Light Scattering and Ink Penetration Effects on Tone Reproduction

Li Yang, Björn Kruse and Reiner Lenz
Institute of Science and Technology, Linkping University,
S-601 74 Norrköping, Sweden

Abstract

We develop a framework to account for the effects of light scattering (or Yule-Nielsen effect) and ink penetration on the reflectance and tristimulus values of a halftone sample. We derive explicit expressions for reflectance values of ink dots (R_i), bare paper (R_p), the halftone image (R) and the optical dot gain Δf as functions of properties of materials (paper and ink) and their bilateral interaction. It is predicted that light scattering extends the color gamut of the printed images. The present approach and conclusions are appliable to both AM and FM halftoning schemes.

1. Introduction

Optical and rheological properties of ink, substrate paper and their mutual interaction play important roles in color reproduction. The attempt to explain the optical and color appearences of the printed image in terms of these properties led in the late 30's to the Murray-Davis formula [1] and the Neugebauer Equations [2]. These equations were soon found to be inaccurate in describing experimental results. In 1951 Yule and Nielsen [3] interpreted the discrepancy as the result of light penetration and scattering in the paper substrate, which is now known as the Yule-Nielsen effect. This led to a modification of the Murray-Davis formula which was replaced by

$$R = [R_i^{\frac{1}{n}} f + R_p^{\frac{1}{n}} (1-f)]^n \tag{1}$$

The exponent n is usually obtained by fitting the experimental data (such as optical density). In 1978 Ruchdeschel and Hauser [4] obtained an estimation of the exponent n in terms of point spread function (PSF) describing the scattering of light in paper. The study showed that $1 \leq n \leq 2$. However there is experimental evidence showing many exceptions where $n \geq 2$. In addition, both theoretical and experimental studies have pointed out that the exponent n itself may depend on f, especially in the case of $f > 50\%$.

Recently, the Yule-Nielsen effect has been further studied using numerical, probability approach and analytical methods. Gustavson [5, 6] investigated the Yule-Nielsen effect by direct numerical simulation of the scattering events. Based on a PSF approach, Rogers [7, 8] proposed a matrix approach where the tristimulus values of a halftone image are calculated as the trace of a product of two matrices. In addition, Arney et. al. [9, 10] extended the probability model which was originally introduced by Huntsman [11] to account for the optical dot gain. Similar model has also been reported by Hübler [12]. Nevertheless the effect of ink penetration was approximated roughly which sometimes leads to results not matching expectations [13].

Very recently we established a theoretical approach to account for the effect of ink penetration in the system that the substrate paper has been uniformly covered by ink layers [14]. In the present paper we moved a step forward to study halftone images where light scattering exists coincidently with ink penetration.

2. Model and Methodology

The basic geometry used is shown in Fig. 1 where the surface of the substrate paper has been divided into two sets, Σ_1 the paper under the dots (or ink penetrated paper), and, Σ_2 the bare paper. For simplicity, only the case of a single layer of dots is analyzed. Extension to a multilayer system is straight forward but tedious and therefore will be reported elsewhere. Also for simplicity, we assume that the ink layer has uniform thickness.

2.1. Point Spread Function Approach

Consider an element light source $I_0 d\sigma_1$ that strikes the dot at $\vec{r_1} \in \Sigma_1$, the flux of light detected at $\vec{r_2} \in \Sigma_2$ due to scattering of the incident light from $\vec{r_1}$ may be written as

$$d^2 J_{12} = p(\vec{r_1}, \vec{r_2}) T I_0 d\sigma_1 d\sigma_2 \tag{2}$$

The transmittance of the ink is T and $T I_0 d\sigma_1$ is thus the amount of light entering the substrate under the dot. The PSF, $p(\vec{r_1}, \vec{r_2})$, is the probability that photons enter the paper under the dot at position $\vec{r_1}$ and exit from the bare paper at position $\vec{r_2}$. The total amount of light that enetrs Σ_1 and is scattered into Σ_2 may be written as:

$$J_{12} = I_0 T \int_{\Sigma_1} \int_{\Sigma_2} p(\vec{r_1}, \vec{r_2}) d\sigma_1 d\sigma_2 \tag{3}$$

The double integral in Eq. (3) is the overall probability of photons scattered from Σ_1 into Σ_2. This is therefore a measure of the Yule-Nielsen effect.

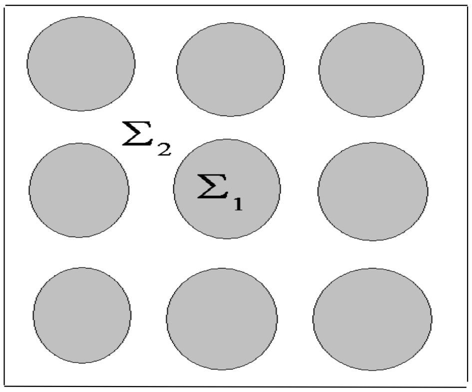


Figure 1: Overview over the halftone image. The surface is subdivided into two groups: points under ink dots (Σ_1) and between dots (Σ_2)

If one exchanges the position of the lighting source with that of the detector by puting the light source $I_0 d\sigma_1$ at $\overrightarrow{r_2}$ and puting the detector at $\overrightarrow{r_1}$, and keeps other conditions unchanged. Then one gets

$$d^2 J_{21} = p(\overrightarrow{r_2}, \overrightarrow{r_1}) T I_0 d\sigma_1 d\sigma_2 \tag{4}$$

From the optical reciprocity one obtains the following relation,

$$p(\overrightarrow{r_1}, \overrightarrow{r_2}) = p(\overrightarrow{r_2}, \overrightarrow{r_1}) \tag{5}$$

and therefore

$$J_{12} = J_{21} \tag{6}$$

It means that under the uniform illumination onto the whole halftone sample, the amount of light being scattered from Σ_1 (halftone dots) into Σ_2 (bare paper) is equal to that of the light scattered from Σ_2 into Σ_1. Eq. (3) can be further expressed as,

$$J_{21} = J_{12} = I_0 T \overline{p} f(1-f) \tag{7}$$

where $\overline{p}$ is the mean value of the integrated PSF defined as

$$\overline{p} = \frac{1}{f(1-f)} \int_{\Sigma_1} \int_{\Sigma_2} p(\overrightarrow{r_1}, \overrightarrow{r_2}) d\sigma_1 d\sigma_2 \tag{8}$$

Evidently $\overline{p}$ depends not only on the physical properties of the substrate paper and the ink, but also on the geometric and spatial distribution of the dots. As shown later on, $\overline{p}$ is closely related with the optical gain, and are therefore experimentally measurable.

2.2. Probability Approach

Light propagartion can be formulated in another way ie. by the so-called probability approach. If a photon enters the surface of the paper under the dot (Σ_1 in), we define P_{11} as the probability that it returns the surface under the dots (Σ_1 out. Note it is not necessarily to be the same dot as it enters), and P_{12} as the probability that it leaves the surface of the paper between the dots (Σ_2 out). Similarly we define P_{21} and P_{22} as propabilities that a photon enters the surface from paper between the dots (Σ_2 in) and then exits the surface under the dots (Σ_1 out) and between the dots (Σ_2 out), respectively. The probabilities fulfil the following constrain conditions[15]

$$\begin{aligned} P_{11} + P_{12} &= R_i^0 && (9) \\ P_{21} + P_{22} &= R_p^0 && (10) \end{aligned}$$

where R_i^0 is the reflectance of the paper under the dots (ink penetrated paper) and R_p^0 that of bare paper. In the case of no ink penetration, there is $R_i^0 = R_p^0$.

If the percentages of ink dots and the bare paper are f and $(1-f)$, respectively, and if the intensity of irradiance onto the whole system is I_0, the flux of photons striking the Σ_1 and Σ_2 areas are $I_0 f$ and $I_0(1-f)$, respectively.

Then the flux J_{ij} of photons entering Σ_i and then leaving Σ_j $(i, j = 1, 2)$ may be expressed as,

$$\begin{aligned} J_{11} &= T^2 I_0 f P_{11} && (11) \\ J_{12} &= T I_0 f P_{12} && (12) \\ J_{21} &= T I_0 (1-f) P_{21} && (13) \\ J_{22} &= I_0 (1-f) P_{22} && (14) \end{aligned}$$

Here the ink layer is approximated as a filter with transmittance T. Comparing Eqs. (12, 13) with Eq. (7), one can obtain the the following expressions,

$$\begin{aligned} P_{21} &= \overline{p} f && (15) \\ P_{12} &= \overline{p}(1-f) && (16) \end{aligned}$$

Evidently probabilities P_{12} correlates with P_{21} by,

$$P_{21}(1-f) = P_{12} f \tag{17}$$

The total flux of photons emerging from the bare paper J_p and from ink dots J_i may be written as

$$\begin{aligned} J_p &= I_0[T P_{12} f + P_{22}(1-f)] && (18) \\ J_i &= I_0 T [T P_{11} f + P_{21}(1-f)] && (19) \end{aligned}$$

Correspondingly the reflectance values of the dots R_i and the paper between dots R_p can be calculated by,

$$\begin{aligned} R_p &= \frac{J_p}{(1-f) I_0} = R_p^0 - \overline{p} f (1-T) && (20) \\ R_i &= \frac{J_i}{f I_0} = T[\gamma R_p^0 T + \overline{p}(1-f)(1-T)] && (21) \end{aligned}$$

where $\gamma = R_i^0/R_p^0$ describes the effect of ink penetration upon the reflectance of the substrate paper. Because of stronger absorption from the ink penetrated paper, γ is generally smaller than unit. The value of $\overline{p}$ depends on the size and the spatial distribution of the printed dots and the optical properties of the materials involved. Thus the knowledge of $\overline{p}$ is of critical importance in predicting and reproducing the desired reflectance. The variable R_p does not depend on γ and therefore R or R_i should be used when parameters must be fitted to experimental data.

3. Discussion and Examples

The equations (20) and (21) describe how the reflectance values depend on the material properties and geometry. They reveal the following important facts:

1. R_p and R_i are no longer constants as they were assumed in Murray-Davis Equation, as soon as light scattering has to be taken into account.
2. In the case where $\overline{p}$ is independent on the dot coverage f, R_i and R_p vary linearly with f. In the other words, the non-linearity of R_p and R_i with f provides information about the f-dependence of the light scattering effect (or $\overline{p}$).
3. Because the analysis was not restricted to any specific type of halftoning, all of expressions and conclusions are appliable to both AM and FM halftoning schemes.

3.1. Reflectance of a Halftone Image and Optical Dot Gain

The reflectance of the halftone sample is given by

$$R = R_i f + R_p(1-f) \qquad (22)$$

which is a quadratic function of f if $d\overline{p}/df = 0$ as can be seen from Equations (20) and (21). For $d\overline{p}/df \neq 0$, this is no longer the case and the relation between R and f depends on the form of $\overline{p}$.

Substituting the expressions of R_p and R_i (Equations (20) and (21)) into Equation (22), one gets

$$R = R_{MD} - \Delta R \qquad (23)$$

where

$$R_{MD} = R_i^0 T^2 f + R_p^0(1-f) \qquad (24)$$

is the reflectance of the halftone sample without light scattering (i.e. the Murray-Davis value). Light scattering inside the substrate paper is described by

$$\Delta R = (1-T)^2 \overline{p} f(1-f) \qquad (25)$$

Because $\Delta R > 0$ the true reflectance R is smaller than its Murray-Davis value R_{MD} and the halftone image appears to have larger dot coverage than predicted when scattering is ignored. It is why this effect is known as optical dot gain. If scattering is not modeled then the measured reflectance R seems to originate from a dot size $f + \Delta f$ instead of the true dot size f. From $R(f) = R_{MD}(f+\Delta f)$ one can then obtain the optical dot gain, Δf, as the function of the optical properties of the materials and ink penetration:

$$\Delta f = \frac{\Delta R}{R_p^0(1-\gamma T^2)} = \frac{(1-T)^2 \overline{p} f(1-f)}{R_p^0(1-\gamma T^2)} \qquad (26)$$

From the measured optical dot gain profile, one can therefore estimate $\overline{p}$ and obtain valuable information about the PSF.

The maximum of the optical dot gain can be obtained from equation,

$$\overline{p}' f(1-f) + \overline{p}(1-2f) = 0 \qquad (27)$$

where $\overline{p}' = d\overline{p}/df$. For $\overline{p}' = 0$, the optical gain has a single maximum at $f = 50\%$ and has a symmetric profile around the maximum.

We now illustrate the influence of the form of $\overline{p}$ on the reflectance functions and the optical dot gain. In the first example we consider $\overline{p} = R_p^0$. In the case of no ink penetration ($\gamma = 1$), this corresponds to the Yule-Nielsen model with the exponent $n = 2$. In this case (see Fig. 2a), the reflectances R_i and R_p vary linearly with the dot area f. The mean reflectance R of the whole image, on the other hand, varies quadratically. The calculated optical dot gain has a parabolic profile with a single maximum at $f = 50\%$. In the second example we adopt

$$\overline{p} = R_p^0(1 - f^m(1-f)^{1-m}) \qquad (28)$$

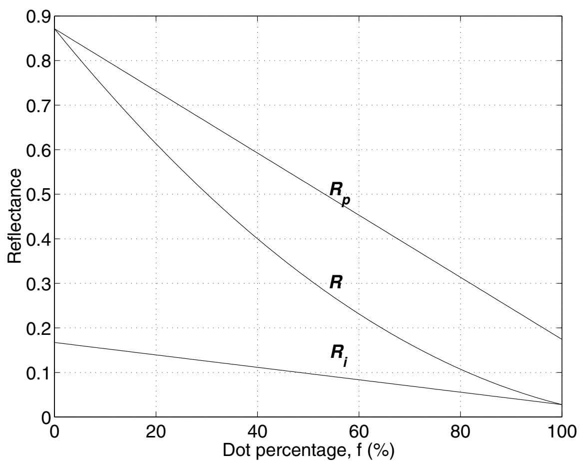
with $m = 0.7$. Now all reflectance, especially R_p and R_i, are highly nonlinear functions of the dot coverage (see Fig. 3a). Their behavior is characteristic for typical AM halftone images as can be seen from the measurements[9]. The sharp decrease of R_p as f increases shows that the probability (P_{22}) of a photon entering and then exiting from the bare paper decreases dramatically when the area of the bare paper becomes relatively much smaller than the area of the ink dots. Accordingly, the maximum of the computed optical dot gain has been shifted downwards to $f = 37\%$.

The dependence of the reflectances on the ink penetration has also been shown in Fig. 3 which we will explore further in the next subsection. We use these two examples only to illustrate the power and flexibility of the model developed above. In a real application one can either determine $\overline{p}$ by fitting the measured data (such as a optical dot gain profile) or compute it by integrating the PSF over the halftone pattern using Equation (8).

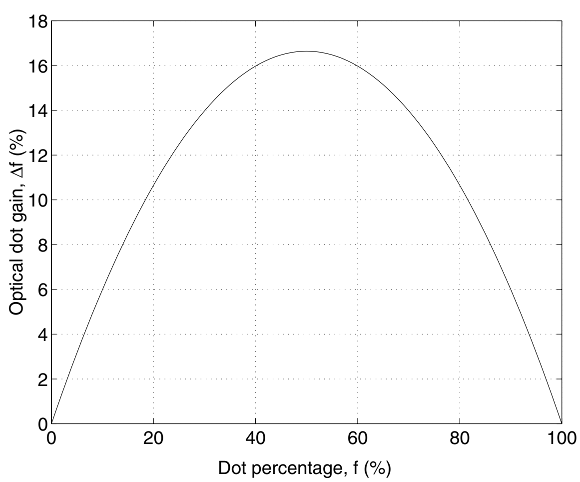
3.2. Optical Effects of Ink Penetration

Assume the thickness and transmittance of the printed ink layer are d and T_0, respectively, and the reflectance of the paper is R_p^0. The penetration of a part of the printed ink into the substrate paper has two optical effects: the transmittance of the pure ink layer, T_i, becomes bigger due to the thinner ink layer. On the other hand, the reflectance of the ink penetrated paper, R_i^0, becomes smaller due to the strong absorption of the penetrating ink. We recently [14] showed that $T_0^2 R_p^0 \geq T_i^2 R_i^0$.

To illustrate the influence of this effect on the properties of the halftone image, we computed the reflectances, $R_p,\ R_i,\ R$ and the optical dot gain, Δf, with and without considering the effect of ink penetration. In the case of ink penetration, we have assumed that $T_i = 1.3T_0$ and $\gamma = R_i^0/R_p^0 = 0.8$. As shown in Fig. 3, all the reflectance values increase when ink penetration is taken into account (dotted lines). The largest relative increase is obtained for the reflectance of the ink dots, R_i. Due to light scattering the ink penetration also changes the reflectance of the bare paper, R_p. Ink penetration also decreases the optical dot gain. Ink penetration also decreases the optical dot gain.

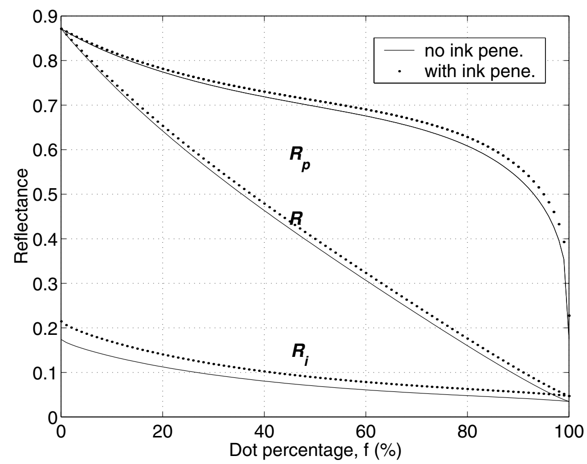


(a) Computed reflectance

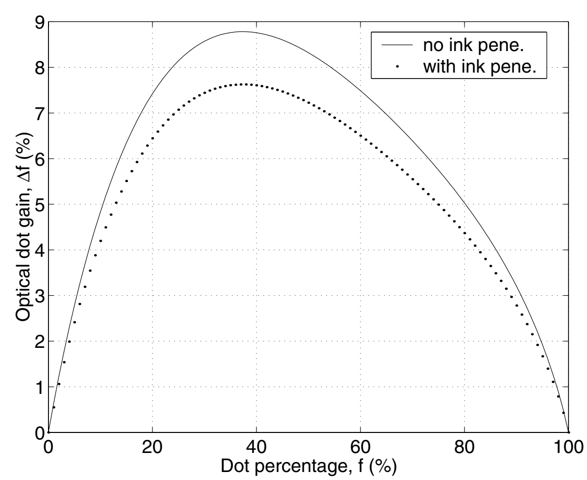


(b) Computed optical dot gain

Figure 2: Computed reflectance and optical dot gain with $\overline{p} = R_p^0 = 0.87$ and $T = 0.2$



(a) Computed reflectance



(b) Computed optical dot gain

Figure 3: Computed reflectance and optical dot gain with $\overline{p}$ given in Equation (28), where $R_p^0 = 0.87, m = 0.7$. Solid lines - no ink penetration, $T_i = T_0 = 0.2,\ \gamma = 1$; dot lines - with consideration of ink penetration, $T_i = 1.3T_0,\ \gamma = 0.8$

An explanation is probably that the absorption of the ink penetrated paper decreases the probability of photon exchange between the areas Σ_1 and Σ_2. The dependence of the optical dot gain profile on the ink penetration could be used to obtain estimates of material properties such as $\overline{p}$ and γ.

3.3. Investigation of Color Printing

Finally we sketch how the methodology developed above can be used in the investigation of tone reproduction. Similar to Equation (22) we get the tristimulus values W ($W = X, Y, Z$) of the halftone sample

$$W = W_i f + W_p(1-f) \tag{29}$$

where

$$\begin{aligned} W &= \int R(\lambda)S(\lambda)\overline{w}(\lambda)d\lambda \\ W_i &= \int R_i(\lambda)S(\lambda)\overline{w}(\lambda)d\lambda \\ W_p &= \int R_p(\lambda)S(\lambda)\overline{w}(\lambda)d\lambda \end{aligned}$$

To simplify notation we use $\overline{w}(\lambda)$ to represent one of the tristimulus functions $\overline{x}, \overline{y}$ or $\overline{z}$. In addition we use notations, $R_i(\lambda), R_p(\lambda)$ and $R(\lambda)$ to denote explicitly their dependence on the wavelength of the illuminating light.

Equation (29) has a similar structure to that of Neugebauer-Equations. However the interpretation is rather different. Here both W_p and W_i depend on f because of the f-dependence from R_p and R_i. Therefore the linearity assumption underlying the original Neugebauer Equations [2] does no longer hold. Moreover W_p is not only a function of the color coordinates of the paper since scattering from the ink-area has to be taken into account.

Following the approach used in Equation (23) the influence of the light scattering on the tristimulus values can be described by

$$W = W_{MD} - \Delta W \tag{30}$$

where

$$W_{MD} = \int R_{MD}(\lambda)S(\lambda)\overline{w}(\lambda)d\lambda \tag{31}$$

is the contribution from light following the Murray-Davis' assumption, and

$$\Delta W = \int \Delta R(\lambda)S(\lambda)\overline{w}(\lambda)d\lambda \tag{32}$$

corresponds to the Yule-Nielsen effect. From the non-negativity of ΔW we find that for any tristimulus value W_0 we have:

$$W_0 - W \geq W_0 - W_{MD} \tag{33}$$

This holds especially for the the tristimulus values of the paper (i.e. W_0 is the tristimulus values of the bare paper). Condisering the fact that $W_0 - W$ means the range of color presentation of the image, we can therefore draw the conclusion that the light scattering leads to a larger color gamut in halftone printing. This is in line with results of numerical simulations reported in [5, 6].

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Biography

Li Yang is a PhD student in Media Group at Department of Science and Technology, Campus Norrköping, Linköping University. He had physics in background. His research interest is in color reproduction of printing in general and the mechanism of Yule–Nielsen effect and ink-penetration in particular.