Evaluating Quality Factors of Hypothetical Spectral Sensitivities

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Abstract

It is well known that the spectral sensitivities (*SS*) for color imaging devices should satisfy the Luther condition which requires *SS* to be linear combinations of the CIE color matching functions. In practice, it is difficult to construct an *SS* exactly to meet this condition. Some quality factors such as q-factor for single *SS* by Neugebauer and μ-factor for a set of *SS* by Vora et al. were introduced in order to describe the deviation of *SS* from their nearest color mixture curves. In this paper, a simple method is introduced to implement an evaluation platform for the above two quality factors. A series of hypothetical spectral sensitivities are constructed with cubic spline functions with shape and peak position of the *SS* parametrically varied. The evaluation platform is used to optimize these *SS* parameters to obtain a maximum quality factor. Furthermore, the improvement of quality factor by adding a fourth *SS* is discussed in the paper as well.

Introduction

Capturing color images with digital camera is widely spreading. The principle of such a camera is usually a charge-coupled device or complementary metal-oxide-semiconductor (CCD/CMOS) sensor array with a set of filters before it. Human visual responses to color stimuli have been determined by psychophysical experiments and are officially recommended as color matching functions by the Commission Internationale de l'Eclairage (CIE). It characterizes spectral distributions of object colors by tristimulus values since the human eye has three types of cones with different spectral sensitivities. Most imaging systems are therefore set up with three channels and the device sensitivities are initially designed to mimic human visual system.

The spectral sensitivity evaluation and design problem has been studied before. Ohta started the evaluation and optimization of spectral sensitivities in subtractive color photography.[10,11] The spectral sensitivities for color imaging devices (digital cameras, color scanners etc.) should satisfy the Luther condition, that is, the spectral sensitivities need not be exact duplicates of the color-matching functions but need be only a nonsingular transformation of them. In practice, it is not always possible to manufacture filters that satisfy Luther condition due to the physical limitations of fabricating process. Measurement noise also plays an important role and will degrade the color accuracy even when spectral sensitivities fulfil Luther condition. A measure of goodness or quality factor for evaluating and designing spectral sensitivities for color imaging devices is therefore desirable.

The first quality factor, Q-factor, proposed by Neugebauer, is limited to the evaluation of single filter.[1] Lately, Vora and Trussell extended the quality factor to filter sets with an arbitrary number of filters.[2] This factor, μ-factor, describes the difference between the orthonormal subspaces of color matching functions and the spectral sensitivity space. These measures can all be related to a mean-squared error metric in CIEXYZ space. Recently, Wolski et al[5] proposed the use of local linearization of CIELAB space to reduce the computational complexity with preserving the desirable property of perceptual uniformity. Sharma and Trussell[4] presented a new figure of merit for color scanners, which is also based on an error metric in linearized CIELAB space but incorporates a model for measurement noise. It has high degree of perceptual relevance and also accounts for noise performance of different filters.

Tajima[3] proposed a totally new quality factor ("*T*-factor") without satisfying Luther condition by taking account of the object color spectral characteristics. His metric is based on that each object spectral characteristic can be restored from three sensor signals due to the fact that almost all object spectral reflectance can be reconstructed by three or four principal components. Then accurate tristimulus values (XYZ) are estimated from the restored object spectral reflectance and known color-matching functions.

It is still arguable that the spectra of object color can be satisfactorily restored by only three or four sensor measurements. Since most cameras use only three channels and colorimetric matching is still the goal of most imaging devices with tradeoff of cost, we will only discuss the q-factor and μ-factor in our paper. Furthermore, the approach used in the paper is methodically applicable when we consider some additional practical issues, such as noise, multi-illuminant color correction etc.

The higher the μ-factor for the imaging device, the more accurate color reproduction is expected. One approach to improve the color accuracy other than satisfying Luther condition with three channels is to use an increased number of color channels. As the number of color filters is increased, additional information about the object color is obtained, but cost and fabrication difficulty is also increased. Hence four-channel is a good tradeoff. Our paper demonstrated a method to compute the optimal transmittance of a fourth filter by maximizing the total μ-factor of the system dramatically.

In this paper, at first, we addressed the hypothetical spectral sensitivity to be used. The hypothetical spectral sensitivity function is modeled as smooth cubic spline functions with single peak, which is proposed first by Ohta [11]. We discussed the meaning of q-factor and μ-factor through least square approach and evaluated hypothetical spectral sensitivities by these criteria. We then optimized a fourth spectral sensitivity with constraints to maximize the μ-factor of the color imaging system.

In our paper, we use finite dimensional representations of all continuous spectral functions. All spectral distributions are sampled at 10*nm* intervals from 400*nm* to 700*nm* and represented as 31-element column vectors.

The Hypothetical Spectral Sensitivity

We define the spectral sensitivity of color imaging systems as the product of the spectral sensitivity of imaging device and the transmittance of the filter. The hypothetical spectral sensitivities formed by the combination of cubic spline functions were widely used by N. Ohta to simulate the practical spectral sensitivity in color photography.[11] In general, the spectral sensitivity is assumed to be a smooth single-peaked curve in visible range with nonnegative value of no more than one. The peak position and width vary considerably for real spectral sensitivities in color reproduction, however they can be simulated by a combination of smooth cubic spline functions for instance peak position at $\lambda=\lambda_0$ written as:

$$C(\lambda) = \begin{cases} \dfrac{w^3+3w^2(w-|\lambda-\lambda_0|)+3w(w-|\lambda-\lambda_0|)^2-3(w-|\lambda-\lambda_0|)^3}{6w^3} & |\lambda-\lambda_0|\le w \\ \dfrac{(2w-|\lambda-\lambda_0|)^3}{6w^3} & w\le|\lambda-\lambda_0|\le 2w \\ 0 & |\lambda-\lambda_0|\ge 2w \end{cases} \tag{1}$$

where $2w$ is the width of the cubic spline function. For example, Figure 1 shows a spectral sensitivity function whose peak locates at 550*nm* and width $2w$ is 80*nm*.

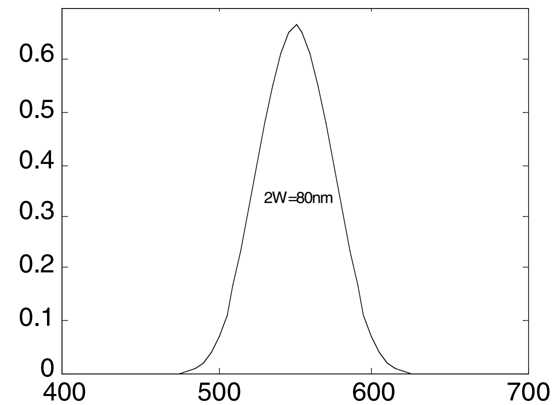

Figure 1. Typical hypothetical spectral sensitivity

Q-Factor of Spectral Sensitivity

In measuring colors a light sensitive receiver is required whose spectral response is equivalent to color matching functions (*cmf*). But in many cases, this is not usually fulfilled owing either to imperfections of color filters used or to other conflicting conditions imposed on the filters. In particular, it is difficult to construct designed scanning ss exactly, and any errors in the construction will change the space spanned by the spectral sensitivities, resulting in an error in the measurement of the expected projection. This error will lead to the error in the reproduction. This error will occur even if the measurements are noise free in all other respects. A compromise is required and it would be of great help to have a method of evaluating the deviation of a *ss* curve from the nearest *cmf.*

Let $\bar{x}(\lambda),\bar{y}(\lambda),\bar{z}(\lambda)$ be the CIE color matching functions, for convenience, we define A=[$\bar{x}(\lambda),\bar{y}(\lambda),\bar{z}(\lambda)$] as the human visual subspace (HVSS). In an attempt to measure the goodness of *SS, Neu*gebauer's q-factor for an *SS m* can be defined as following:

Assume m can be mostly approximated by the linear combination of *cmf*s (Af), where f is a 3×1 vector, that is,

$$\min\|Af-m\|_F^2$$

where the Frobenius norm

$$\|X\|_F = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}|x_{ij}|^2}$$

for matrix $X \in R^{m\times n}$ This is a least-square problem, and we can obtain $f=(A^TA)^{-1}A^Tm$ with pseudo inverse knowledge. Thus the residue

$$\Delta = \|Af - m\|_F^2 = (Af - m)^T (Af - m) = (f^T A^T - m^T)(Af - m)$$
$$= f^T A^T Af - m^T Af - f^T A^T m + m^T m$$
$$= m^T A(A^T A)^{-1} A^T A(A^T A)^{-1} A^T m - m^T A(A^T A)^{-1} A^T m$$
$$- m^T A(A^T A)^{-1} A^T m + m^T m$$
$$= m^T m - m^T A(A^T A)^{-1} A^T m$$

and

$$\frac{\Delta}{\|m\|^2} = \frac{m^T m - m^T A(A^T A)^{-1} A^T m}{m^T m} = 1 - \frac{m^T A(A^T A)^{-1} A^T m}{m^T m} = 1 - q(m)$$

Therefore,

$$q(m) = \frac{\|P_V(m)\|^2}{\|m\|^2} = \frac{\|A \cdot (A^T A)^{-1} A^T \cdot m\|^2}{\|m\|^2} = \frac{m^T A(A^T A)^{-1} A^T m}{m^T m} \quad (2)$$

which is Neugebauer's q factor of spectral sensitivity m.

As an extreme case, for example, when spectral sensitivity $m(i)$ is a δ-function peak at λ_0 belonging to ($400nm$, $700nm$), that is, $m(i) = (0,0,\ldots,1,\ldots0)^T$, where i is the position of 1 in the N-vector. We can obtain the corresponding q-factor as $\mathrm{diag}(A(A^TA)^{-1}A^T,i)$, which is the i^{th} diagonal element of the matrix $(A(A^TA)^{-1}A^T$. Figure 2 shows the q-factors of a series of spectral sensitivities $m(i)$ where i changes from 1 to N. The q-factor curve presents three peaks at about $450nm$, $540nm$ and $600nm$ with corresponding q-factors 0.2263, 0.1756 and 0.1858. Comparatively, the q-factor of full-pass spectral sensitivity $m(i) = (1,1,\ldots,1,\ldots,1)^T$ is about 0.7224.

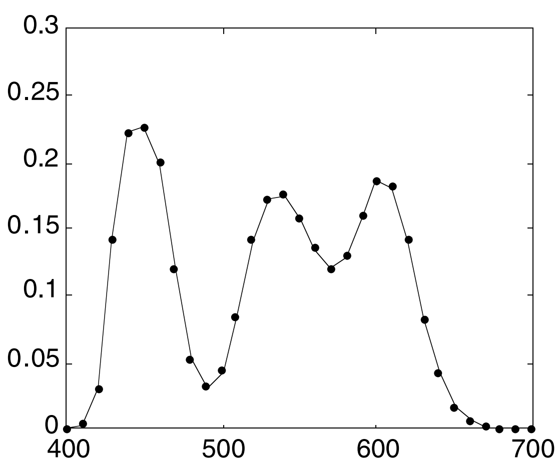

Figure 2. q-factor of δ-functions using CIE 2° *cmf*

Notice that $0 \le q(m) \le 1$, and the closer the value of $q(m)$ to unity, the better the color-scanning *ss m* performs in color reproduction. If the value of $q(m)$ is small compared with unity, the filter measurement does not give much information about the measured signal, and hence the *ss* is not appropriate for color scanning. The q-factor is a reasonable quality measure for spectral sensitivities not in the HVSS, because $\|m\|^2$ $(1 - q(m))$ is the square of the Euclidean distance of m from HVSS as we derived above.

Now we evaluate the q-factors of the hypothetical spectral sensitivity function. We let the peak position λ_0 of the cubic spline curve changes from $400nm$ to $700nm$ by 10nm (31 different positions), and the half width w changes from 10nm to 90nm by 20nm (5 different widths). In each combination, we calculated the corresponding q-factor.

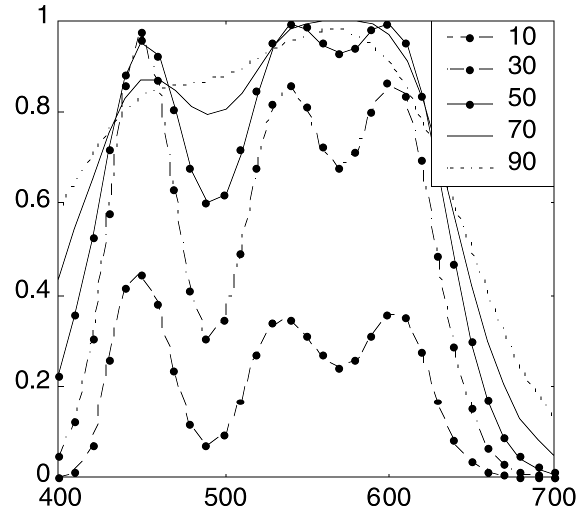

Figure 3. q factor of hypothetical spectral sensitivities with different peak λ_0 *and width 2w*

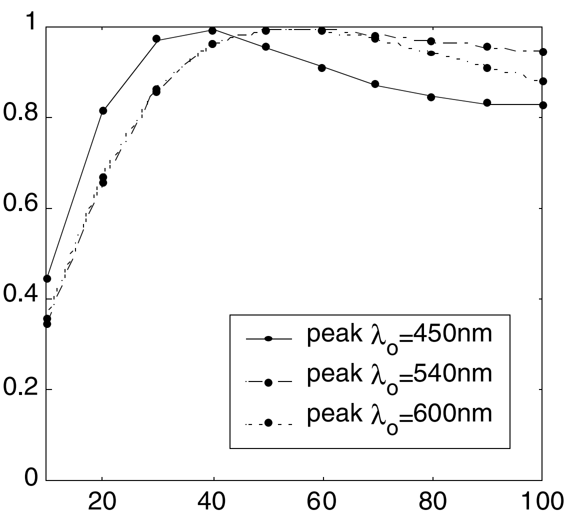

Figure 4. The effect of width on maximum q factor

Figure 3 shows how the q-factors change as we change the peak position λ_0 and the width variable w of the hypothetical spectral sensitivity. In the figure, when w is not so large, for example, $w \le 70nm$, each curve gives a series of varying q factors and there are 3 obvious peaks of q factors. The wavelength positions of SS with maximal q-factors are almost consistently located at $450nm$, $540nm$ and $600nm$.

In these wavelength positions, there exists some optimal width that maximizes the q-factor to near one. We can find the optimal width is about 40~60nm. Figure 4 shows how the maximum q-factor changes with width. When the width goes to large enough, say 100nm, the three peaks of q factor disappear and the curve becomes flat. On the contrary, the q-factor in the middle part of the curve is not very small but changes slowly along wavelength, so it is not optimal to choose very wide spectral sensitivity in color reproduction. In fact, when the width is small enough, it can be modeled as a δ-function, while it gets wide enough, it is a full-pass function. So the curve is very similar to Figure 2 when width is small and the curve becomes flat when it is large. In a limitation condition when $w\to\infty$, as SS is like full-pass spectral sensitivity, the q-factor at any position is about 0.7.

μ-Factor of a Set of Spectral Sensitivities

A major disadvantage of the q factor is that it was designed to evaluate only single SS. A measure that extends the idea of the q-factor to evaluate a set of color-scanning spectral sensitivities would be useful.

Another disadvantage of the q factor is that it cannot be used to evaluate set of more than three spectral sensitivities. Current trends show that more than three spectral sensitivities may be used to improve the quality of the color reproduction. First, in many cases, three parameters are not enough to define sufficiently the visual stimulus of an N-dimensional for color correction. Second, the constraint of feasibility on the spectral sensitivities might imply that no set of three feasible spectral sensitivities could span the HVSS, although a set of four feasible spectral sensitivities could be constructed so that the required projection would be obtained. When more than three parameters (four scanning spectral sensitivities, for example) are necessary, the q factor is not an effective measure of the goodness. For example, suppose that $\{s_1,s_2,s_3,s_4\}$ is a set of scanning spectral sensitivities. It is possible that the HVSS is contained in the span of the set of four spectral sensitivities, but $q(\mathbf{s}_i)<1$ for i=1,2,3,4. Such a system could provide perfect color scanning, although the individual q factor would not be high.

Let S denotes the matrix of r scanning spectral sensitivities, $S=[s_1\ s_2\ \ldots\ s_r]$. Let $A=[a_1\ a_2\ \ldots\ a_s]$ denote the human visual space (color matching functions) to be approximated. An orthonormal basis for A is defined by $U=[u_1\ u_2\ \ldots\ u_\alpha]$. Such a basis may be obtained by the Gram-Schmidt orthogonalization procedure. The number of orthonormal vectors, α, is the rank of A and α equals s if A is a linearly independent set. Similarly, an orthonormal basis for S is defined by $O=[o_1\ o_2\ \ldots\ o_\beta]$. Also notice that β is the rank of S and that β equals r if S is linearly independent set. The orthonormal basis U and O need not represent realizable spectral sensitivities. It can be derived that $S(S^TS)^{-1}S^T = OO^T$ and $A(A^TA)^{-1}A^T = UU^T$.[6]

Our purpose is to approximate A by the linear combination of S, that is, to minimize $\|A - SQ\|_F^{\ 2}$, where Q is the variable matrix to be optimized. This is a least-square issue as well. Similarly, we can obtain $Q = (S^TS)^{-1}S^TA$ by pseudo inverse operation. And the residue:

$$\mu_U(S) = \frac{\sum_{i=1}^{\beta} q(o_i)}{\alpha} = \mu_U(O)$$

Therefore,

$$\frac{\Delta}{\|A\|_F^2} = \frac{Trace\{A^TA\} - Trace\{A^TS(S^TS)^{-1}S^TA\}}{Trace\{A^TA\}}$$
$$= 1 - \frac{Trace\{A^TS(S^TS)^{-1}S^TA\}}{Trace\{A^TA\}}$$
$$= 1 - \mu_A(S)$$

where

$$\mu_A(S) = \frac{Trace\{A^TS(S^TS)^{-1}S^TA\}}{Trace\{A^TA\}} \tag{3}$$

is the goodness measure of a set of ss S against A, and $Trace\{X\}$ is the sum of diagonal elements of X. When we use the orthonormal vectors U instead of A, we have

$$\begin{aligned} &Trace\{U^TU\} = Trace\{I_\alpha\} = \alpha, \\ &Trace\{U^TS(S^TS)^{-1}S^TU\} = Trace\{U^TOO^TU\} \\ &= \|O^TU\|_F^2 = \|U^TO\|_F^2 = Trace\{O^TUU^TO\} \\ &= \sum_{i=1}^{\beta} o_i^TUU^To_i = \sum_{i=1}^{\beta} q(o_i) \end{aligned} \tag{4}$$

so

$$\mu_U(S) = \frac{Trace\{O^TUU^TO\}}{\alpha} = \frac{\sum_{i=1}^{\beta} q(o_i)}{\alpha} \to \mu_U(O) \tag{5}$$

which is the definition of μ-factor for a set of SS [2].

This equation can be rewritten as:

$$\begin{aligned} \mu_U(S) &= \frac{Trace\{O^TUU^TO\}}{\alpha} = \frac{Trace\{S^TUU^TS\cdot(S^TS)^{-1}\}}{\alpha} \\ &= \frac{Trace\{S^TA(A^TA)^{-1}A^TS\cdot(S^TS)^{-1}\}}{\alpha} \end{aligned} \tag{6}$$

In this equation, $Q_S = S^TUU^TS$ is the q-factor matrix (diagonal elements are q-factors of original ss, off-diagonal elements are inter-product pseudo q-factors), $\rho = S^TS$ is the correlation between the original spectral sensitivities. The operation $\rho^{-1} = (S^TS)^{-1}$ is a de-correlation process, that is, it remove the correlation between the set of spectral sensitivies, and obtain a "pure" uncorrelated (orthonormal) ss to calculate the goodness measure. Hence, we see that

$$\sum_{i=1}^{r} q(m_i) \text{ cannot be used instead of } \sum_{i=1}^{\alpha} q(o_i)$$

as a measure because spectral sensitivities with high value of correlation

$$\frac{\left|s_i^T s_j\right|}{\left|s_i\right| \cdot \left|s_j\right|}$$

for $i \neq j$ may have high q-factors but poor joint performance. Ensuring that the spectral sensitivities O are orthogonal removes the correlation effect

$$\frac{\left|s_i^T s_j\right|}{\left|s_i\right| \cdot \left|s_j\right|}$$

for $i \neq j$.

Table 1. Peak positions of *ss* with maximal μ-factor at different width

Width (*nm*)	*μ-factor*	*Blue ss peak* (*nm*)	*green ss peak* (*nm*)	*red ss peak* (*nm*)
10	0.380	450	540	600
20	0.713	450	540	600
30	0.895	450	540	600
40	0.965	450	540	600
50	0.978	450	540	600
60	0.963	450	540	600
70	0.935	460	540	600
80	0.901	470	530	600
90	0.865	490	510	600
100	0.824	490	510	600

Evaluation on a Set of Hypothetical Spectral Sensitivities with μ-Factor

Now we employ the aforementioned hypothetical spectral sensitivities to evaluate their μ-factor. There are three spectral sensitivities denoted as R, G and B with peaks λ_0 locating at 600-700*nm*, 500-600*nm* and 400-500*nm* by 10nm individually. Considering their mutual combinations, we have totally 11^3=1331 sets of spectral sensitivities. We also vary their width 2*w* so as to check its influence on the measure of goodness. We let *w* changes from 10*nm* to 100*nm* by 10*nm*, which generates totally 10×1331 combinations to be verified. We can obtain three peak positions of R, G and B with maximal μ-factor among the 1331 indexed combinations for each width. We found the maximal μ-factor locates almost always at coded index=50, that is, the corresponding peak position of R spectral sensitivity function is at 600*nm*, that of G spectral sensitivity function is at 540*nm*, and that of B spectral sensitivity function is at 450*nm* (Table 1). This result is consistent with the properties of *q* factor of a series of spectral sensitivities. Three spectral sensitivities with high *q*-factors own high μ-factor if they are uncorrelated as possible as they can. Some second smaller peaks give other combinations of spectral sensitivities that have comparatively high μ-factors, but these peaks are very close to the above three principle peaks, for example, {610*nm*, 530*nm*, 460*nm*} etc.

As we noted above, μ-factor is the sum of *q*-factor of orthogonal sensitivities. The width of the spectral sensitivities affects their μ-factor. There exists an optimal width for the maximum μ-factor when peak positions are fixed. Very interestingly, here again (Figure 5), the optimal width w=50*nm* with corresponding μ-factor=0.9779.

When the peak positions and widths of two spectral sensitivities are fixed, and only one *SS* changes its peak position, how does the μ-factor change? Since μ-factor is an extension of *q* factor, the peak position should be consistent with that of *q* factor. Figure 6 shows that it's correct. We let one *SS* change its λ_0, say, 400-500*nm* by 10*nm*, the other two fix their peak positions and width when we obtain the maximum μ-factor (540*nm*, 600*nm*, and width=50*nm*). These figures graphically describe the behavior of μ factor of changing single *SS* just like that of *q*-factor. The peak positions of *SS* with maximum μ-factor locate at about λ_0 = 450*nm*, 530~540*nm*, and 600~610*nm*.

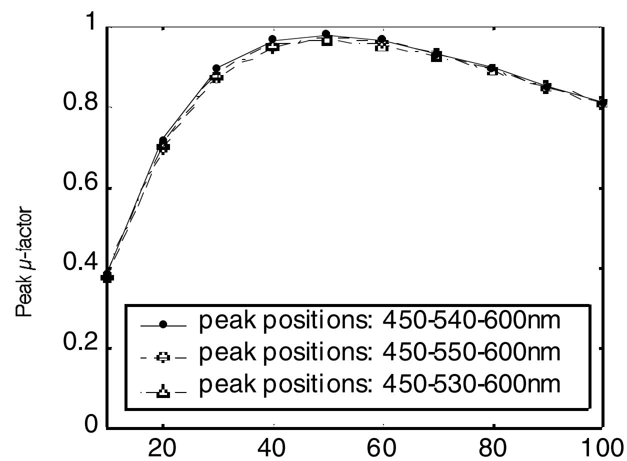

Figure 5. Effect of width of SS on peak μ-factor Optimal width is about w=45-55nm

More Discussion on μ-Factor

It is expected that the color filters and its number affect the accuracy of recording an original image. The use of more than three filters in the recording process is an alternative approach when three filters cannot span the human visual space effectively (low μ-factor) because of cost or manufacturing difficulty. The following simulated example demonstrates how a fourth filter dramatically improve the μ-factor of a camera. The three hypothetical spectral sensitivities have width 50*nm* with peak positions 650*nm*, 550*nm* and 450*nm* individually. Its μ-factor is 0.742. We let the fourth hypothetical spectral sensitivity changes peak positions from 400*nm* to 700*nm* by 10*nm*, and width from 10*nm* to 100*nm* by 10*nm*, there are 310 combinations. We find the maximal μ-factor of the four-channel system is 0.973, and the fourth filter has width of 60*nm* and peak position of 590*nm*. The corresponding *q*-factors of the four spectral sensitivities are 0.953, 0.982, 0.297 and 0.997 respectively.

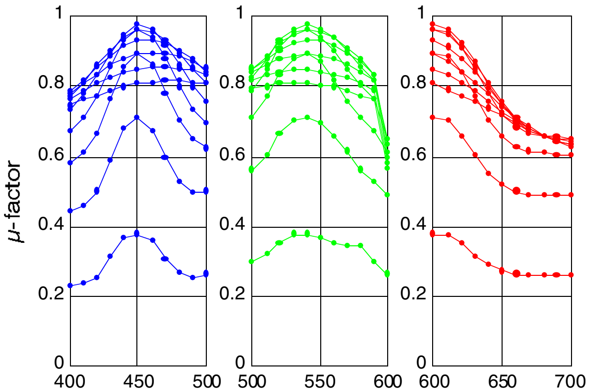

Figure 6. μ-factor of a set of ss: one ss changes peak λ_0, the other two ss fix peak λ_0, that is, change one of (450nm, 540nm, 600nm) by 10nm and vary width from 10nm to 100nm by 10nm

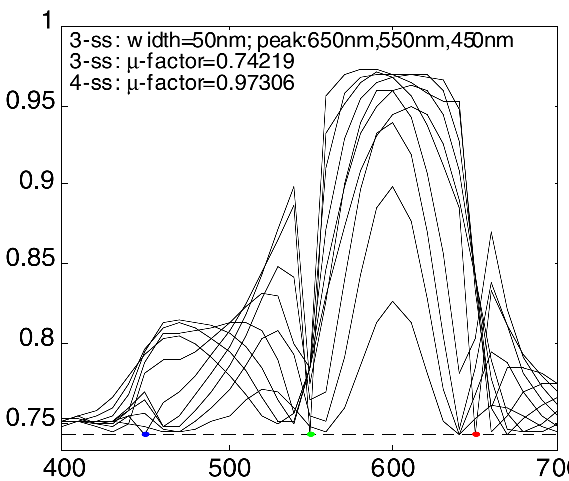

Figure 7. Chooing a fourth spectral sensitivity

Since μ-factor is not based on a perceptually uniform color space, such as $L^*a^*b^*$, a high μ-factor doesn't always lead to a small $L^*a^*b^*$ error. But the average $L^*a^*b^*$ error over an ensemble of reflectance is usually highly correlated with the μ-factor of the camera system.[2] On the other hand, some research indicates that the condition μ-factor≈1 is a sufficient condition but not a necessary one for color reproduction,[3] therefore some color imaging devices with poor μ-factor can generate a good image reproduction. Another issue of μ-factor is that it doesn't consider the measurement noise, which exists in real world and may contaminate the output signal thus lead to a big color difference in measurement. And finally, with the development of multi-spectral imaging system, a measure of goodness to evaluate its quality is desired.[3]

Conclusion

The metrics of goodness of spectral sensitivities including Neugebauer's *q*-factor for single *SS* and Vora et al's μ-factor for a set of *SS* were analyzed in the paper based on least square approach. Hypothetical spectral sensitivities with peak position and width varied were evaluated on these criteria. The disadvantage of *q*-factor has been overcome by μ-factor. But the latter has the disadvantage of without considering practical issues such as measurement noise and it is not based on a perceptual uniform color space.

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Biography

Shuxue Quan received his B.S. and M.S. degree in Optical Engineering from Beijing Institute of Technology in 1994 and 1997. Since 1997 he has been a Ph.D. candidate in Imaging Science at Rochester Institute of Technology (RIT). His work primarily focuses on the design and optimization of spectral sensitivities for color imaging systems.

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