

Quantification of Banding, Streaking and Grain in Flat Field Images

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Abstract

In this paper, we discuss techniques for the quantification of random and periodic noise present in flat field images. The types of noise studied are: (1) One-dimensional periodic noise (banding); (2) One-dimensional random noise (streaking) and, (3) Two-dimensional random noise (grain). A spectral separation technique was developed which allows the estimation of the individual noise power spectrum (NPS) of each artifact, based on the original flat field data. We proceed by assuming that the components are additive. First, the one-dimensional periodic components are identified, quantified, and removed from the flat field scan. Then the residual one-dimensional and two-dimensional random noise components are estimated via analysis of the two-dimensional NPS. This technique is useful in the characterization of digital cameras, printers, and scanners.

Introduction

The noise power spectrum (NPS) provides a statistical description of random fluctuations, based on image data from uniform areas (flat fields). The original applications of NPS (or Wiener spectrum) in imaging science were aimed at the characterization of photographic granularity,[1] but it has also been applied to the analysis of electronic image acquisition[2] and printing. Requirements for the measurement of the NPS include stationarity, or shift-invariance of the noise pattern statistics, and a finite spatial mean square value.

Photographic and electrophotographic granularity, and many sources of electronic noise, tend to produce two-dimensional random noise patterns that satisfy these requirements. If periodic fluctuations are present in the data, however, conventional NPS measurements provide inaccurate information about the magnitude of these components.[3] If the NPS measurement is modified to accurately report the magnitude of these periodic components, then the random noise is not properly characterized.

Digital devices may generate additional unwanted fluctuations in the image. For example, random variations in the gains of individual pixels in a linear scanning array (input or output) may lead to streaking, a one-dimensional random noise. Periodic placement errors in the position of a linear array may lead to one-dimensional periodic artifacts (banding).[4-6] Such patterns lead to NPS that are considerably more complex than those typically arising from photographic or electrophotographic granularity.[7]

Noise Power Spectrum Analysis

For two-dimensional isotropic random noise, a one-dimensional slice through the two-dimensional NPS surface can be obtained by scanning the noise pattern with a long, narrow slit, and applying the following estimator to the one-dimensional flat field scan data: [1]

$$NPS_{1D}\left(v_j\right) = \left(\frac{L\Delta x}{MN}\right)\sum_{m=1}^{M}\left|\sum_{n=1}^{N}\left(d_m(n\Delta x) - \bar{d}\right) * e^{\left(-i2\pi v_j n\Delta x\right)}\right|^2 \qquad (1)$$

where v_j is the j^{th} spatial frequency, L is the length of the measuring slit, N is the number of points per segment, m is the segment index, M is the number of segments, Δx is the sampling increment, $d(x)$ represents a scan across a flat field, and $\bar{d}$ is the estimate of the mean data value. In this case, a stable NPS estimate can be obtained that is independent of measuring slit length and segment length.

For one-dimensional artifacts such as banding or streaking, scanned perpendicular to the deterministic direction, application of Eq. (1) yields an NPS estimate that scales linearly with the slit length L.[3,7] In addition, the estimates for patterns containing one-dimensional periodic structures (banding) scale linearly with the segment length N. Despite the instability of the NPS estimate with respect to L and N in the presence of one-dimensional artifacts, useful information is still obtainable. For example, in the case of streaking, one may simply take the difference between one-dimensional NPS estimates in the x and y directions (assuming that streaking is present in one or the other direction, but not both) to arrive at the effective streaking NPS. Note that because of the one-dimensional nature of the streaking noise, the units of the NPS are (variance.mm), which corresponds to variance per (one-dimensional) unit spatial frequency interval, while the units of the two-dimensional NPS are (variance.mm^2). In cases where streaking and banding are present in both the x and y directions, along with two-dimensional random noise, a one-dimensional analysis is not sufficient to separate the artifacts. A frequency domain analysis of these more complex patterns requires the full two-dimensional NPS surface, which can be estimated as follows:[8]

$$NPS_{2D}(v_{xj}, v_{yk}) = \left(\frac{\Delta x \Delta y}{MN_x N_y}\right) \sum_{m=1}^{M} \left| DFT_{2D}\left\{ d_m(x,y) - \overline{d} \right\} \right|^2 \qquad (2)$$

where $DFT_{2D}\{\ \}$ is the two-dimensional discrete Fourier transform, given by

$$DFT_{2D}\{g(x,y)\} = \sum_{p=1}^{N_x} \sum_{q=1}^{N_y} g(\Delta x \Delta y) \cdot e^{\left(-i 2\pi v_{xj} p \Delta x\right)} \cdot e^{\left(-i 2\pi v_{yk} q \Delta y\right)}$$

Here v_x and v_y are the spatial frequencies, N_x and N_y are the number of data points in the x and y directions, respectively, for each two-dimensional block, M is the number of blocks, and Δx, Δy are the sampling increments in the x and y directions, respectively.

The simplest assumption is that the banding, streaking and two-dimensional random noise components are additively superimposed, i.e.,

$$d(x,y) = \overline{d} + g_{2D}(x,y) + s_x(x) + s_y(y) + b_x(x) + b_y(y) \qquad (3)$$

where $d(x,y)$ represents a two-dimensional data trace across a flat field, $g_{2D}(x,y)$ is a two-dimensional zero mean ergodic random process representing the image granularity, $s_x(x)$ and $s_y(y)$ are one-dimensional zero mean ergodic random processes representing the streaking in the x and y directions, respectively, and $b_x(x)$ and $b_y(y)$ are one-dimensional zero mean periodic functions representing the banding in the x and y directions, respectively. The banding is assumed to be characterized by the following model (written here for the x direction):

$$b_x(x) = \sum_j a_j \cos\left(2\pi v_j x + \phi_j\right) \qquad (4)$$

where a_j, v_j, and ϕ_j are the amplitude, spatial frequency and phase of the j^{th} component, respectively. A similar form is assumed for $b_y(y)$.

In the spatial frequency domain, the random components in Eq. (3) lead to the following NPS:

$$NPS_{rand}\left(v_x, v_y\right) = G_{2D}\left(v_x, v_y\right) + S_x\left(v_x\right)\delta\left(v_y\right) + S_y\left(v_y\right)\delta\left(v_x\right) \qquad (5)$$

where $G_{2D}(v_x, v_y)$ is the two-dimensional NPS of the image granularity, $S_x(v_x)$ and $S_y(v_y)$ are the one-dimensional NPS of the streaking in the x and y directions, respectively, and $\delta(v)$ is the Dirac delta function. Thus the image granularity produces a two-dimensional spectrum, while the streaking produces a continuous one-dimensional spectrum along each axis.

To understand the general appearance of these components in the two-dimensional NPS estimate, consider the Fourier transform of Eq. (4) (ignoring the phase angle ϕ_j):

$$B_x\left(v_x\right) = \left(\frac{1}{2}\right) \sum_j a_j \left[\delta\left(v_x + v_j\right)\delta\left(v_y\right) + \delta\left(v_x - v_j\right)\delta\left(v_y\right)\right] \qquad (6)$$

The one-dimensional periodic components generate a series of delta functions (e.g., line components) along the x and y axes, assuming that the banding is oriented along these directions.

Banding Removal and Quantification

Figure 1 shows a synthetic flat field pattern containing banding, streaking, and two-dimensional random noise on a constant background. The banding and streaking are present along both horizontal and vertical directions, and multiple banding components are present. The two-dimensional random noise is simulated using a pseudo-random number generator in combination with a transformation that results in a Gaussian distribution. The streaking is simulated via the same process, with the following modification: to simulate streaking in the vertical direction (for example), one row of random numbers is replicated for all lines in the field. Here the streaking was spatially filtered to produce different spectral bandwidths along the two orthogonal directions. Banding components were added in accord with Eq. (4).

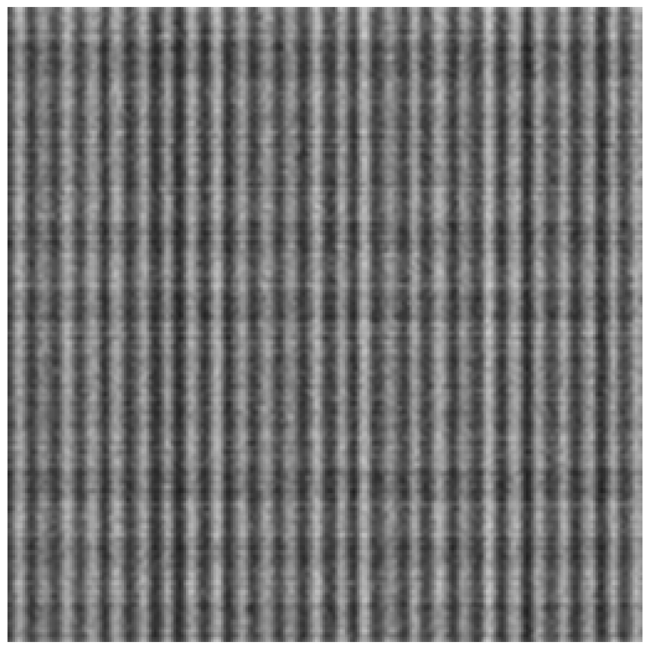

Figure 1. Synthetic Flat Field Pattern

The two-dimensional NPS of this synthetic pattern is shown in Fig. 2. Banding components are present at 0.5 cycles/mm along the horizontal (x) direction, and at 1.5 cycles/mm and 3.0 cycles/mm along the vertical (y) direction. The streaking components, as predicted by Eq. (5), give rise to the ridges along the x and y directions, which exhibit the intended variations in spectral bandwidth.

The presence of both banding and streaking in the same direction results in a further complication to the analysis of the NPS surface. The magnitude of the spectral lines due to banding leads to spectral leakage that is on the order of the NPS component caused by the streaking. Thus, the two effects are confounded in the spectral domain. The two components must, therefore, be separated prior to spectral estimation.

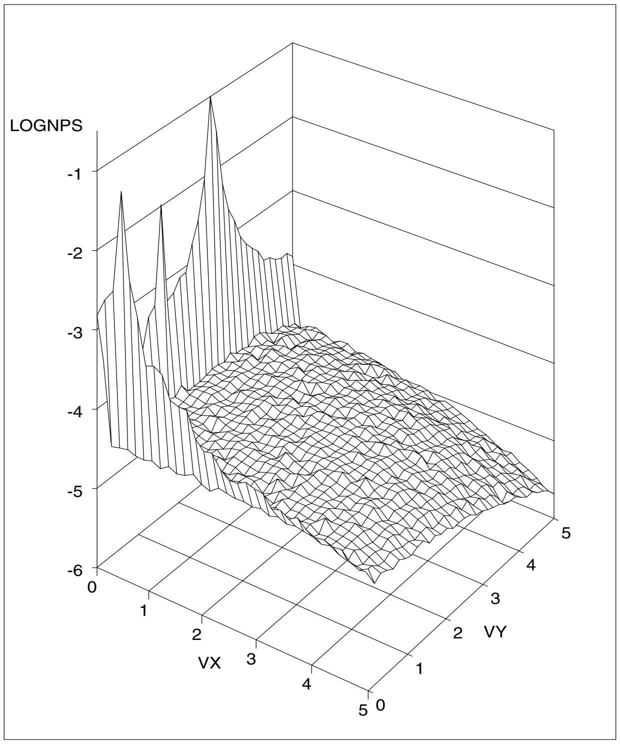

Figure 2. Two-dimensional NPS of Synthetic Pattern. The frequency axes are [0-5 cy/mm].

The amplitudes and frequencies of the banding components can be estimated from the flat field data in a number of ways. We chose the technique previously described by Bouk and Burningham.[9] This technique uses an iterative center-of-mass approach to estimate the banding frequency from the NPS. Once the banding frequency is determined, a two-dimensional regression procedure is used to fit a sum of sinusoidal components [as in Eq. (4)] to the flat field data. The final results of the regression yield the desired banding frequencies and amplitudes. Once the results are known, the fitted equation is subtracted from the original flat field data to arrive at a set of flat field data (the residual or error component of the regression procedure), containing only the original streaking and two-dimensional noise. The residual is then further analyzed to separate the two-dimensional noise and streaking. For the synthetic noise shown in Fig. 1, the banding frequency and amplitude estimates agreed well with the simulation input values, to within the precision of the measurement.

Granularity and Streaking NPS

The spectral power along the axes of the two-dimensional spectrum, after the banding components have been removed, represents the sum total of the streaking artifact and the granularity (see Eq. (5)). Because both artifacts result in continuous spectra, it is not possible to separate their individual contributions without an independent estimate of at least one of the artifact spectra. Whereas the granularity is the only artifact with a two-dimensional nature, it is possible to use the off-axis power of the two-dimensional spectrum to derive an independent estimate of the axial power distribution for the granularity. This is accomplished by taking a region adjacent to one of the axes, taken to be the *x*-axis for the sake of specificity. In this example, the band used is $0 < \nu_y < 3$ cycles/mm for all values of the ν_x -axis of the two-dimensional NPS. The points in this ν_y band, for each value of ν_x, are fit by a quadratic model and then extrapolated to $\nu_y = 0$, i.e., the granularity NPS along the ν_x-axis. The granularity NPS along the ν_y-axis is arrived at similarly.

With independent estimates of the granularity NPS along the ν_x and ν_y axes in hand, and again assuming additivity of artifacts. The next step is the determination of the streaking NPS along the *x* and *y*. From Eq. (5), we have

$$\hat{S}_x(v_x) = NPS_{filt}(v_x,0) - \hat{G}_{2D}(v_x,0) \tag{7}$$

and

$$\hat{S}_y(v_y) = NPS_{filt}(0,v_y) - \hat{G}_{2D}(0,v_y) \tag{8}$$

where NPS_{filt} refers to the filtered two-dimensional NPS of the flat field scan (after banding removal), and $G_{2D}(\nu_x, 0)$, $G_{2D}(0, \nu_x)$ are the axial estimates of the two-dimensional granularity NPS, described above. In practice, the streaking NPS estimates should be low-clipped at zero, since the NPS is by definition a positive number. Because of random error in the NPS estimates (which is a decreasing function of the number of segments or blocks), it is possible for the differences in Eq. (7) and Eq. (8) to result in a negative value at frequencies where the streaking NPS approaches zero. Finally, the streaking NPS estimates of Eq. (7) and Eq. (8) must be divided by the block length of the two-dimensional NPS estimate along the orthogonal direction (i.e., $S_x(\nu_x)$ should be divided by $N_y\Delta y$ and $S_y(\nu_y)$ should be divided by $N_x\Delta x$, in order to obtain a properly scaled one-dimensional streaking NPS estimate from the two-dimensional data. This scaling relationship is:

$$NPS_{1D}(v_{xj}) = \frac{NPS_{2D}(v_{xj},0)}{N_y \Delta y} \tag{9}$$

That is, the NPS of the one-dimensional pattern can be obtained from the k = 0 slice of the two-dimensional NPS estimate, divided by the block length in the orthogonal direction. Note that this one-dimensional estimate does not include a slit correction, as expected for one-dimensional patterns.

Figure 3 shows the component one-dimensional estimates for the streaking and granularity NPS of the synthetic pattern. These show excellent agreement with the known NPS of the input components. Note that the periodic components have been successfully filtered, along with the attendant spectral leakage. There is some evidence of a downward spike in the streaking NPS estimates at 1.5 cycles/mm, possibly caused by a slight overestimation of the amplitude of this banding component.

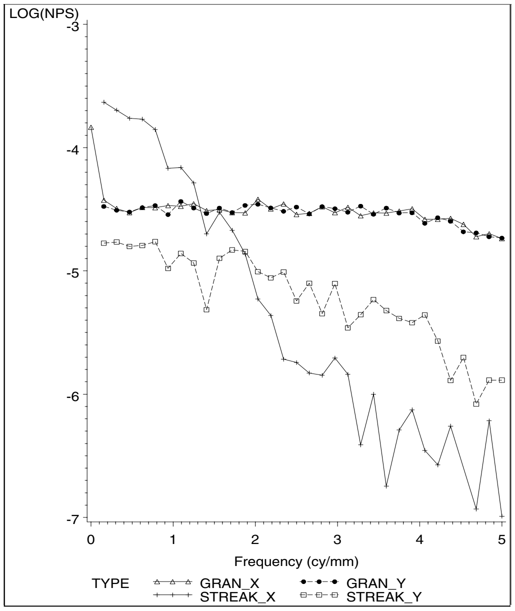

Figure 3. Streaking and Granularity NPS for Synthetic Pattern

Digital Printer Example

We now turn to an example of a measurement made on a flat field print from a digital printer, to demonstrate the typical features and interpretation of the NPS of such devices. A flat field of visual density 0.8, produced by a commercially available, electrophotographic laser printer, was scanned on a reflection microdensitometer, using a 50 micrometer square aperture at the specimen plane. An array of 1280 by 1280 points was scanned at 50 micrometer sample spacing, using an optical filter pack designed to simulate a photopic visual response cascaded with a CIE D5000 illuminant. The two-dimensional NPS estimate was computed with square blocks of length 128.

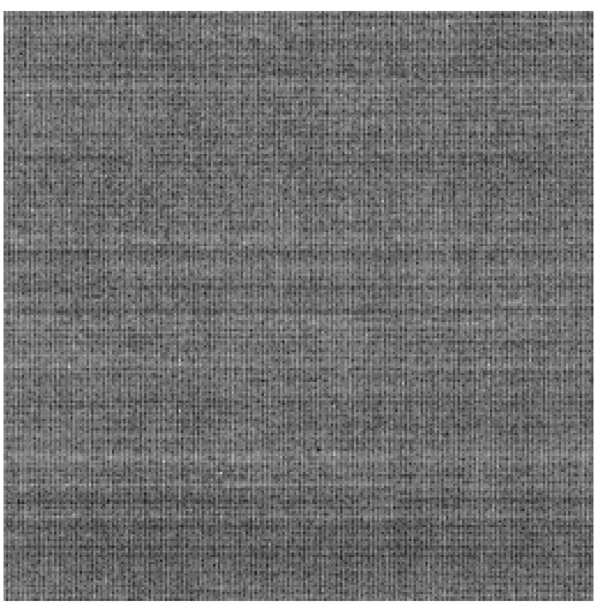

Figure 4. Flat Field from Digital Printer

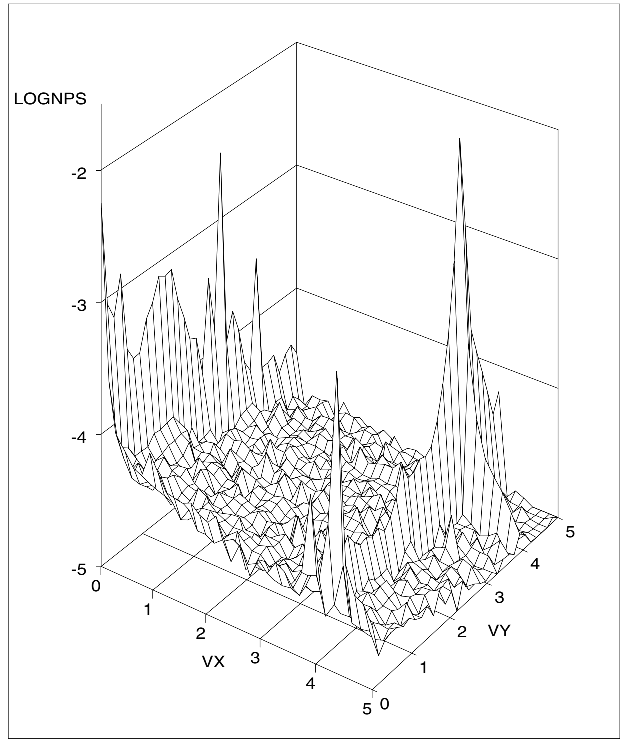

Figure 5. Two-dimensional NPS of Digital Printer Flat Field.

Figure 4 shows the flat field data. Prominent horizontal streaking, and possibly banding, are visible in the image.

Figure 5 shows the two-dimensional NPS. The horizontal streaking produces a corresponding ridge in the two-dimensional NPS parallel to the v_y frequency axis. A triplet of spectral lines appears at roughly 4 cycles/mm, along both frequency axes and on the diagonal. The fact that these spectral lines appear in a regular two-dimensional grid (as opposed to along one frequency axis only) is interpreted to be the result of a two-dimensional periodic pattern. The fact that there is no visually obvious one-dimensional banding in the field supports this hypothesis. In this case, an attempt would still be made to filter the spectral lines along the frequency axes, in order to avoid spectral leakage and the resulting bias in the streaking NPS. However, the amplitude of the fitted one-dimensional periodic components would not be interpreted as the result of a banding pattern, but rather as a two-dimensional periodic pattern. Finally, the two-dimensional random noise floor appears isotropic, again in agreement with direct observations.

The banding components near 4 cycles/mm have comparable amplitudes that agree well with visual observations, and with amplitude estimates obtained directly from the axial NPS slices. The component along the v_y direction near 3 cycles/mm is also prominent. In addition, a component at 0.1 cycles/mm along the direction of the v_x axis, not obvious in the two-dimensional NPS plot, was identified using a one-dimensional NPS estimate with higher spectral resolution. The granularity NPS estimates (see Fig. 6) are reasonable, compared with the two-dimensional noise floor seen in Fig. 5.

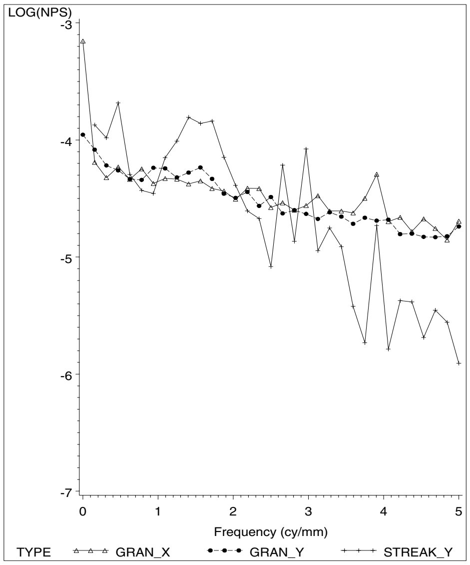

Figure 6. Streaking and Granularity NPS for Digital Printer Flat Field

The streaking NPS estimate along the direction of the v_y axis (Fig. 6), which has been scaled per Eq. (9), shows some evidence of residual banding, although the amplitudes have been greatly reduced, to perceptually negligible levels. The streaking NPS in the direction of the v_y axis is significant, and explains the majority of the line patterns seen in this flat field.

Conclusion

We have described a spectral separation procedure that allows the estimation of individual banding, streaking and image granularity NPS from flat fields containing one or more of these components. This frequency-domain procedure was tested with synthetic data sets, as well as data obtained from flat field prints or captures from digital devices, and has been found to produce results that are in basic agreement with visual observations. In this way, sources of stochastic and non-stochastic error can be included in a framework of general imaging system analysis.

References

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Biography

Paul Kane received the B.S. in Physics from the University of Scranton and the MS in Optics from the University of Rochester been with Eastman Kodak since 1986. He is currently a Research associate in the Imaging Science Division, where he is Group Leader of the System Image Quality Modeling Group.