On Color Edge Detection

Reiner Lenz
Institute Science and Technology
Campus Norrköping, Linköping University, Bredgatan
SE-60174 Norrköping
Sweden
email: reile@itn.liu.se
www.itn.liu.se/~reile

Abstract

We describe a new color edge detection scheme and a related diffusion process based on hyperbolic coordinates in color space. We derive intensity, hue and saturation based edge detectors and show that the intensity and hue based diffusion processes are governed by the heat equation familiar from conventional scale space theory. A qualitatively new differential equation is derived for the saturation component.

1. Introduction

Edge detection in gray-value images is one of the most important (and probably also one of the most intensively investigated) problems in image processing. Due to the increasing importance of color and other vector-valued images the design of edge detection methods for color and vector-valued images has received much attention recently.

Apart from many heuristical approaches the theoretically most convincing methods are based on differential geometry. The oldest method [2] views the image as a map from the grid into a manifold. On this manifold (or rather its tangent-spaces) there exists a distance measure. This distance measure is pulled-back to the image grid where it defines a new (pseudo-)metric. This metric is defined by a symmetric, positive-semidefinite matrix. The edge-strength and the edge-orientation are then defined in terms of the eigenvectors and eigenvalues of this matrix.

Recently [5, 4, 7] another similar framework was introduced by Sochen et. al.. They describe the image as a surface in a (n+2)-dimensional space where n is the dimension of the pixel vectors and the additional two coordinates are the pixel coordinates. They then view the image as an embedding of the two-dimensional coordinate manifold into an (n+2)-dimensional feature manifold. On both the coordinate manifold and the feature manifold exist Riemann-metrics. From high-energy physics they introduce the concept of a norm which depends on: (1) the metric in the coordinate manifold, (2) the metric in the feature manifold and (3) the mapping. This is then used to introduce a general framework for edge-preserving smoothing and other diffusion-based image processing methods. This is a very general framework since a point on the feature manifold can represent an intensity value, a color vector, computed features like result vectors from filter operations or a combination of them.

Before these methods can be applied the user has to choose the two matrices which define the geometries of the coordinate and the feature manifold. One natural selection for the geometry of the coordinate space is to assume that it is euclidean. In this paper we will mainly adopt this selection and concentrate on the selection of the metric in the feature space. We will only consider the case where the points in feature space describe color vectors and we will argue that the geometry of the feature space should reflect the geometrical properties of color space. We will use results from our previous study of the Karhunen-Loéve expansion of color spectra and argue that hyperbolic geometry provides a natural geometry of the feature manifold.

2. Hyperbolic structure of color space

Traditional color processing methods describe colors with three parameters (such as RGB or the CIE-XYZ, CIE-Lab etc.). The parameters are chosen to give a good description of color characteristics of the human color vision system which is based on three color sensors or, in technical applications, they may be optimized for the hardware used. Spectral based color processing methods try to avoid these restrictions by working on the spectra directly. Many of these methods describe the spectra by the coefficients in an eigenvector expansion. In most cases the eigenvector system is computed from the measured spectra of color chips in one or more color systems. Typical systems used are the Munsell and the NCS system.

In our experiments we used a database consisting of reflectance spectra of 2782 color chips, 1269 from the Munsell system and the rest from the NCS system. For each of the 1269 chips of the Munsell System the spectra were measured from 380 nm to 800 nm at 1 nm steps, while the 1513 samples from the NCS system were measured from 380 nm to 780 nm at 5 nm intervals. These measurements were combined in one set consisting of 2782 spectra (sampled in 5 nm steps from 380 nm to 780 nm). These spectra were used to to compute the eigenvector system. When we expanded the spectra in the databases we found that the coefficients are all located in a cone, with the exception of one spectrum which lies directly on the border 1. A detailed description can be found in [6].

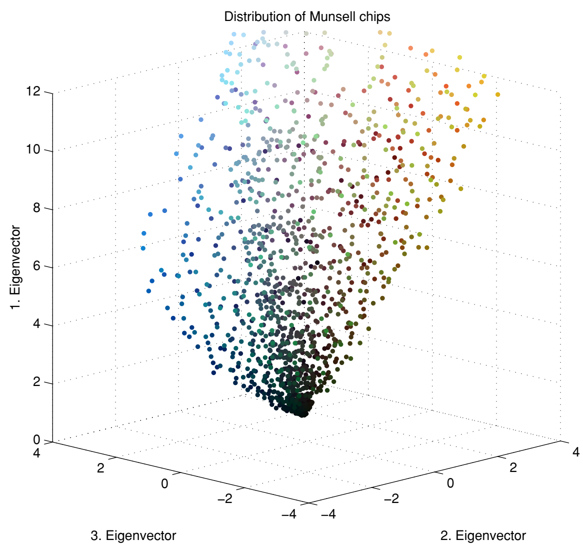


Figure 1: *Eigenvector expansion of Munsell color chips*

Mathematically we compute N eigenvectors $\boldsymbol{b}_m(\lambda)$ ($0 \leq N < 10$ are used in typical applications) and approximate the spectrum $\boldsymbol{s}(\lambda)$ as:

$$\boldsymbol{s}(\lambda) \approx \sum_{n=0}^{N} c_n \boldsymbol{b}_m(\lambda). \quad (1)$$

For the spectra of all chips in the database we find then that

$$|c_0|^2 - \sum_{n=1}^{N} |c_n|^2 \geq 0 \quad (2)$$

In the cone we can introduce a natural coordinate system consisting of three parameters (ρ, α, ϕ) defined as:

$$\begin{aligned} c_0 &= \mathrm{e}^{\rho} \cosh \alpha \\ c_1 &= \mathrm{e}^{\rho} \sinh \alpha \cos \phi \\ c_2 &= \mathrm{e}^{\rho} \sinh \alpha \sin \phi \end{aligned} \quad (3)$$

The three coordinate axes describe: (a) rays through the origin (ρ), (b) circles (ϕ) and (c) hyperbola (α). In the following we will view the cone as the product of the real line and the unit disk and express this by using the coordinates (ρ, ξ, η) defined as:

$$\begin{aligned} \rho &= \log c_0^2 - c_1^2 - c_2^2 \\ \xi &= \frac{c_1}{c_0} = \tanh \alpha \cos \phi \\ \eta &= \frac{c_2}{c_0} = \tanh \alpha \sin \phi. \end{aligned} \quad (4)$$

This leads to the natural coordinate system:

$$(\alpha, \phi) \mapsto \xi + i\eta = \tanh \alpha \mathrm{e}^{i\phi} \quad (5)$$

on the unit disk.

The natural metric on the unit disk (in the (α, ϕ) coordinate system, see page 378 in [8]) is defined by the matrix

$$\boldsymbol{H}_D = \begin{pmatrix} 1 & 0 \\ 0 & \frac{\sinh 2\alpha}{2} \end{pmatrix} \quad (6)$$

3. The norm of a color image

We now give a formal description of the strategy introduced by Sochen et.al. in [7]. They consider a color (or more general a vector-valued) image as a mapping between two Riemann-manifolds. The manifolds defined on the domains $\mathcal{S}$ and $\mathcal{T}$ are the coordinate and the feature manifold respectively. Points in the set $\mathcal{S}$ are described by coordinates $x and y$ which are collected in the vector $\boldsymbol{x}$. The coordinate vector on $\mathcal{T}$ will be denoted by $\boldsymbol{u}$. The metrics on $\mathcal{S}$ and $\mathcal{T}$ are given by the symmetric matrices $\boldsymbol{G}$ and $\boldsymbol{H}$. In general these matrices vary over the manifold since we are dealing with Riemann-manifolds: $\boldsymbol{G}(\boldsymbol{x})$ and $\boldsymbol{H}(\boldsymbol{u})$. The image $I : \mathcal{S} \to \mathcal{T}$ is a mapping from $\mathcal{S}$ to $\mathcal{T}$. The Jacobian matrix of I is denoted by $\boldsymbol{J}$ and the inverse, the transpose and the trace of a general matrix $\boldsymbol{M}$ are given by $\boldsymbol{M}^{-1}, \boldsymbol{M}'$ and $\mathrm{tr}(\boldsymbol{M})$.

For such a triple $(\mathcal{S}, \boldsymbol{G}), I, (\mathcal{T}, \boldsymbol{H})$ the norm is defined as:

$$\begin{aligned} E &= E((\mathcal{S}, \boldsymbol{G}), I, (\mathcal{T}, \boldsymbol{H})) \\ &= \int_{\mathcal{S}} \sqrt{\det \boldsymbol{G}(\boldsymbol{x})} \times \ldots \\ &\quad \mathrm{tr}\left(\boldsymbol{G}^{-1}(\boldsymbol{x}) \boldsymbol{J}'(\boldsymbol{x}) \boldsymbol{H}(I((\boldsymbol{x}))) \boldsymbol{J}(\boldsymbol{x})\right) d\boldsymbol{x} \end{aligned} \quad (7)$$

This definition of the norm ensures that its value depends only on the geometry of the objects involved, not on the particular coordinate system.

In the general case we have a measurement vector for each point in the set $\mathcal{S}$. The elements of the feature vectors $I()$ can then consist of a the raw measurements and/or computed feature values derived from these raw measurements. Using the measurement vectors we can thus design the feature space $\mathcal{T}$ and the matrices $\boldsymbol{G}(\boldsymbol{x})$ and $\boldsymbol{H}(I(\boldsymbol{x}))$ which define the metric structures. In this paper we will only consider the simplest cases in which the feature vectors $I(\boldsymbol{x})$ describe the coordinate vector $\boldsymbol{x}$ and the color at position $\boldsymbol{x}$.

$$\begin{aligned} I : \mathcal{S} &\rightarrow \mathcal{T} \\ \boldsymbol{x} &\mapsto \left(x,\ y,\ \rho(\boldsymbol{x}),\ \phi(\boldsymbol{x}),\ \alpha(\boldsymbol{x})\right) \end{aligned} \quad (8)$$

The Jacobian $\boldsymbol{J}(\boldsymbol{x})$ is thus given by:

$$\boldsymbol{J}'(\boldsymbol{x}) = \begin{pmatrix} 1 & 0 & \rho_x(\boldsymbol{x}) & \phi_x(\boldsymbol{x}) & \alpha_x(\boldsymbol{x}) \\ 0 & 1 & \rho_y(\boldsymbol{x}) & \phi_y(\boldsymbol{x}) & \alpha_y(\boldsymbol{x}) \end{pmatrix} \quad (9)$$

where $f_x(\boldsymbol{x}) = \frac{\partial f}{\partial x}(\boldsymbol{x})$.

We also choose the euclidean geometry on $\mathcal{S}$. Its metric is thus given by the identity matrix $\boldsymbol{I} : \boldsymbol{I} = \boldsymbol{G}(\boldsymbol{x})$. The norm simplifies in this case to

$$E((\mathcal{S}, \boldsymbol{I}), I, (\mathcal{T}, \boldsymbol{H})) = \int_{\mathcal{S}} \operatorname{tr}\left(\boldsymbol{J}'(\boldsymbol{x}) \boldsymbol{H}(I((\boldsymbol{x}))) \boldsymbol{J}(\boldsymbol{x})\right) d\boldsymbol{x} \quad (10)$$

Selecting the euclidean metric also on the feature manifold (given by the identity matrix $\boldsymbol{I}$) we get the norm

$$\begin{aligned} &E((\mathcal{S}, \boldsymbol{I}), I, (\mathcal{T}, \boldsymbol{I})) = \\ &\int_{\mathcal{S}} \operatorname{tr}\left(\boldsymbol{J}'(\boldsymbol{x}) \boldsymbol{J}(\boldsymbol{x})\right) d\boldsymbol{x} = \\ &\int_{\mathcal{S}} 2 + \left(\rho_x^2(\boldsymbol{x}) + \rho_y^2(\boldsymbol{x})\right) + \left(\phi_x^2(\boldsymbol{x}) + \phi_y^2(\boldsymbol{x})\right) + \ldots \\ &\left(\alpha_x^2(\boldsymbol{x}) + \alpha_y^2(\boldsymbol{x})\right) d\boldsymbol{x} = \\ &\int_{\mathcal{S}} 2 + |\nabla \rho(\boldsymbol{x})|^2 + |\nabla \phi(\boldsymbol{x})|^2 + |\nabla \alpha(\boldsymbol{x})|^2 \ d\boldsymbol{x} \end{aligned} \quad (11)$$

where $|\nabla f(\boldsymbol{x})|^2 = (f_x^2(\boldsymbol{x}) + f_y^2(\boldsymbol{x}))$ is the squared length of the gradient of f at $(\boldsymbol{x})$.

The expression $\operatorname{tr}(\boldsymbol{J}'(\boldsymbol{x}) \boldsymbol{J}(\boldsymbol{x}))$ is often used as a measurement of edge strength in the processing of vector-valued images (see [3])

Geometrically the feature manifold consists of three components: the coordinate manifold (represented by the coordinates x, y), the intensity component (represented by the variable ρ) and the chromaticity disk (represented by the variables α, ϕ). It is thus natural to define the following metric on this manifold (see also the definition of the metric on the unit disk defined in Eq. 6):

$$\boldsymbol{H}_D((\boldsymbol{x})) = \begin{pmatrix} A & 0 & 0 & 0 & 0 \\ 0 & A & 0 & 0 & 0 \\ 0 & 0 & B & 0 & 0 \\ 0 & 0 & 0 & C & 0 \\ 0 & 0 & 0 & 0 & \frac{C \sinh 2\alpha}{2} \end{pmatrix} \quad (12)$$

The constants A, B, C define the weighting between the three different components. This leads to the final norm derived from the hyperbolic geometry in color space:

$$\begin{aligned} &E((\mathcal{S}, \boldsymbol{I}), I, (\mathcal{T}, \boldsymbol{H}_{\mathcal{D}})) = \\ &\int_{\mathcal{S}} 2A + B |\nabla \rho(\boldsymbol{x})|^2 + \ldots \\ &C \left(|\nabla \phi(\boldsymbol{x})|^2 + \frac{\sinh 2\alpha(\boldsymbol{x})}{2} |\nabla \alpha(\boldsymbol{x})|^2 \right) d\boldsymbol{x} \end{aligned} \quad (13)$$

4. A non-euclidean scale space

The general energy function E defined in Eq. 7 is used in [7] to define a generalization of scale-space theory.

The energy function E defines a variational problem which is solved for the image function I. As an example consider the case where $A = C = 0$, $B = 1/2$ in Eq. 13. This leads to

$$E_\rho = \int_{\mathcal{S}} \frac{1}{2} |\nabla \rho(\boldsymbol{x})|^2 \ d\boldsymbol{x} \quad (14)$$

in Eq. 13. From the calculus of variation (page 165 in [1]) it is known that the function $\widehat{\rho}(\boldsymbol{x})$ with the smallest energy E_ρ must satisfy the heat equation

$$\frac{\partial^2 \widehat{\rho}(\boldsymbol{x})}{\partial x^2} + \frac{\partial^2 \widehat{\rho}(\boldsymbol{x})}{\partial y^2} = \Delta \widehat{\rho}(\boldsymbol{x}, t) = 0 \quad (15)$$

The original scale-space equation

$$\frac{\partial \widehat{\rho}(\boldsymbol{x}, t)}{\partial t} = -\Delta \widehat{\rho}(\boldsymbol{x}, t) \quad (16)$$

can thus be interpreted as the gradient-descent of a diffusion process under the control of the energy function E_ρ. In this way the original function $\rho(\boldsymbol{x})$ is embedded in a scale-space $\rho(\boldsymbol{x}, t)$ where the relation between the different scales is controlled through the gradient of the energy function. Since ρ describes the intensity coordinate we found that ordinary scale space theory for gray scale images is a special case contained in this framework.

As second example consider the "hyperbolic part"

$$E_\alpha = \int_{\mathcal{S}} \frac{\sinh 2\alpha(\boldsymbol{x})}{2} |\nabla \alpha(\boldsymbol{x})|^2 \ d\boldsymbol{x} \quad (17)$$

The Euler-Lagrange equation for this variational problem is then given by (see page 165 in [1]):

$$\sinh(2\alpha)(\alpha_{xx} + \alpha_{yy}) + \cosh(2\alpha)\left(\alpha_x^2 + \alpha_y^2\right) = 0 \quad (18)$$

or

$$\sinh(2\alpha)\,\Delta\alpha + \cosh(2\alpha)\,|\nabla\alpha|^2 = 0 \quad (19)$$

Dividing this by the positive function $\cosh \alpha$ leads to the new differential equation

$$\tanh(2\alpha)\,\Delta\alpha + |\nabla\alpha|^2 = 0 \quad (20)$$

5. Implementation and Examples

The color processing procedure described so far is based on the assumption that we can describe color by the first few eigenvector coefficients of the underlying spectra. Today multispectral imaging techniques are only used in special applications. The eigenvector coefficients are therefore rarely available in practice.What is given instead is usually only a color image in RGB-format.

In the case of RGB images we observe first that all vector components are positive and the vectors are thus all located in one octant. In this octant we use the main diagonal as the first coordinate axis. Around this axis we construct a cone, with top at the origin, which encloses the whole octant and we introduce hyperbolic coordinates (ρ, α, ϕ) as described in Eq. 4 and 5. These coordinates describe the intensity, the saturation and the hue as described before.

In the experiments we implemented the filtering operations with the simple Matlab filters which approximate the gradients with simple 3×3 filter kernels. The original image and the different edge detection schemes in are shown in Figure 2.

6. Summary and conclusions

The principle component analysis of a large database of color spectra shows that the most important coefficients in the eigenvector expansion are all located in a cone. This seems to be a general property of color spectra in general which reflects the non-negativity of the spectra. Therefore we convert RGB color vectors to a conical coordinate system for further processing. When using these coordinates one has to remember that this coordinate system is based on the physical properties of color spectra relevant for color vision. They do not into account the physiological and psychological processes which are important in the understanding of human color perception.

Based on a hyperbolic coordinate system in spectral color space we introduce a metric in this space of color spectra which is natural in the geometrical sense derived from a general group theoretical framework. This natural metric is then combined with the theory of the Beltrami flow introduced by Sochen et. al. to construct a new color edge detector and a related diffusion process based on gradient decent. We showed that the intensity and hue based diffusion processes are based on the Laplacian and that the diffusion in the saturation component follows a new type of differential equation. Diffusion in the hue variable is also different from conventional scale space processes since the (hue-) values of the process are located on the unit circle and not on the real axis as usual.

7. Biography

Reiner Lenz received a Diploma in mathematics from the university in Göttingen (Germany) in 1976 and a PhD degree from Linköping University (Sweden) in 1986. Since 1991 he is docent at Linköping University. He held positions as invited researcher at Zeiss Research Labs, Germany, ATR, Kyoto, Japan, the Mechanical Eng. Laboratories in Tsukuba Japan and Rutgers University, USA. He is interested in mathematical models for vision and color image processing.

Acknowledgements

Reiner Lenz was supported by the Swedish Research Council for Engineering Sciences TFR and the CENIIT program of Linköping University.

8. References

[1] R. Courant and D. Hilbert. *Methoden der mathematischen Physik*. Springer-Verlag, 1993.

[2] Silvano di Zenzo. A note on the gradient of a multi-image. *Computer Vision, Graphics and Image Processing*, Vol. 33, pages 116–125, 1986.

[3] Bernd Jähne. *Digital image processing : concepts, algorithms and scientific applications*. Springer, Berlin, 1997.

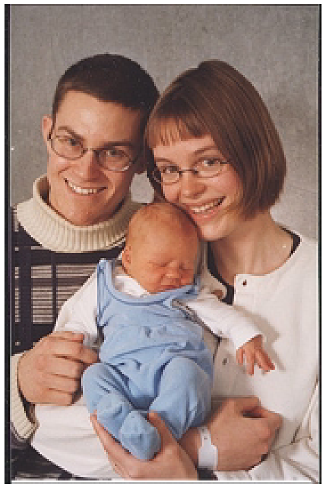
[4] R. Kimmel, N. Sochen, and R. Malladi. From high energy physics to low level vision. In *Scale-Space Theory in Computer Vision*, Lecture Notes Computer Science, pages 236–247. Springer, 1997.

[5] R. Kimmel, N. Sochen, and R. Malladi. Images as embedding mapps and minimal surfaces: movies, color and volumetric images. In *Proceedings CVPR-97*, pages 350–355, 1997.

[6] R. Lenz and P. Meer. Non-euclidean structure of spectral color space. In *Polarization and Color Techniques in Industrial Inspection*, volume 3826 of *Proceedings Europto Series*, pages 101–112, 1999.

[7] N. Sochen, R. Kimmel, and R. Malladi. A general framework for low level vision. *IEEE-Transactions Image Processng*, Vol. 7, No. 3, pages 310–318, 1998.

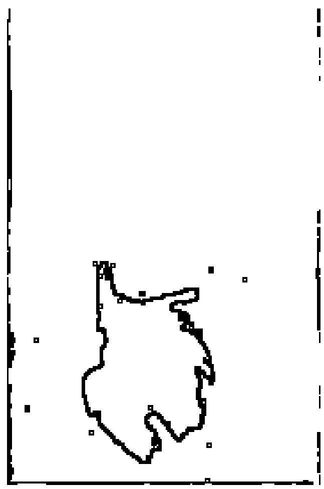
[8] Antoni Wawrzynczyk. *Group Representations and Special Functions*. D. Reidel Publishers, 1984.



(a) Original Image

(b) Combined Edge Detector

(c) Intensity edges



(d) Hue edges

(e) Saturation edges

Figure 2: The original image and the different edge images