Optimality of Blue Noise Mask Binary Patterns

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Abstract

The Blue Noise Mask (BNM) is a halftone screen that pro-duces unstructured dot patterns and visually pleasing images. The starting binary patterns or “seed” patterns play a very important role in the mask generation process. As we can show, the original filtering technique used to generate this pattern reaches a lower limit in terms of perceived mean square error (MSE). A new algorithm is proposed to break this limit. This new algorithm is based on a model of electrostatic forces between charges. By locally enforcing this vector process at the end of filtering, more visually pleasing patterns are generated. Another observation is that this process eventually converges to a very structured dot pattern. This leads us to another related and important discussion regarding the degree of randomness that is optimal in Blue Noise Masks. At one extreme, we have white noise, at another extreme, we have a highly structured pattern. The white noise pattern is visually annoying, but the highly structured pattern is a poor “seed” for adding or subtracting minority pixels for the construction of neighboring gray levels in a dither matrix. What pattern between white noise and structured patterns constitutes an optimal blue noise pattern? A series of patterns is presented and their power spectra is analyzed. A discussion of optimality for using these individual patterns as “seed” patterns is also presented.

Introduction

Stochastic screening has been an active research field in recent years. Blue noise halftone screens were first developed by Mitsa and Parker[1] in 1991. Work on filtering and constructing BNMs continues to be an active area (Yao & Parker,[2] Ulichney.[4] The BNM combines the blue noise characteristic of error diffusion[3] and the fast speed of ordered dither.

Briefly, the BNM can be constructed one level at a time from some intermediate starting binary pattern, or “seed”. At each level, filter is used to identify and eliminate low frequency structures (large “clumps”) incompatible with the desired blue noise power spectrum.

In his Ph.D. thesis,[3] Yao gave a detailed mathematical analysis of the BNM construction based on a human visual model, which provides insights to the filtering process and also prescribes the locations of the dots that will result in a binary pattern of minimum perceived error when swapped. The analysis of the filtering technique put a lower bound on the lowest MSE we can achieve assuming a human visual system based filter is used to measure perceived MSE. As Yao pointed out, the difference between the local filtered output of the largest white clump and the largest black clump must be greater than a certain value T in order for the MSE to be further reduced. T is given by:

$$T = 1/(4 * \pi * \sigma^2) \tag{1}$$

whereas σ is the sigma of the adaptive filter based on a human visual model.

The binary “seed” pattern is also generated using this same filtering technique, where the starting pattern can be a white noise random pattern. Therefore, we face the same MSE limit problem. In the following section, we will present a new algorithm to break this limit. By locally enforcing a vector process at the end of filtering, we are able to get generate more visually pleasing binary patterns.

New Algorithm for Modifying Binary Patterns

This new algorithm is based on the model of electrostatic force between charges.

Point charges of same polarity will repel each other while charges of different polarity will attract each other. Therefore, in case of a binary pattern whose pixel values are either 1 or 0, if we assume “+” polarity for all the minority pixels, then there will be interactive force between all the pixels. If we let those minority pixels move freely under the net electrostatic force on them from nearby pixel charges, we should expect a nearly homogeneous distribution of those minority pixels after certain iterations of movement. Since every pixel has some force acting on them, we should set a threshold such that only when the net force on a pixel surpasses the threshold value, will that pixel move in the direction which would minimize the net forces on that minority “charge”. Remember that the starting pattern for this new algorithm is the one we obtain from the filtering process, which is already free of clumps. Thus if we plot the histogram of the net force (either in horizontal direction or vertical direction) on every minority pixel, we will get a distribution that is highly peaked near 0. Therefore, we can assume the mean value of net force on every minority pixel to be 0. In this case, we calculate the force on each minority pixels from their individual neighborhoods and the standard deviation (SD) of these force, then we set the threshold (T) value as:

$$T = V * SD \tag{2}$$

V is a variable that will be adaptive to the gray level and iteration number.

As binary patterns are 2-D, we will do this force calculation and pixel movement in horizontal and vertical direction (or X and Y direction) respectively.

The steps of this new algorithm are outlined as follows:

1. Set the neighborhood size and initial V value according to the gray level of the pattern. The maximum value of V and increments should also be set.
2. Make a copy of the starting pattern 1 and denote this as pattern 2.
3. For pattern 1, calculate the net force on each minority pixel from their neighborhood in X and Y direction, $(f_x)_j$, $(f_y)_j$, j represents each minority pixel.
4. Calculate the standard deviation of these two groups of forces and set the threshold T_x and T_y.
5. For every minority pixel j of pattern 1, compare the absolute value of $(f_x)_j$ against T_x, if it is greater than T_x, move the corresponding pixel of pattern 2 one pixel in the direction based on the sign of $(f_x)_j$. Otherwise, make no movement. The same procedure is done in Y direction for that same pixel.
6. After this comparing and moving is done for every minority pixel in pattern 1, we obtain a new pattern 2 that is different from pattern 1 because of the pixel movement. The MSE of pattern 1 and pattern 2 are compared. If MSE of pattern 2 is less than that of pattern 1, pattern 2 is accepted and used to update pattern 1, and another iteration is called starting at step 2. Otherwise, V is increase by certain amount. If V is less than the maximum value set in step 1, go back to step 2. Otherwise, the process is terminated.

A different force-relaxation model for adaptive halftoning of images was proposed by Eschbach and Hauck.[5]

Experimental Results

To illustrate the procedure, we apply the filtering technique of Yao & Parker[2] to a random white noise pattern (P1) of gray level 245, where the sigma for the filter is 2.4 in this case. Figure 1 shows the filter-weighted MSE drop vs. iteration number and Figure 2 shows the difference between the largest white clump and the largest black clump (DWB) for each iteration. As we can see, since we start from a white noise pattern, the DWB is quite large, so the MSE keeps going down in each iteration. After a certain number of iterations, the DWB approaches the lower bound we set in Equation 1, in this case approximately 0.0137, then the filter can no longer improve the binary pattern. Figure 3(a) shows the binary pattern (P2) obtained from the filtering process with MSE value of 0.263.

We then use this pattern as the starting pattern for our new algorithm. We define the neighborhood as 13 by 13 and set starting value of V as 1.5. Figure 3(b) shows the binary pattern (p3) after just 5 iterations with MSE of 0.165.

It is quite obvious that by locally enforcing our new vector process, we can further reduce the perceived error and get progressively more ordered dot patterns.

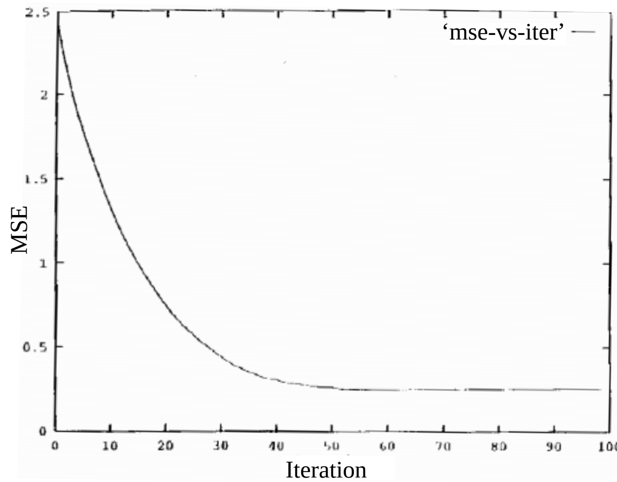

Figure 1. MSE drio vs. iteration in filtering process.

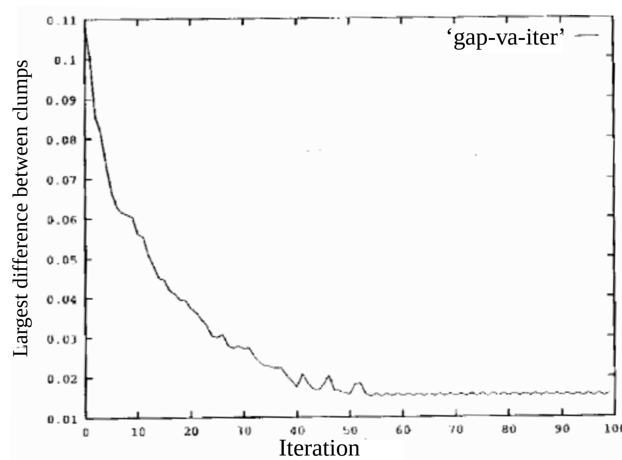

Figure 2. Difference between the largest white clump and the largest black clump in the filtering process

Discussion

We note that our new algorithm does converge after further iterations. Figure 3(c) shows the final pattern (P4) obtained when the new algorithm converges after 75 iterations. The final MSE is 0.087. As we can see, this is a very structured pattern.

One thing we should keep in mind is that the purpose of this new algorithm is not just to get a visually pleasing binary pattern but a visually pleasing "seed" from which a halftone mask will be generated later. Therefore, the decision which pattern to choose as the "seed" should be based on their performance in mask generation.

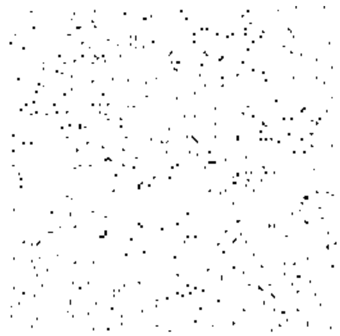

Figure 3. (a) White Noise Pattern (P1)

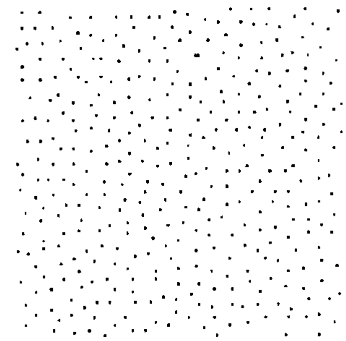

Figure 3. (b) Pattern after filtering Process (P2)

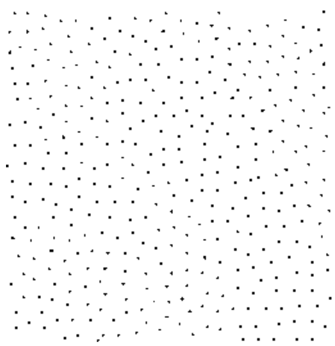

Figure 3. (c) Pattern after 5 iterations of force-relaxation (P3).

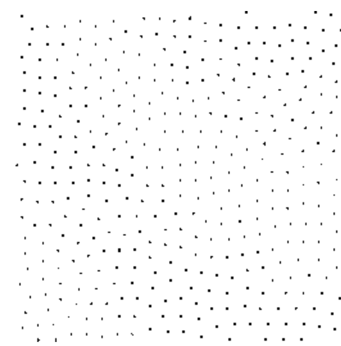

Figure 3. (d) Pattern after 75 iterations of force-relaxation (P4).

Obviously, a white noise pattern cannot be a candidate. However, we also find out that the highly structured pattern is also a poor candidate. If we use it as the "seed", those binary patterns with gray level in its neighborhood will be very visually annoying due to noticeable disruption of the semi-regular patterns established by the "seed" pattern. This leads to the question: What pattern between white noise and structured patterns constitutes an optimal blue noise pattern?

Figure 4 shows the power spectra of all the patterns we presented above. P1 has the typical white noise characteristic, P2 and P3 have the typical blue noise characteristic, P4 has a very high peak at the principal frequency. If we plot the power spectrum of all intermediate patterns, we could see the trend that starts from white noise, generally moves into blue noise, and end up with a concentration of energy around the principal frequency of the gray level. Therefore, in carrying out our new algorithm, we should set a criterion that will enable us to terminate the process once an excessive concentration of energy is reached. Current research is investigating in this area.

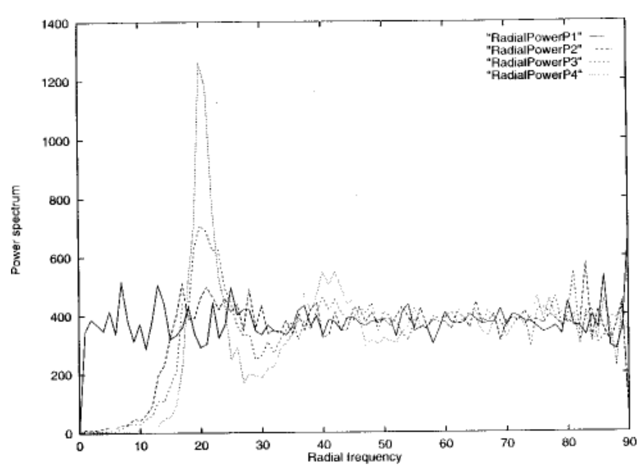

Figure 4. Radially averaged power spectrum of binary patterns

Conclusion

In this paper, we proposed a new algorithm based on electrostatics model, which if combined with current filtering technique, will generate visually pleasing blue noise pattern to be used as a “seed” in stochastic screen design.

Reference

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4. R. A. Ulichney, *Digital Halftoning*, MIT Press, Cambridge, Mass. (1987).
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