Abstract

The Yule-Nielsen effect, also called optical dot gain, has often been modeled based on convolutions between half-tone dot patterns and a point spread function, PSF, characteristic of the paper. The form of the PSF is generally assumed or measured empirically. An alternative approach to modeling the Yule-Nielsen effect employs a probability function $P_p$, which describes the fraction of reflected light emerging between halftone dots and under dots. The probability model is shown to fit experimental data on the Yule-Nielsen effect quite well. Moreover, the model can be implemented with simple algebraic expressions rather than the convolution or Fourier calculations required for PSF models. In addition, the quantitative relationship between $P_p$ and PSF is demonstrated.

Introduction

Most printing processes print ink onto paper at only one level, and gray scale is achieved by printing patterns of dots and varying the fraction $F$ of the paper covered with the dots. Murray and Davies modeled gray scale in printed halftone dots with the following expression,  

$$R(F) = FR_{i} + (1 - F)R_{p},$$ (1)

where $R$, $R_{i}$, and $R_{p}$ are, respectively, the mean level reflectance of the image, the reflectance of the printed ink, and the reflectance of the not printed paper. However, variation from this simple linear model is typically observed and is often called the Yule-Nielsen effect. Figure 1 illustrates the Yule-Nielsen effect for a 65-lpi, clustered-dot-halftone gray scale printed with a wax thermal printer with 300-dpi addressability.

Yule and Nielsen suggested the following function to model the effect,

$$R(F) = [FR_{i}^{1/n} + (1 - F)R_{p}^{1/n}]^{n},$$ (2)

where $n$ is an empirical constant adjusted to fit the experimental data, as illustrated in Fig. 1. Subsequent work by Yule and others has examined the fundamental relationship between the $n$ factor and independently measurable parameters of the ink and the paper. The focus of the current report is to contribute to this understanding by exploring a probability-based model of the Yule-Nielsen effect similar to that suggested by Huntsman.

The assumptions employed in this report include (1) clustered dot halftones, (2) no penetration of the dot into the paper substrate, (3) negligible scattering within the dot, and (4) negligible effects from multiple specular reflections between the ink dots and the paper. The intent is to lay the foundations for later investigations of the impact of varying from these assumptions. Experimental data in this report were obtained using a calibrated CCD camera, microscope optics, frame grabber, and analysis software as described previously.

Light Scattering

Yule and Nielsen pointed out that the fundamental reason for the nonlinear relationship between $R$ and $F$ is the lateral scattering of light within the paper. Light that enters the paper between halftone dots scatters laterally, and this lateral motion of the light in the paper increases the probability the light will encounter an ink dot and be absorbed. Published reports relating the Yule-Nielsen effect to paper and ink parameters have employed one of two models: The first and most common modeling approach describes the lateral motion of the light in...
paper with a point spread function PSF or the Fourier equivalent, the modulation transfer function MTF. Oittinen and Engeldrum have derived the MTF from Kubelka-Munk theory, but a useful empirical model of the paper MTF and its corresponding PSF are as follows:

\[
MTF(\omega) = \frac{1}{1 + (k_p \omega)^2},
\]

(3)

\[
PSF(x) = \frac{1}{k_p K_0} \left( \frac{2 \pi x}{k_p} \right),
\]

(4)

where \(x\) is the distance in mm from the point where light enters the paper, \(w\) is spatial frequency in mm\(^{-1}\), \(k_p\) is a constant proportional to the mean lateral distance light travels in the paper, and \(K_0\) is a Bessel function of the third kind. The interaction between the light and the halftone dots can be modeled using a convolution integral between the dot pattern \(f(x)\) and the point spread function \(PSF(x)\). Several excellent examples of this approach to halftone modeling have been published, including studies of the impact of the Yule-Nielsen effect on the color gamut in printed halftones.

**A Probability Model**

An alternative approach for modeling the Yule-Nielsen effect is based on probability functions for photon behavior in paper, as suggested by Huntsman. For example, as illustrated in Fig. 2, a photon entering the paper between halftone dots will have a finite probability \(R\) of returning to the paper surface as a reflected photon. The term \(R\) is also the optical reflectance factor of the paper. The photon can emerge between dots, (A) in Fig. 2, or under a dot, (C) in Fig. 2. As pointed out by Huntsman, we can identify the probability \(P_p\) of these photons returning to the surface under a halftone dot and a probability \(1 - P_p\) of their returning to the surface between the dots. We may also consider the photons that strike the halftone dots of transmittance \(T_i\). These photons have a probability \(T_i\) of entering the paper, and then a probability \(R_i\) of returning to the surface of the paper. Of those returning to the surface, there is a probability \(P_i\) of returning to the paper surface under a halftone dot and a probability \(1 - P_i\) of returning to the surface between the dots. These probabilities are summarized in Table I. Based on probability accounting of this kind, Huntsman was able to derive the Murray-Davies Eq. 1. Moreover, Huntsman's derivation demonstrated \(R_i\) and \(R_p\) are not constants as assumed in both Eqs. 1 and 2, but are functions of the dot area fraction \(F\).

Experimental observations of the variation of both \(R_i\) and \(R_p\) with \(F\) have been reported, and the data in Fig. 3 are an example. The data were obtained for a 65-lpi clustered-dot halftone printed with a wax thermal printer at 300-dpi addressability, and the data were measured with a CCD camera, microscope, frame grabber, and analysis software as described previously. By analogy with Yule-Nielsen, the data in Fig. 3 were fit to the following empirical functions for \(R_i\) and \(R_p\) by varying an arbitrary power factor \(w\). The value \(T_i\) is a constant equal to the transmittance of the ink dot.

\[
R_p = R_i [1 - (1 - T_i) (1 - (1 - F)^w)],
\]

(5)
\[ R_i = R_f T_i [1 - T_f F^w]. \] (6)

The resulting \( R_i \) and \( R_f \) values were used with Eq. 1 to model the overall reflectance \( R \) versus \( F \).

Earlier work demonstrated a relationship between the empirical model of Eqs. 5 and 6 and the MTF/PSF model represented by Eqs. 3 and 4. In particular, the power factor \( w \) and the mean free path \( k_p \) are related as follows:

\[ w = 1 - e^{-A k p f}, \] (7)

where \( f \) is the dot frequency of the halftone pattern. The constant \( A \) was determined empirically by fitting Eq. 7 to experimentally measured values of \( w \) versus \( k_p f \). As suggested in previous work by Huntsman,\(^6\) the value of the constant \( A \) depends on the shape and distribution of the halftone dots. An experimental study of this effect is currently underway. In the current report, the physical significance of Eqs. 5 and 6 is examined by further development of the probability model suggested by Huntsman.

**Paraphrasing the Huntsman Model**

The notation used in the following derivation differs from Huntsman,\(^6\) but the thrust of the arguments is the same. We begin with an incident irradiance \( I_o \) onto the halftone sample. The relative flux of photons striking the paper and the paper between the dots is \( F I_o \) and \( (1 - F) I_o \), respectively. The dot decreases the flux of photons entering the paper to \( F I_o \). With absorption by the paper \( R_p \) and scattering governed by the probabilities \( P_i \) and \( P_p \), the number of photons emerging as reflectance at the surface of the paper can similarly be expressed as shown in Table II. The total flux of photons emerging from the paper between the dots, \( I_o \), is the sum of the terms in cells (c) and (d) of Table II.

\[ I_o = R_o I_o [F I_o (1 - P_i) + (1 - F) (1 - P_p)]. \] (8)

In order to obtain the observed reflectance of the paper between the dots, we divide Eq. 8 by the flux incident on the paper between the dots, \( I_o (1 - F) \). The result shows \( R_o \) as a function of the dot area fraction, \( F \).

\[ R_o = R_o T_i \left[ T_i (1 - P_i) \left( \frac{F}{1 - F} \right) + (1 - P_p) \right]. \] (9)

Similarly, the reflectance of the dot is as follows.

\[ R_i = R_f T_i \left[ P_i T_i + P_p \left( \frac{1 - F}{1 - F} \right) \right]. \] (10)

Thus, if we knew the probability functions, \( P_i \) and \( P_p \), then \( R_i \) and \( R_o \) could be calculated, and then with Eq. 1 the overall halftone reflectance could be calculated.

Intuitively, the two probabilities \( P_i \) and \( P_p \) must be related. We can gain some insight into this relationship by examining a special case of \( T_i = R_i = 1 \). In this case we know \( R_i = R_o \) so we can equate Eqs. 9 and 10. By rearranging the result we obtain a relationship between \( P_i, P_p, \) and \( F \).

\[ P_i = 1 - P_p \left( \frac{1 - F}{F} \right). \] (11)

This relationship is true for \( T_i = R_i = 1 \). The generality of Eq. 11 can be extended by recognizing that \( P_p \) and \( P_i \) are independent of the total number of photons scattering about in the paper, so that the probabilities are independent of \( T_i \). We also will assume they are independent of \( R_i \), although it has been shown the mean free path in paper is a function of absorption as well as scattering.\(^{13,14} \) Thus, Eq. 11 is assumed true for all values of \( T_i \) and \( R_i \).

By substitution of Eq. 11 into Eqs. 9 and 10, we obtain the following two expressions for \( R_i \) and \( R_p \).

\[ R_i = P_i T_i [1 - P_i (1 - T_i)], \] (12)

\[ R_i = R_p F_i [1 - P_i (1 - T_i)]. \] (13)

Equations 1 and 11 through 13 provide a model for tone reproduction. Three things are needed: constants for \( R_i \) and \( T_i \) and knowledge of the probability function \( P_p \). If \( P_p \) is known, then the function \( P_i \) is determined with Eq. 11. Then \( P_i \) and \( P_p \) are used with Eqs. 12 and 13 to determine the functions \( R_i \) and \( R_p \) which then are used with Eq. 1 to calculate the mean level reflectance.

**Modeling the \( P_p \) Probability Function**

Models based on light scattering in paper generally use an empirical expression for MTF or PSF such as shown in Eqs. 3 and 4. Similarly, a model based on probability may employ an empirical expression for \( P_p \). For example, the data in Fig. 3 show the measured reflectance between the dots, \( R_p \) and the reflectance of the dots, \( R_i \), versus \( F \).

![Table II. Summary of Reflected Photon Flux Between and Under Halftone Dots.](image)

The data are reasonably well fit by Eqs. 5 and 6. Comparing Eqs. 5 and 6 with Eqs. 12 and 13 suggests the following empirical expressions for \( R_p \) and \( R_i \).

\[ \text{Due to typographic error, } P_i | R_o (1 - F) \text{ should be read as } P_i | R_f (1 - F). \]
Recent Progress in Digital Halftoning II

Figure 4. Empirical model with Eqs. 14, 11, 13, 12, and 1 fit to data from Fig. 3 on paper reflectance $R_p$ and ink reflectance $R_i$.

Figure 5. Empirical model with Eqs. 15, 11, 13, 12, and 1 fit to data from Fig. 3 on paper reflectance, $R_p$, and ink reflectance $R_i$.

Figure 6. Empirical model with Eqs. 22, 11, 13, 12, and 1 fit to data from Fig. 3 on paper reflectance $R_p$ and ink reflectance $R_i$.

However, these two expressions do not relate to each other through Eq. 11 as they should. Thus, we might select Eq. 14 and construct $P_p$ from Eq. 11. Then we can model the data as shown in Fig. 4, where $w$ has been adjusted to provide the best fit to $R$ versus $F$. Unfortunately, $R_i$ versus $F$ does not fit well. As an alternative, we can select Eq. 15 and solve Eq. 11 for $P_i$. Then we can model the data as shown in Fig. 5, where $w$ has been adjusted to provide the best fit to $R_i$ versus $F$. Unfortunately, $R_p$ versus $F$ does not fit well. Thus, the empirical model suggested previously does not quite agree with the requirements of the probability model.

Equation 14 seems to model the data well as $F$ approaches 1, and Eq. 15 seems to model the data as $F$ approaches 0. A combination of Eqs. 14 and 15 can be constructed that is in agreement with the probability model. This is done by first defining the following functions:

$$PP_1 = 1 - (1 - F)^w,$$  \hspace{1cm} (16)

$$PI_1 = 1 - PP_1 \left( \frac{1 - F}{F} \right),$$  \hspace{1cm} (17)

$$PI_2 = F^w,$$  \hspace{1cm} (18)

$$PP_2 = (1 - PI_2) \left( \frac{F}{1 - F} \right).$$  \hspace{1cm} (19)

We then combine them so Eq. 14 dominates at high $F$ and Eq. 15 dominates at low $F$.

$$P_p = F \cdot PP_1 + (1 - F) PP_2,$$  \hspace{1cm} (20)

$$P_i = F \cdot PI_1 + (1 - F) PI_2.$$  \hspace{1cm} (21)

With some algebra it can be shown that these two expressions for $P_p$ and $P_i$ do relate as they should through Eq. 11 and thus are consistent with the probability model of halftone imaging. Equation 20 can be developed into the following equivalent expression:

$$P_p = F[1 - (1 - F)^w + (1 - F^w)].$$  \hspace{1cm} (22)

Then Eq. 22, combined with Eqs. 11, then 5 and 6, and then 1 results in the fit shown in Fig. 6. This model fits the data as well as empirical Eqs. 5 and 6 and is also consistent with the probability model.

Conclusion

Probability-based models of the Yule-Nielsen effect offer some advantages over models based on the paper MTF or PSF functions. They are more intuitively described and understood, and they are generally expressed as simple, algebraic functions. The disadvantage of probability-based modeling is that the form of the probabili-
ity function $P_p$ will be different for various halftone patterns. However, this may not be a significant drawback compared to traditional models based on PSF because a geometric function for the specific halftone pattern must be known to apply a PSF model. In addition, deriving the $P_p$ function from a convolution between the geometric halftone pattern and the paper PSF should be possible. An alternative possibility is the development of experimental relationships, such as Eq. 7, to relate the probability models for a particular digital halftone pattern of interest to the spread function of light in paper. Further work is underway in this laboratory to explore these relationships experimentally.

References

17. Peter G. Engeldrum, *J. Imag. Sci. Technol.* **38**, 545 (1994); *see pg. 383, this publication*.