A Prospective Research on Using Fuzzy Sets for Determining Color Sorting Groups More Accurately

Fabiana Rodrigues Leta(1) and Raquel Mello Araújo(2)
Universidade Federal Fluminense, Escola de Engenharia(1)
SENAI/CETIQT – FEITex – Fac. de Engenharia Industrial Têxtil(2)
Rio de Janeiro, Brazil

Abstract

Assessing color difference is still a very hard task to fulfill. It must take into account not only the readings of a spectrophotometer (considering that all the reading environment and calibration must be designed and controlled) but as well some personal feelings and understanding. The recognition of color comes from individual perception, which is acquired during life. When presenting a sample to be assessed, one may say that a sample is lighter or brighter or even deeper or shallow than the original; else it may come as a romantic or sober hue; a red car can be assessed as an adventure or wild or even fascinating and dynamic color. Depending on the mood the person has at the time of evaluation, his sense of the color, can be changed. So, how to make a standard language out of it? It is a problem that the standard logic cannot cope with. Multivalued logic can be solved with the fuzzy logic theory. Using fuzzy logic, we insert the uncertainty of the natural world into the field of rational, exact numbers. The feeling of a person projected on a color can be quantified using the membership function (or grade of membership) of a fuzzy set. The similarity of colors can be calculated from a computer program designed to range membership functions of colors to one simple image word, which will consider the characteristics used by the evaluator. This work will deal with different hues of colors, varying from different shades. Those colors will be selected from Trumatch Swatching System. Those sample colors will be presented to a group of 50-60 people from different society level groups; the language to classify the characteristics of a color will also be specified. Then a method for designing the correspondence of the subjective evaluation and the instrumental evaluation of sorting will be developed.

Introduction

The rational world of engineers has become a little bit more uncertain after the fuzzy set theory created by Lofti Zadeh. Since his paper in 1965, scientists started to accept medium terms in their evaluations such “maybe not so cold, but as cold as…” and so on and so forth. Multivalued analysis does not cope with these sort of not precise terms, turning subjective language into a code that had to be put into the frame of logic words, such as numbers and percentages. As due to its subjective nature, color suffered a lot with the concept of rational analysis, principally when the first standards were created, after the 1931 CIE Conference, when the standard observer was set, altogether with the illumination sources tables and equations correlating the amounts.

Perceiving color is a subjective assessment by nature. Of course, when we grew more experienced, we can name colors in a more appropriate standard way using a color book or a gallery created according to our company’s preferences. But even characterizing a color by numbers (L, a, b values) read by a spectrophotometer, one might have a different color sensation.

Interesting to not is although with all the environment assessment, to a certain extent, well controlled, the observers tend to use uncertain verbal communication.

Depending on what kind of product the observer is evaluating, his/her sensation changes: if the color of a car is to be assessed among some million of new choices, the observer might bear in mind ideas of success, popularity, well being, classy, status and so on; the same goes to clothes or textile material; when dealing with appliances, i.e. refrigerators, ovens, cupboards, one might think about not greasy colors; when it comes to wall paint colors, we may want to feel warm or calm or cool, depending on the season of the year.

In this paper, we try to bring about some of those uncertainties and also a procedure to have some results out of a controlled chaos of names.

We do not intend to have a specific procedure to solve this sort of problems in industry but only a new to look at the designing of color groups using fuzzy logic. The material to be assessed here was the coated paper edition of the Trumatch Swatching System.

Theoretical Background

Basic Theory of Fuzzy sets

B-1. Representation of Fuzzy Sets

If U is a collection of objects denoted generically by x then a fuzzy set A in U can be represented by three methods:
(I) Zadeh's method where

$$A = \frac{\mu_A(x_1)}{x_1} + \frac{\mu_A(x_2)}{x_2} + \dots \frac{\mu_A(x_n)}{x_n} = \sum_{i=1}^{n} \frac{\mu_A(x_i)}{x_i} \qquad \text{(B-1)}$$

where $\mu_A(x)$ is called the membership function or grade of membership (also degree of compatibility or degree of truth), and the terms $\mu_A(x_i)/x_i$ denote the relation between x_i and its membership function $\mu_A(x)$ instead of fractions.

(2) Ordered pairs method

$$A = \left\{(x, \mu_A(x)) \middle| x \in U\right\} \qquad \text{(B-2)}$$

(3) Vector method

$$A = \left\{(\mu_A(x_1)), (\mu_A(x_2)), \dots, (\mu_A(x_3))\right\} \qquad \text{(B-3)}$$

B-2. Fuzzy Relations

Let X, $Y \subseteq \Re$ be universal sets, then

$$R(X,Y) = \left\{((x,y), (\mu_R(x,y)) \middle| (x,y) \in (X,Y)\right\} \qquad \text{(B-4)}$$

is called a fuzzy relation on X x Y. The membership function of the fuzzy relation can be denoted as

$$\mu_R(x,y) : X\,x\,Y \to [0,1] \qquad \text{(B-5)}$$

The definition can be extended to a more general space: $X = X_1 \; x \; X_2 \; x \; \dots \; x \; X_n$; and the membership function of the fuzzy relation $R(X_1, X_2, \dots, X_n)$ can be expressed as:

$$\mu_R(x_1, x_2, \dots, x_n) : X_1 \; x \; X_2 \; x \; \dots \; x \; X_n \to [0,1]$$

B-3. Fuzzy Matrices

Let R be fuzzy relation on X x Y, and X has m elements and Y has n elements, then R can be defined by the following relational matrix

$$\begin{matrix} r_{11} & r_{12} & & r_{1n} \\ r_{21} & r_{22} & & r_{2n} \\ & & & \\ r_{m1} & r_{m2} & & r_{mn} \end{matrix} \qquad \text{(B-7)}$$

whose elements are given by

$$r_{ij} = \mu_R(x_i, y_j), X\,x\,Y \to [0,1] \qquad \text{(B-8)}$$

Such a matrix is called a fuzzy relational matrix or a fuzzy matrix in brief.

B-4. Operations for Fuzzy Sets

(1) Classical set operations

The complement, union, and intersection operations of a fuzzy set are corresponding to the linguistic variables of "no", "or", and "and", respectively. Their definitions are given by

(i) Complement operation

The membership function of the complement of a fuzzy set A, $\mu_{CA}(x)$, is defined by

$$\mu_{CA}(x) = 1 - \mu_A(x), x \in X \qquad \text{(B-9)}$$

(ii) Union operation

The membership function $\mu_D(x)$ of the union D = A U B is pointwise defined by

$$\mu_D(x) = \max\{\mu_A(x), \mu_B(x)\} x \in X \qquad \text{(B-10)}$$

(iii) Intersection operation

The membership function $\mu_C(x)$ of the intersection $C = A \cap B$ is pointwise defined by

$$\mu_C(x) = \min\{\mu_A(x), \mu_B(x)\} \; x \in X \qquad \text{(B-11)}$$

The min and max are not the only operators that could have been chosen to model the intersection or union of fuzzy sets respectively. Other authors have suggested alternative definitions for set theoretic operations.

(a) Definitions of Yager

The intersection of fuzzy sets A and B is defined by

$$A \cap B = \left\{x, \mu_{A \cap B}(x) \middle| x \in X\right\}$$

where

$$\begin{aligned} &\mu_{A \cap B}(x) = \\ &1 - \min\{1, ((1-\mu_A(x)^p) + (1-\mu_B(x)^p)^{1/p})\}, p \geq 1 \end{aligned} \qquad \text{(B-12)}$$

The union of fuzzy set is defined as

$$A\,U\,B = \{x, \mu_{A \cup B}(x) | x \in X\}$$

where

$$\begin{aligned} &\mu_{A \cup B}(x) = \min\{1, (\mu_A(x)^p + (\mu_B(x)^p)^{1/p}\}, \\ &p \geq 1 \end{aligned} \qquad \text{(B-13)}$$

The intersection operator converges to the min-operator for $p \to \infty$, and this union operator to the max-operator for $p \to \infty$.

(b) Definitions of Dubois and Prade

The intersection of two fuzzy sets A and B is defined as

$$A \cap B = \{x, \mu_{A \cap B}(x) | x \in X\}$$

where

$$\mu_{A \cap B}(x) = \frac{\mu_A(x) \times \mu_B(x)}{\max\{\mu_A(x), \mu_B(x), \beta\}} \qquad \text{(B-14)}$$

This intersection operator is decreasing with respect to β and lies between max $\{\mu_A(x), \mu_B(x)\}$ (which is the resulting operation for $\beta = 0$) and the algebra product $\mu_A(x)$. $\mu_B(x)$ (for $\beta = 1$).

For the union of two fuzzy sets A and B, defined as

$$A\,U\,B = \{(x, \mu_{A \cup B}(x) | x \in X\}$$

where

$$\mu_{A\cup B} = \frac{\mu_A(x)+\mu_B(x)-\mu_A(x).\mu_B(x)-\min\{\mu_A(x),\mu_B(x),(1-\beta)\}}{\max\{1(-\mu_A(x)),(1-\mu_B(x)),\beta\}}$$
$$\beta \in [0,1] \quad \text{(B-15)}$$

(2) Compositions of fuzzy relations

Let $P(x, y)$, $(x, y) \in X \, x \, Y$ and $Q(y, z)$, $(y, z) \in Y \, X \, Z$ be two fuzzy relations. The max-min composition P max-min Q is then the fuzzy set

$$P.Q = \{[(x,z),\ \max\{\min\{\mu_P(x,y),\ \mu_Q(y,z)\}\}]| x \in X, y \in Y, z \in Z\}$$

whose membership function is defined as

$$\mu_{P.Q}(x,z) = \max\{\min\{\mu_P(x,y), \mu_Q(y,z)\}\} \quad \text{(B-17)}$$

B-5. Fuzzy clustering

The task to divide n objects $x \in X$ characterized by p indicators into c, $2 \leq c \leq n$, categorically homogeneous subjects called "clustering". The objects belonging to any one of the clusters should be similar and the objects of different clusters as dissimilar as possible. The important problem of feature extration, that is, the determination of the characteristics of the physical process, the image of other phenomena that are significant indicators of structural organization, can be solved by clustering analysis.

The most important question to be answered before applying any clustering procedure are which mathematical properties of the data set, for example, distance, connectivity, intensity, and so on should be used and in what way should they be used in order to identify clusters.

(I) Representation of sample indicator

Assume there are n samples to be categorized, and each sample possesses m features (or indicators), y_1, y_2, ... , y_m. Then the indicators for these samples can be expressed by x_{ij} which denotes the j-th indicator for i-th sample. The i-th sample can be expressed in the following vector form:

$$X_i = (x_{i1}, x_{i2}, x_{i3}, \ldots, x_{im}) \quad \text{(B-18)}$$

(2) Construction of the similarity relation

The similarity relation between the samples X_i and X_j can be expressed by the correlation coefficient of the sample, $d(X_i, X_j)$, as follows:

$$d(X_i, X_j) = r_{ij} = \frac{\sum_{k=1}^{m} |x_{ik} - x_i| \cdot |x_{jk} - x_j|}{\sqrt{\sum_{k=1}^{m}(x_{ik}-x_i)^2} \cdot \sqrt{\sum_{k=1}^{m}(x_{jk}-x_j)^2}}$$

where

$$x_i = \frac{1}{m}\sum_{k=1}^{m} x_{ik} \qquad x_j = \frac{1}{m}\sum_{k=1}^{m} x_{jk} \quad \text{(B-l9)}$$

After the similarity relations have been calculated, we can get a square fuzzy matrix in rank $n \; X \; n$, and further get the compatibility relations for a α-cut.

B-6. α-cut or α-level set

The set of elements that belong to the fuzzy set A at least to the degree α.

$A_\alpha = \{x \in X \mid \mu_A(x) \geq \alpha\}$ is called the "α-cut" or "α-level".

Experimental Procedures

This study was performed with the color samples reference Trumatch 1; Trumatch12; Trumatch19, Trumatch30, Trumatch36; Trumatch44. Those were presented to 36 evaluators (this is a preliminary phase of the procedure): 24 women (from 19 to 36 years of age) and 12 male (20-38 years of age).

Decision of colors were taken randomly, trying to get some of the universe of colors in daily life.

The following table describes the colors and its characteristics:

Color number	Color Tone
Trumatch1-a	Magenta (Y=40, M=90,C=0, K=0)
Trumatch 12	Yellow (Y=100, M=0, C=0, K=0
Trumatch19	Green (Y=100, M-0, C=100, K=0)
Trumatch30	Blue (Y=0, M=0, C=100, K=0)
Trumatch 36	Purple (Y-0, M=70, C-100, K=0)
Trumatch 44	Pink (Y=0, M=100, C=30, K=0)

Methodology

To perform the precise task of the instrumental controlling of colors with reliability and straight-forwardness, it is necessary to set the tolerance limits very carefully, based on previous systematic visual evaluations made by very apt person in discriminating colors. The methodology to be applied must be very critical in order to establish the tolerances appropriately. A brief description of the method should be as the following: a number of samples are chosen and assessed by a group of selected observers. The same samples will be read by a spectrophotometer to obtain the first numerical characterization of the structure of the sample. Later, the pass/fail visual proceeding is performed among the selected observers, among the approved/reproved samples against a standard (one for each color). The standard could have been chosen as the average of the read samples. After this step, the observers are asked to assess the same sample again, now against a defined standard. The samples should be presented randomly in order to assure that the observer have a clue of if he had approved or reproved the sample previously. This task shall be performed using a standard illumination booth, with a selected standard light source (generally D65).

In brief, the proceeding could be described as such:

1. Selection of color samples
2. Selection of observers
3. Measurement of the samples
4. Presenting the samples to the observers in a controlled environment
5. Reading of the evaluations.
6. Remeasurement of the samples having the subjective evaluations

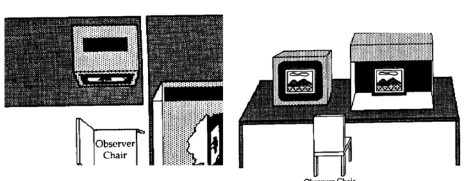
The classical process to cluster color samples is performed through the choosing among a universe of observers. There is a deterministic approach to select the color samples: the samples are read by an instrument and clustered to the best approximate match, which could be an average of the samples in the working set.

Procedure to Present Color to the Observers

It was prepared a booth of 60x60 cm, 30 cm thickness, painted in neutral gray, with a source lamp Phillips D65.

The samples were presented by range, i.e., Trumatch samples belonging to the same number in the same page were put together, in pairs to be assessed.

The observer should read each color pair twice in a different hour of the day, preferably in a different day from the first assessment. The observer should sit close to the booth, looking 45 degrees at the sample, with his head out of the booth (as in the picture).



Decision of Image Words

Image words are used by the evaluators to describe their feelings in relation to the color presented. The evaluators used some different names for different hues of colors such as light, dull, soft and some others. The kind of linguistic variables are endless. The following table show the names used for the presented color hues and their different values.

1. Vivid
2. Intense
3. Colorful
4. Strong
5. Dazzling
6. Tropical
7. Dimmed
8. Dull
9. Soft
10. Romantic
11. Tranquil
12. Tender
13. Old-fashioned
14. Rustic
15. Simple
16. Wild
17. Neutral
18. Sporty

These color names were used in the questionnaire of evaluation (in the Annex). They were collected to form a set of color images as follows:

$$\mathrm{LC} = \{lc1, lc2 \ldots lc5\}$$

Only 5 of those eighteen words were selected for the evaluation.

Measurement of the Membership Function

In the questionnaire presented, the range established to the evaluators was: completely different match (cdm), not such a good match (nsvgm), very close match (vcm), very good match (vgm), exact match (em), corresponding to: 0%, 25%, 50%, 75% and 100% (0, 0.25, 0.50, 0.75, 1.0).

So, the fuzzy evaluation set C was defined as:

$$C = \left\{ \frac{0}{cdm}, \frac{0.25}{nsvgm}, \frac{0.50}{vcm}, \frac{0.75}{vgm}, \frac{1.0}{em}, \right\}$$

The judgment on the C set was based on the assessment received from the questionnaires presented, given a result ranging from 0 to 1. After this step, a fuzzy matrix was build for the color image with a rank 10x5 (K_{10x5}).

Numerical Procedures

The Fuzzy Library of Matlab was used to build some of the simulations. The total results are still to be published. At this point, only the numerical algorithm was tested, giving tentative results.

The union and intersection sets for the bundle of variables were built. We used the assumptions that most of the evaluators, when not perceiving the color difference and willing to express their opinion, said the color was neutral or dull in comparison to its counterpart or that a specific color sample tended to be dull or neutral.

The graphics tended to be in the range of 0.50 of the membership function. More detailed analysis should be done in the months to come.

Measures of the Fuzziness of the Fuzzy Set for Color Image

The results showed that we still have to train the observers to have more accredited results, because for their recent evaluation of the colors, we tended to be on the average of the color range. The fuzzy matrix had the following presentation:

	$lc1$	$lc2$	$lc3$	$lc4$	$lc5$	
$y1$	.50	.58	.65	.71	.54	.
$y2$	.48	.38	.46	.65	.45	.
$y3$	.44	.56	.34	.67	.56	.
$y4$	.38	.56	.50	.63	.37	.
$y5$	.80	.50	.75	.65	.48	.
$y6$	.51	.56	.74	.53	.43	.
$y7$	.48	.58	.47	.53	.56	.
$y8$	.49	.36	.65	.68	.69	.
$y9$	.65	.54	.76	.32	.43	.
$y10$	.38	.65	.52	.56	.63	.

The standard deviation of the membership function is defined by:

$$S = \sqrt{\frac{\sum_{i=1}^{n}(x_i - x_{av})^2}{n}}$$

The color samples were read by a handheld Minolta 9992-970 spectrophotometer and the readings were compared to the visual determinations taken by the observers. The CMC(1:2) was used, setting the tolerances to be according to the grade of membership established in this article. The questionnaire used for the research was the following:

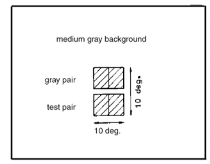
Example:

Questionnaire for Color Evaluation

Image words	Grade of membership				
	0	0.25	0.50	0.75	1.0
Vivid					
Intense					
Colorful					
Strong…					

The color samples were read using D65 source and it was done before the observers made their evaluation. The readings were made with specular excluded filter.

Below a picture to show the presentation of the sample to be read:



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Biography

Raquel Mello Araújo is a chemical engineer, with an ongoing Master Degree on Computer Graphics. Since 1991, she has worked with Applied Research and Technology at SENAI-CETIQT, Rio de Janeiro, Brazil.

Fabiana Rodrigues Leta is a professor at Universidade Federal Fluminense at the Mechanical Engineering Department since 1991. She has got a D.Sc.in Computer Graphics.