

A Series-Expansion Method for the Measurement of the MTF of Paper

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Abstract

Diffusion of photons within the paper substrate of a halftone print significantly affects the halftone color, an effect known as optical dot gain or the Yule-Nielsen effect. In modeling the relation of light, ink, and paper to halftone color, photon diffusion is often accounted for by a point spread function (PSF), or equivalently, the paper's modulation transfer function (MTF). This report introduces a novel technique for measuring the MTF of paper. A square-wave intensity pattern is projected onto the paper surface, and the intensity of the reflected light is recorded as a function of position. The MTF is given by the coefficients of the Fourier series of the reflectance data expanded at the harmonics of the square-wave frequency. This technique is similar to the edge-gradient method, except that it averages over several edges, and avoids taking the derivative of noisy data.

Introduction

Models that simulate the physical interaction of light with ink and paper can help to increase our understanding of the halftone process[1]. In constructing such a "first principals" model, it is necessary to account for the scatter and diffusion of light within the paper substrate. Diffusion of light within paper has a significant affect on halftone color; this effect is known as optical dot gain or the Yule-Nielsen effect[3, 2]. Several workers[4, 5, 6, 7, 8] have used a point spread function (PSF) to account for photon diffusion, or equivalently, a modulation transfer function (MTF), which is the modulus of the Fourier transform of the PSF[9]. The MTF measures the frequency response of an optical system, and several techniques have been used to measure the MTF of paper.

Inoue *et al*[10] measured paper MTF by the sine-wave method. In this method, a pattern with sinusoidal variation in intensity is projected onto the paper, and the intensity of the reflected light is measured as a function of position. The MTF is given by the modulation depth of the measured reflectance divided by the projection modulation depth. This is the most direct method for measuring the paper MTF, but a different frequency sine-wave projection and a different set of measurements are required for each frequency at which the MTF is evaluated.

Engeldrum and Pridham[8] measured the MTF of paper using the edge-gradient method. In this method, a "knife-edge" pattern is projected onto the paper, and the intensity of the reflected light is measured as a function of position. The reflectance data is a measure of the edge-spread function. The MTF is obtained as the Fourier transform of the derivative of the edge-spread function[9]. A disadvantage to this method is that one must take the derivative of noisy data. The noise introduces a bias error that increases with increasing frequency[9]. Also, because this method measures a single point, paper inhomogeneities tend to make the measurements difficult to reproduce[8].

A new technique for measuring the MTF of paper – a series-expansion method – is introduced here[11]. In this method, a pattern with square wave variation in intensity is projected onto the paper, and the reflected intensity is measured as a function of position. The reflectance data is then expanded in a Fourier series in multiples of the square-wave frequency, and the MTF is given by the expansion coefficients. The series-expansion method essentially measures the edge-spread function, but has an advantage in that it does not involve taking the derivative. Also, the method measures several edges and therefore averages over paper inhomogeneities. In obtaining the MTF from the harmonics of a single square-wave pattern, the series-expansion method is similar to the technique developed by J. Primot and M. Chambon[12] for measuring the MTF of sampled imaging systems.

In the analysis developed below, it is assumed that the MTF of the recording medium, and the MTF of the imaging system are together much wider than the MTF of the paper, so the effects of the imaging system and recording medium are ignored.

Theory

The square-wave projection onto the paper has the following intensity pattern:

$$I(x) = I_0\,\tau(x) + I_s,$$

where I_0 is the spatially uniform intensity of the light illuminating the target, I_s is an evenly distributed background illumination due to scatter, etc., ($I_s \ll I_0$), and τ is the square-wave target function:

$$\tau(x) = \sum_n \tau_n(x), \qquad (1)$$

with

$$\tau_n(x) = \begin{cases} 1, & na \le x < (n+\frac{1}{2})a \\ 0, & x \ge (n+\frac{1}{2})a \text{ or } x < na \end{cases} \qquad (2)$$

with a the square-wave period. The projection, $I(x)$, is shown in Figure 1 (a). The relative reflectance from the paper at position x is:

$$R(x) = \frac{R_p}{I_0 + I_s} \int_{-\infty}^{\infty} I(x')\,L(x'-x)\,dx' \qquad (3)$$

where $L(x)$ is the line-spread function (LSF) and R_p is the paper reflectance. The LSF is a one-dimensional form of the PSF[9]. It is assumed that the LSF is real, even, and normalized:

$$L(x) = |L(x)|; \quad L(-x) = L(x), \quad \int_{-\infty}^{\infty} L(x)\,dx = 1.$$

If the LSF is a delta-function, the relative reflectance from bright areas ($\tau = 1$) is R_p, and from dark areas ($\tau = 0$) is $R_p I_s/(I_0 + I_s)$.

In the following, Eq. (3) is used to obtain an expression for the MTF in terms of the relfectance $R(x)$. One may express the target function as a Fourier series:

$$\tau(x) = \frac{1}{2} + \frac{2}{\pi} \sum_{n \text{ odd}}^{\infty} \frac{1}{n} \sin(\omega_n x), \qquad (4)$$

with the sum for all positive, odd n, and:

$$\omega_n = \frac{2\pi n}{a},$$

with a the square-wave period. For simplicity, the phase is chosen such that for $n \neq 0$ the cosine terms are all zero, Eq. (2). The development does not, however, depend on this particular choice of phase: the phase is arbitrary. To obtain $\tilde{L}(x)$ in terms of $R(x)$, one carries out the integration in Eq. (3) using Eq. (4). One must evaluate the integral:

$$\int_{-\infty}^{\infty} L(x'-x)\,\sin(\omega_n x')\,dx'.$$

Noting that $L(x)$ is even, one can write this as:

$$\sin(\omega_n x) \int_{-\infty}^{\infty} L(x')\,\cos(\omega_n x')\,dx',$$

which is equal to $\sin(\omega_n x)\,\tilde{L}(\omega_n)$ with $\tilde{L}(\omega)$ the paper's MTF. One then finds that the reflectance can be expressed as:

$$R(x) = \frac{\alpha+\beta}{2} + \frac{2}{\pi}\,(\alpha-\beta) \sum_{n \text{ odd}}^{\infty} \frac{\tilde{L}(\omega_n)}{n}\,\sin(\omega_n x), \qquad (5)$$

where one defines:

$$\alpha = R_p, \quad \beta = \frac{R_p\,I_s}{I_0 + I_s}.$$

The MTF can be expressed in terms of the reflectance, $R(x)$, by multiplying both sides of Eq. (5) by $\exp(-i\omega x)$ and integrating over x to obtain:

$$\tilde{L}(\omega_n) = \left| \frac{i\omega_n}{2} \int_0^a R(x)\,\exp(-i\omega_n x)\,dx \right| (\alpha-\beta)^{-1}, \quad n \text{ odd}. \qquad (6)$$

The MTF can be obtained by a Fourier series expansion of the reflectance data $R(x)$.

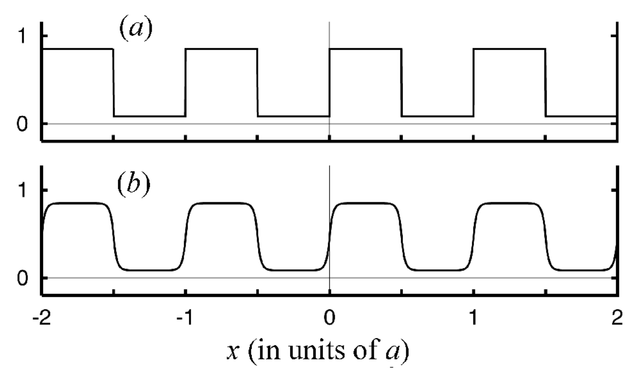

Figure 1. (a) the square-wave projection pattern. (b) the point reflectance with $\langle x \rangle = a/50$ given by Eq. (8).

Experimental Process

In the following, simulated data from a series-expansion MTF measurement are analyzed. The simulated data is obtained as follows: One integrates Eq. (3) using Eq. (2):

$$R(x) = (\alpha - \beta) \sum_n \int_{na}^{(n+1/2)a} L(x' - x)\, dx' + \beta. \quad (7)$$

In generating the simulated data, an exponential LSF is assumed,

$$L(x) = \frac{1}{2\langle x \rangle} \exp(-|x|/\langle x \rangle),$$

where $\langle x \rangle$ is the diffusion length – the average distance a photon diffuses within the paper before exiting. Carrying out the intgration, one finds for $R(x)$:

$$R(x) = (\alpha - \beta) \sum_n \left\{ e[x - na] - e[x - (n + 1/2)a] \right\} + \beta. \quad (8)$$

where $e(x)$ is the edge-spread function, given by:

$$e(x) = \begin{cases} 1 - 1/2\ \exp(-x/\langle x \rangle), & x \geq 0 \\ 1/2\ \exp(x/\langle x \rangle), & x < 0 \end{cases} \quad (9)$$

Figure 1 (b) shows the the reflectance $R(x)$ from Eq. (8) with $\langle x \rangle = a/50$. In order that there be a significant number of harmonics in the Fourier expansion of the reflectance, the square-wave period should be much larger than the diffusion length: $a \gg \langle x \rangle$.

There are N_p data points per period at equal intervals Δ over M periods, with the total number of points $N = N_p M$. The data points are at positions x_k given by:

$$x_k = k\Delta$$

with $k = 0, 1, 2, \ldots, N_p M - 1$, and

$$\Delta = \frac{a}{N_p}.$$

At each point k, Eq. (8) is used to calculate $R_k \equiv R(x_k)$. To this signal is added zero mean noise, n_k, from a random number generator:

$$R'_k = R_k + n_k.$$

The MTF is proportional to the Fourier expansion coefficients, which can be calculated from the discrete Fourier transform of the reflectance data, evaluated at the harmonic frequencies. Before taking the Fourier transform, however, the data was passed through a low pass Gaussian filter to prevent aliasing, with the cut-off frequency of the filter less than the Nyquist critical frequency, f_c. The bandwidth of the signal, B, as can be seen from Eq. (5), is approximately the inverse of the diffusion length: $B \sim 1/\langle x \rangle$, and this should be significantly less than f_c. The Nyquist frequency is one half the sampling rate:

$$f_c = \frac{N_p}{2a}$$

so it follows that:

$$\frac{1}{\langle x \rangle} \ll \frac{N_p}{2a},$$

or

$$\frac{2a}{\langle x \rangle} \ll N_p.$$

With $a/\langle x \rangle = 50$ one chooses $N_p = 400$. Figure 2 shows the signal plus noise after having passed through the filter.

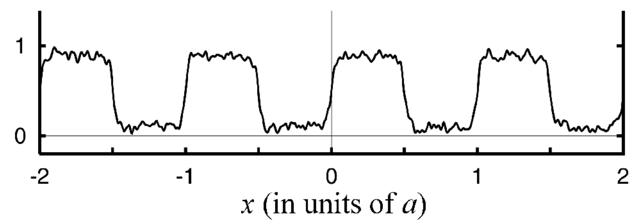

Figure 2. The simulated reflectance signal with noise. The signal is that shown in Fig. 1 (b), with noise added, then passed through a Gaussian filter.

The n^{th} expansion coefficient, r_n, is calculated by the discrete Fourier transform of R'_k at ω_n:

$$r_n = \frac{\Delta}{Ma} \sum_{k=0}^{N-1} R'_k \exp(i\omega_n x_k)$$

Subsituting for Δ, ω_n, and x_k, one obtains:

$$r_n = \frac{1}{N} \sum_{k=0}^{N-1} R'_k \exp\left(\frac{i\, 2\pi n k}{N_p}\right). \quad (10)$$

From Eq. (6), the MTF is proportional to the modulus of r_n:

$$\tilde{L}(\omega_n) = \pi n\, |r_n|\, (\alpha - \beta)^{-1}. \quad (11)$$

Figure 3 shows the MTF as obtained from the Fourier transform of the R'_k data (points), Eqs. (10) and (11), and the MTF obtained by a Fourier transform of the line-spread function (line). The MTF of the exponential LSF is a Lorentzian:

$$\tilde{L}(\omega) = \frac{1}{1 + (\omega \langle x \rangle)^2}.$$

One sees in Figure 3 that even with significantly noisy data, the technique is able to extract a relatively clean MTF.

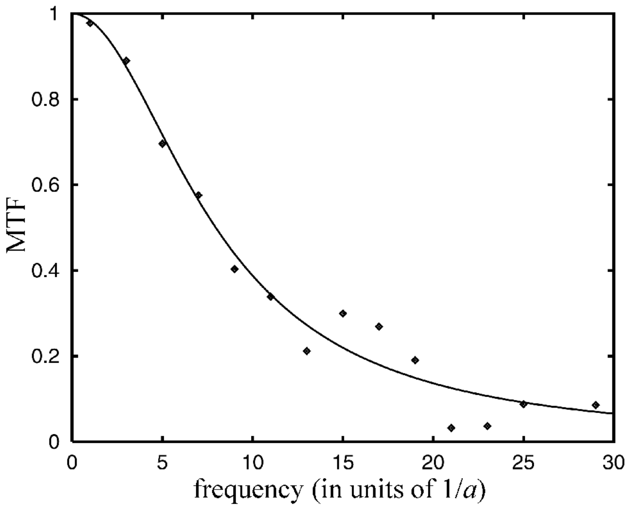

Figure 3. The line is the MTF for an exponential LSF with $\langle x\rangle = a/50$. The dots are the MTF obtained from the Fourier series expansion of the simulated reflectance data shown in Figure 2.

Conclusion

A new procedure for measuring the MTF of paper has been presented. The series-expansion method has advantages over the edge-gradient and sine-wave techniques. The experimental procedure for the series-expansion method is simpler than for the sine-wave in that it requires a single target and a single measurement. The advantage over the edge-gradient technique is that it does not require the taking of the derivative of noisy data and that it simultanously samples a number of points on the paper thus averaging over paper inhomogeneities.

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