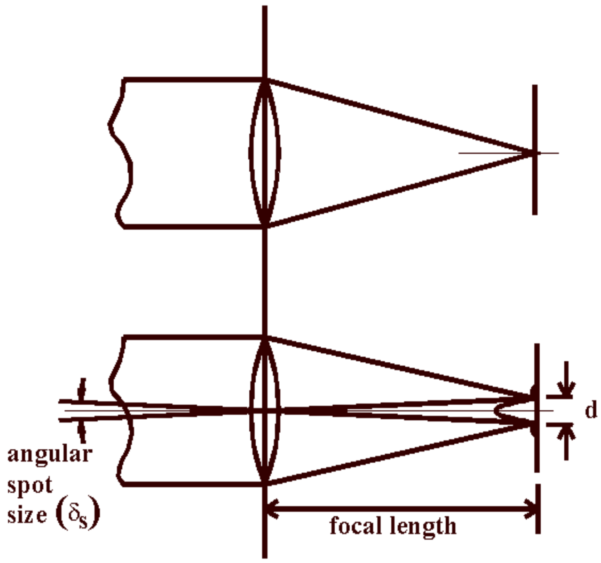

Figure 4. Angular resolution

The angular spot size (δ_s) in radians is the diameter of the spot divided by the focal length of the lens. The focal length in the divisor cancels the focal length in the f/# so δ_s is strictly a function of aperture size.

$$\delta s = \frac{2.44 * \lambda}{a} \tag{2}$$

This relationship shows that once the angular spot size is specified the minimum lens aperture diameter has also been specified. The smallest resolvable feature is about 1/3 the Airy disk width so the smallest resolvable angle (SRA) is 1/3 the angular spot size, which leads to Eq. 3.

$$SRA = 0.813 * \frac{\lambda}{a} \tag{3}$$

Eye

The eye is diffraction limited at about f/8, its focal length is effectively about 17 mm, so this sets the minimum aperture size at 2.125 mm for any system that should have at least the resolution of the eye. This can be a problem for short focal length lenses when light levels are high. A 6 mm lens can't be stopped down past about f/3 (6 mm focal length/2.125 mm aperture) and still have the resolution of the eye. The absolute lower limit for pixel size can be determined from these parameters. The largest relative aperture (smallest f/#) that a lens can have is f/0.5. A f/0.5 lens with a 2.125 mm aperture has a focal length of 1.06 mm. The point-spread function for this lens is 0.6 microns across the central spot; the smallest pixel size should be 1/2 or 1/3 this width or 0.2 to 0.3 microns. At 1/3 the central spot diameter the MTF at the Nyquist frequency because of the point-spread function is about 20%. The fastest lens that can reasonably be built in production is probably f/1.4. This leads to a 3 mm focal length, and 1.8 micron spot size, which makes the smallest reasonable pixel size 0.6 to 0.9 micron at 1/3 to 1/2 the lens point-spread function diameter. To look at this another way, the normal lens will have a horizontal angle of view of about 40 degrees for a 4:5 aspect ratio print. The eye has a resolution of 1 minute of arc so there should be 2400 pixels in 40 degrees. Each pixel should subtend 1 minute of arc. The 3 mm lens discussed previously has to have 0.87 micron pixels for each pixel to subtend 1 minute of arc, which agrees fairly well with the previous argument based on 1/2 the Airy disk size.

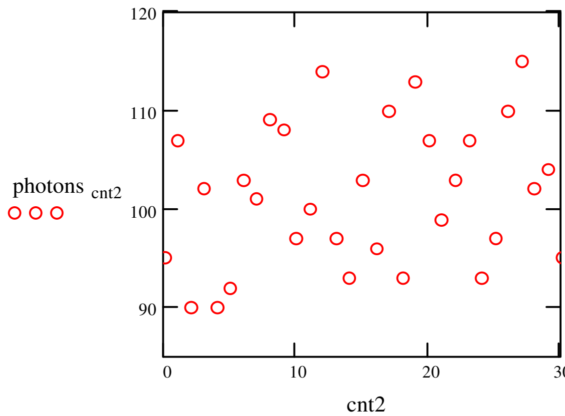

Figure 5. Poisson Distribution

Quantum Effects

Low-light level and high ISO speed images are noisy due to quantum effects [3]. Each place in the scene is not illuminated uniformly by the source. The uniformity over a given time period (like the exposure time) is a function of the average number of photons falling on the surface in the time period. Each rod or cone on the retina, each grain of a photographic emulsion, and each pixel of an electronic imager collects a finite number of photons during the exposure time. The pixel-to-pixel variation for a uniformly reflecting surface will be Poisson distributed. The standard deviation for a Poisson distribution is the square root of the average value, provided the average value is greater than approximately 30. An example of a Poisson distribution is shown below. The average value for this distribution is 100. The signal-to-noise is defined as the average value divided by the standard deviation. In this case, the signal-to-noise is 10 [(average value)/(standard deviation)]. This value is quite low based on the current ISO standard for a signal-to-noise-based speed. The standard calls for a S/N, in an 18% gray, of 40 for high-quality images and 10 for just acceptable images. Adjacent pixels, 30 in this case, could have the distribution of values shown for a uniform (flat field) exposure. A signal-to-noise of 10 corresponds to

100 photons and a signal-to-noise of 40 corresponds with 1600 photons.

The quantum limitation on signal-to-noise ratio can be used to determine the minimum pixel size for given set of specifications. A single equation can be derived for the number of photons/pixel. The signal-to-noise ratio is just the square root of this equation. First, the number of photons per lumen has to be determined. The spectral distribution of a 5500 K source is scaled to produce one lumen using the Eq. 4 below. P[λ] is the source spectral power distribution in watts/nm scaled to make the flux F in lumens equal to 1 lumen. V[λ] is the visual response function. λ1 and λ2 are the limits of integration and are 350 nm and 750 nm respectively because the visual response function is very small outside these limits.

$$F = 680 * \int_{\lambda 1}^{\lambda 2} V[\lambda] * P[\lambda] * d\lambda \tag{4}$$

The energy in each photon is a function of wavelength and can be determined using the equation:

$$e = h\nu \tag{5}$$

h = planks constant $6.62517*10^{-34}$ joule-sec
ν = frequency of the light = c/λ
where c is $3*10^{17}$, the speed of light in nm/sec
λ is wavelength in nm

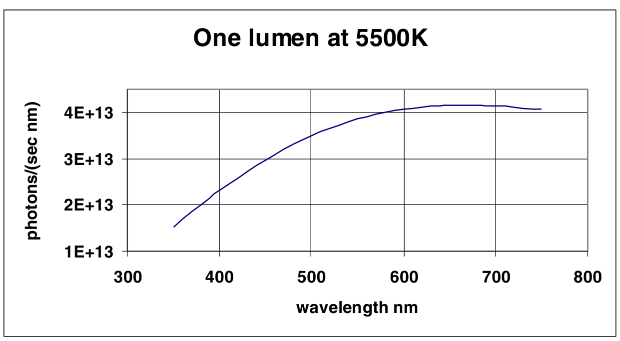

Figure 6. Photons/sec-nm at 1 lumen

The graph above shows the number of photons/(second nm) for a 1 lumen 5500 Kelvin source. Integrating this curve over wavelength from 400 to 700 nm produces the number of photons in a lumen at 5500 Kelvin. The final result is $1.1*10^{16}$ photons/lumen sec. On a per color channel basis there are $2.9*10^{15}$ photons from 400-500 nm, $3.8*10^{15}$ photons from 510-600 nm, $4.5*10^{15}$ photons from 610-700 nm.

The number of photons per pixel can be determined using this data and the image plane illumination. The image plane illumination is a function of the image conjugate f/#, the object illumination and the object reflectance. The equation relating these factors is:

$$Ei = \frac{Eo * Ro}{4 * Ne^2} \tag{6}$$

where, Ei = image plane illumination, Eo = object plane illumination, Ne = effective image plane f/#, Ro = object reflectance. The factor Ne can be replaced with Te for the effective t number, which includes correction for the lens transmission, Te is just Ne divided by the square root of lens transmission. The absolute physical limit for the f/# (Ne) in air is f/0.5. At f/0.5 the image plane illumination equals the object plane illumination times the object reflectance.

Adding a factor, kppls, to convert lumens per meter square to photons per meter square per second. There are about 1.1 x 10^{16} photons in a lumen second so kppls = 1.1 x 10^{16} photons/(lumen second) and ppsm2 = photons/second meter^2.

$$ppsm2 = Ei * kppls = \frac{Eo * Ro * kppls}{4 * Ne^2} \tag{7}$$

Finally, to get to photons/pixel (ppp) where a pixel can be any light sensitive element in an array, like a single photo receptor in the eye, a grain in a silver halide photographic material or a pixel in an electronic array. The exposure time has to be included and the area of the light sensitive element and the quantum yield has to be included to get the effective number of photons per pixel. Ap = pixel area in meters^2, exp = exposure time in seconds, ppp = photons/pixel, QY = quantum yield.

$$effective\_photons/pixel(ppp) = ppsm2 * Ap * \exp * QY$$

$$= \frac{Eo * Ro * kppls * Ap * \exp * QY}{4 * Ne^2}$$

The signal-to-noise ratio is:

$$SN = \frac{signal}{signal\_standard\_deviation}$$

$$= \frac{photons/pixel}{\sqrt{photons/pixel}} = \sqrt{photons/pixel} \tag{9}$$

finally:

$$SN = \sqrt{\frac{Eo * Ro * kppls * Ap * \exp * QY}{4 * Ne^2}} \tag{10}$$

This equation can be solved for pixel size given the other parameters. To simplify the calculation the number of photons/sec nm is assumed the same for all wavelengths. This simplifies the calculation but adds some error. As an example, given a signal-to-noise of 40:1, illumination of 50 lux, 18% reflectance, and an exposure time of 1/50 second, a quantum yield of 0.2 (including color filter array, CFA) and an f/1.4 lens, the minimum pixel size is about 6 microns square. The current ISO speed standard specifies the signal-to-noise at 18% gray.

Another way of looking at this problem is shown in Table 1. The number of photons collected by pixels 1 to 12 microns square is tabulated at various ISO speed values. To start building the table the number of effective photons per lumen second for each channel has to be calculated. First, the spectral power distribution of a 5500 Kelvin 1lumen source is calculated using Eq. 4 above. This source is cascaded with the spectral transmission of any filtration in the system, the spectral transmission of each CFA channel, and the spectral quantum efficiency of the monochrome sensor. For a perfect sensor, 100% quantum efficiency, and 100%CFA transmission, the effective photons/(lumen second) for the red, green, and blue channel are in rows 5 to 7 of the first column of Table 1. For cyan, magenta, yellow (CMY) CFA systems the corresponding channels can be added together to determine the number of photons per CMY channel pixel. The exposure (E) in lux seconds because of an 18% gray for ISO values from 25 to 3200 are shown in the third row of Table 1. These are derived using the Eq. 11, where ISO is the ISO speed. Multiplying the effective photons/(lumen-second) times the exposure in lux-seconds for and 18% gray at each ISO produces the number of effective photons per meter^2 falling on the sensor for each channel (top row). Multiplying these values by the pixel area yields the number of effective photons collected by each pixel size (down columns). The square root of these numbers is the signal-to-noise. Values greater than 1600 have a signal-to-noise better than 40:1 and values less than 100 have a signal-to-noise less than 10:1. The values shown are a limit that can't be exceeded because they are based on 100% quantum efficiency, and 100% system transmission. Typical real systems achieve about 15% of these values.

$$E = \frac{10}{ISO} \tag{11}$$

Conclusion

Based on physical optics, the smallest focal length lens that has the resolution of the eye is about 1 mm focal length at f/0.5 and the smallest pixel is 0.2 micron, about 1/3 of the point-spread function for this lens.

The advantages of short focal lengths drive photographic systems to small formats like disk format and Advanced Photo System format, or to very small imagers in electronic cameras. Small imagers are less expensive because more small imagers fit on a wafer and the cost of processing a wafer is fairly constant regardless of the number of devices on the wafer. In addition, small imager or film sizes lead to convenient small cameras with high system speed, which seems like a win-win situation, but the camera manufacturer pays with tight manufacturing tolerances and difficult to manufacture small f/# lenses.

Quantum noise is another limitation on pixel size. For a given ISO speed the pixel area determines the number of photons that a photosite collects. The highest possible signal-to-noise, disregarding all other noise sources, is the square root of this value. Current systems, with small pixel sizes are approaching quantum limitations when high ISO speeds are selected. Pixels smaller than about 2 microns square can never achieve good signal-to-noise at very moderate ISO speeds even if 100% quantum yield is achieved.

References

1. Rudolf Kingslake, Lenses in Photography, Garden City Books, Garden City, New York, 1951, p. 1-25.
2. Richard B. Wheeler, Brian W. Keelan, U.S. Patent 5,323,204, 1994.
3. Albert Rose, Vision, human and electronic, Plenum Press, New York, 1973.

Biography

Russ Palum received a BS and MS in photographic science in 1979, and an MS in electrical engineering in 1988, both from the Rochester Institute of Technology. He joined Eastman Kodak Company in 1977 and has worked on process development for molded glass optics, asphere metrology, scanner light source design, lens design, antialiasing filter design and most recently image data path software for small CMOS image arrays.

Table 1

			Assuming a quantum yield of 1 and a CFA and system transmission of 1							
	ISO speed		25	50	100	200	400	800	1600	3200
(effective photons/sec) /lumen	lux seconds for an 18% gray exposure		0.4	0.2	0.1	0.05	0.025	0.0125	0.00625	0.003125
4.54686E+15	red photons/m^2		1.819E+15	9.094E+14	4.547E+14	2.273E+14	1.137E+14	5.684E+13	2.842E+13	1.421E+13
3.79969E+15	green photons/m^2		1.520E+15	7.599E+14	3.800E+14	1.900E+14	9.499E+13	4.750E+13	2.375E+13	1.187E+13
2.88425E+15	blue photons/m^2		1.154E+15	5.768E+14	2.884E+14	1.442E+14	7.211E+13	3.605E+13	1.803E+13	9.013E+12
pixel size in microns	area in meters^2									
1	red	1E-12	1818.7	909.4	454.7	227.3	113.7	56.8	28.4	14.2
	green		1519.9	759.9	380.0	190.0	95.0	47.5	23.7	11.9
	blue		1153.7	576.8	288.4	144.2	72.1	36.1	18.0	9.0
2	red	4E-12	7275.0	3637.5	1818.7	909.4	454.7	227.3	113.7	56.8
	green		6079.5	3039.8	1519.9	759.9	380.0	190.0	95.0	47.5
	blue		4614.8	2307.4	1153.7	576.8	288.4	144.2	72.1	36.1
3	red	9E-12	16368.7	8184.4	4092.2	2046.1	1023.0	511.5	255.8	127.9
	green		13678.9	6839.4	3419.7	1709.9	854.9	427.5	213.7	106.9
	blue		10383.3	5191.6	2595.8	1297.9	649.0	324.5	162.2	81.1
5	red	2.5E-11	45468.6	22734.3	11367.2	5683.6	2841.8	1420.9	710.4	355.2
	green		37996.9	18998.5	9499.2	4749.6	2374.8	1187.4	593.7	296.9
	blue		28842.5	14421.2	7210.6	3605.3	1802.7	901.3	450.7	225.3
7	red	4.9E-11	89118.6	44559.3	22279.6	11139.8	5569.9	2785.0	1392.5	696.2
	green		74474.0	37237.0	18618.5	9309.3	4654.6	2327.3	1163.7	581.8
	blue		56531.2	28265.6	14132.8	7066.4	3533.2	1766.6	883.3	441.7
9	red	8.1E-11	147318.4	73659.2	36829.6	18414.8	9207.4	4603.7	2301.9	1150.9
	green		123110.1	61555.0	30777.5	15388.8	7694.4	3847.2	1923.6	961.8
	blue		93449.6	46724.8	23362.4	11681.2	5840.6	2920.3	1460.1	730.1
10	red	1E-10	181874.6	90937.3	45468.6	22734.3	11367.2	5683.6	2841.8	1420.9
	green		151987.8	75993.9	37996.9	18998.5	9499.2	4749.6	2374.8	1187.4
	blue		115369.8	57684.9	28842.5	14421.2	7210.6	3605.3	1802.7	901.3
12	red	1.44E-10	261899.4	130949.7	65474.9	32737.4	16368.7	8184.4	4092.2	2046.1
	green		218862.4	109431.2	54715.6	27357.8	13678.9	6839.4	3419.7	1709.9
	blue		166132.6	83066.3	41533.1	20766.6	10383.3	5191.6	2595.8	1297.9