The Physics Behind the Yule-Nielsen Equation

J. S. Arney
Center for Imaging Science, Rochester Institute of Technology
Rochester, New York

S. Yamaguchi
Nippon Paper Industries, Imaging Technology Center
1551 Higashidaira, Higashimatsuyama, Saitama, Japan

Abstract

The Yule-Nielsen equation has been applied extensively and successfully over the past half century to calibrate halftone printing with technologies ranging from web-offset to desk top non-impact in color as well as monochrome. The Yule-Nielsen equation has also been used as a starting point for modeling complex colorant-paper interactions in a variety of printing technologies. The utility of the Yule-Nielsen equation has lead many to believe it may represent some physical reality in the halftone process, and this belief has lead many to search for a physical rational for the equation. In this report we examine the physical properties of halftone systems and their impact on the Yule-Nielsen "n" factor. Yule-Nielsen equation is not a description of the physical mechanism of halftone imaging, but it is a useful empirical approximation. The result of this study indicate that in addition to physical and optical dot gain, the n factor which most closely approximates halftone behavior is strongly influenced by the sharpness of the halftone dot edges and by scattering of light within the ink itself.

Introduction

The Yule-Nielsen equation may be expressed in terms of reflectance as shown in equation (1). The original utility of this equation was for calibrating a halftone printing operation.[1,2,3,4] If the printer measured the reflectance of the paper in his press, R_g, and

$$R = \left[F_s \cdot R_k^{1/n} + (1-F_s) \cdot R_g^{1/n}\right] \qquad (1)$$

the reflectance of a solid patch on ink on that paper, R_k, then the reflectance of the halftone image, R, could be predicted by knowing the halftone dot area fraction, F_s, on the film used to make the printing plate. The calibration constant, n, was found by trial and error.

The Yule-Nielsen equation is equally useful for calibrating halftones produced with digital, non-impact technologies by defining F_s as the dot area fraction that the computer commands to printer to print. Moreover, extension of the Yule-Nielsen equation to color systems has been quite successful in several forms.[5,6,7] In all cases the n factor is found by trial and error. A survey of literature reports indicates the value of n can range from 1 to infinity and depends strongly on the paper, the ink, the print technology, and the printing conditions. The extensive success of the Yule-Nielsen equation has lead many researchers over the past half century to search for ways to determine the calibration constant, n, from more fundamental physical parameters of the printing system. This report summarizes the current state of this search.

A Statement of Physics

Halftone imaging can be described as a subset of the law of conservation of energy. The total photon energy reflected from the halftone image must be the sum of the photon energy from the dots and from the paper between the dots. Expressed as a ratio of the incident photon energy, this can be expressed as equation (2), often called the Murray-Davies equation.[4]

$$R = F \cdot R_i \cdot (1-F) \cdot R_p \qquad (2)$$

R is same as the R in equation (1), but R_i and R_p represent the reflectance of the ink and paper within the halftone image, and not the reflectance of the solid ink, R_k, and un-printed paper, R_g. Moreover, F is the actual dot area fraction produced on the printed paper, not the fraction commanded by the printing process, F_s. In the 1950s, direct measurement of R_i, R_p, and F were difficult, so by assuming $R_i = R_k$ and $R_p = R_g$, printers often estimated the actual dot area fraction, F, by measuring the halftone image reflectance, R and solving equation (2) for F.[1,2] The difference, F - F_s, was defined as "Dot Gain". By combining equations (1) and (2), the relationship between the calibration constant, n, and the "Dot Gain" of the printing process can be obtained, again assuming $R_i = R_k$ and $R_p = R_g$.

$$\text{Dot\_Gain} = \left[\frac{R_g - R}{R_g - R_k}\right] + \left[\frac{R_g^{1/n} - R^{1/n}}{R_g^{1/n} - R_k^{1/n}}\right] \quad (3)$$

The Yule-Nielsen Effect

Yule and Nielsen examined dot gain experimentally and noted that if one measures the physical area of halftone dots, F, and assumes $R_i = R_k$ and $R_p = R_g$, then equation (2) leads to a predicted reflectance that is smaller than the measured reflectance, R. This effect has been called "Optical Dot Gain", or the "Yule-Nielsen effect".[8] If we assume the law of conservation of energy to be true, then equation (2) must be true, so clearly the assumption that $R_i = R_k$ and $R_p = R_g$ must be incorrect. This has been demonstrated to be true both experimentally [9,10] and theoretically.[8,11,12,13]

It is difficult to measure R_i and R_p directly, so Yule further demonstrated the utility of his empirical equation (1), with R_k and R_g, by replacing F_s with the actual value of F, and adjusting n by trial and error to correct for optical dot gain.[1] In this way the empirical n factor was shown to be a useful and accurate parameter for calibrating a printing process to the effects of optical dot gain. In the remainder of this report, we will concentrate only on the value of n as a metric of optical dot gain and equation (1) with $F_s = F$.

A Special Case of Optical Dot Gain

A special case of the Yule-Nielsen effect is easy to demonstrate for an ideal halftone described by the three assumptions in Table I. If light is completely scattered and scrambled in the paper of such and ideal halftone, then simple addition of the light coming back from the ink dots and the paper between the dots leads equations (4) and (5) for the reflectance of the ink and the paper between the dots, R_i and R_p. This was first demonstrated in calculations by Yule and Nielsen in 1951.[1]

TableI: Assumptions for Ideal Halftone Dots

Assumption	
(A)	Ink is printed at constant thickness at all values of F.
(B)	Ink is held perfectly on the surface of the paper.
(C)	The ink obeys Beer-Lambert (zero scattering in the ink)

$$R_i = F \cdot R_k + (1-F) \cdot \sqrt{R_k \cdot R_g} \quad (4)$$

$$R_p = F \cdot \sqrt{R_k \cdot R_g} + (1-F) \cdot R_g \quad (5)$$

Applying R_i and R_p from equations (4) and (5) to equation (2) leads to equation (1) with n = 2 and $F_s = F$. This seems to suggest the Yule-Nielsen equation might be a correct physical expression of optical dot gain for all other cases at any degree of light scattering, $1<n<2$. However, more thorough analysis has subsequently shown this is not the case.

The General Theory of Optical Dot Gain

A more general analysis of the ideal halftone system of Table I involves describing the spatial distribution of the ink as a spatial transmittance function, T(x,y). One may then take the convolution integral of this function with the point spread function, PSF(x,y), for light scatter in the paper.[8,11,12,13,14] The result is again attenuated by the ink function, T(x,y), to result in the total reflectance function for the halftone image. Equation (6) summarizes the calculation with operators (•) and (*) representing multiplication and convolution, respectively.

$$R(x,y) = T(x,y) \cdot \left[T(x,y) * PSF(x,y)\right] \quad (6)$$

Taking the average of R(x,y) over a large area of dots leads to the mean reflectance of the image, R. In the limit of PSF(x,y)=1 (total scatter) one again obtains equation (1) with n = 2. In the limit of PSF(0,0) = 1, but zero everywhere else, this calculation leads to equation (2). However, for intermediate cases of scattering, equation (6) does not lead to the Yule-Nielsen equation. In other words, the Yule-Nielsen equation is not an expression of physical reality. It is, as suggested by Yule, only an empirical approximation.[1]

Approximation for Ideal Halftones

Although the "n" factor is only an empirical approximation of the behavior of halftones, it is nevertheless a very useful approximation, and it is useful to relate n approximately to real physical parameters. Thus it has been shown that for reasonably dark inks on reasonably white paper, one can approximate n with equation (7).[14,15]

$$n = 1 + \exp(-A \cdot k_p \cdot \omega) \quad (7)$$

Here, A is a constant characteristic of a given halftone geometry, ω is spatial frequency term to the dots per mm in AM halftones or the inverse of the diameter of the dot in an FM system, and k_p is the mean distance light scatters in the paper between entering and reflecting back out of the paper. Equation (7) is exactly correct in the limit of complete scattering where k_p = infinite, so n = 2. Equation (7) is also exactly correct in the limit of zero scattering where $k_p = 0$, so n = 1. For finite values of k_p, equation (7) provides a useful estimate of n for equation (1) for optical dot gain.

Real Halftones

Application of equation (7) often under-estimates the value of n needed to provide a useful, empirical correction for optical dot gain. Indeed, it is sometimes observed that the value of n needed to fit experimental data is considerably larger than n = 2. An example of this is shown in Figure 1 for a halftone gray scale (35 LPI clustered dot) printed with an HP LaserJet III ™ electrophotographic printer. The mean value of the image reflectance is shown as R, and the dotted line through this data was drawn by plotting the Yule-Nielsen equation (1) with $R_k = R_i$, $R_g = R_p$, $F_s = F$, and n = 3.8. It is convenient to define difference between the reflectance of the image, R, and the reflectance one would expect in the absence of optical dot gain, $R_o = FR_i + (1 - F)R_p$. This difference, $\Delta R = R_o - R$, is shown as a function of F in Figure 2 with the modeled line for the Yule-Nielsen equation at n = 3.8 and also at n = 2.

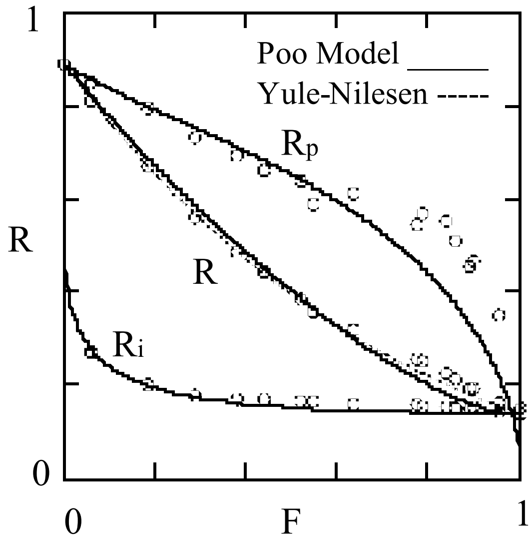

Figure 1. R, R_i, and R_p versus F for a 35 LPI clustered dot halftone from an HP LaserJet III ™ printer. Dotted line is the Yule-Nielsen model with n = 3.8. Solid line is the probability model with w = 0.27 , v = 0.1, and k = 1.

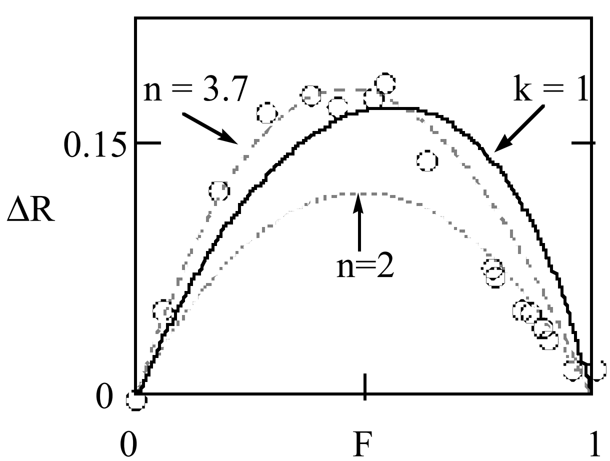

Figure 2: Difference in reflectance between R from equation (1) with n = 1 and the mean reflectance measured experimentally (O), modeled with Yule-Nielsen at n = 2 and 3.8 (------) and modeled with the probability model at w = 0.27 , v = 0.1, and k = 1 (_____).

For an ideal halftone as defined in Table I, the value of n can not exceed 2. The observed fit with n = 3.8 in Figures 1 and 2 suggests this halftone is not well described by the assumptions in Table I. Thus, a model which accounts for non-ideal behavior is needed. A useful non-ideal halftone model has been developed from a simplified version of the model in equation (6).

A Probability Model of Ideal Halftones

Instead of using the point spread function of light, PSF(x,y), and a convolution calculation as shown in of Eq. (6), one may model the Yule-Nielsen effect with a description of the average probability, P_{00}, that light which enters the paper between halftone dots will return to the surface between halftone dots.[15,16] The other probabilities (P_{01}, P_{10}, and P_{11}) describe the probability that light which enters between the dots emerges under a dot, etc., where subscripts 0 and 1 represent paper and dot respectively. Equation (8) has been shown to be a useful approximation for P_{00} for clustered halftone dots.[16,17]

$$P_{00} = 1 - F \cdot \left(2 - F^w - (1-F)^w\right) \tag{8}$$

In equation (8), w = (-A k_p ω) and thus from equation (7) we have n = w + 1. In addition, the value of P_{11} is related to P_{00} as follows,

$$P_{11} = 1 - (1 - P_{00}) \cdot \left[\frac{1-F}{F}\right] \tag{9}$$

and the values of R_p and R_i are given by equations (10) and (11).[16,17]

$$R_p = Rg \cdot T_p \cdot \left(P_{00} \cdot (T_p - T_i) + T_i\right) \tag{10}$$

$$R_i = Rg \cdot T_i \cdot \left(P_{11} \cdot (T_i - T_p) + T_p\right) \tag{11}$$

The transmittance T_p is the transmittance of the layer of colorant over the paper between the dots. This value is T_p = 1 since there is no colorant between the dots in an ideal halftone. The transmittance of the ink layer, T_i, is related to the measured values of R_k and R_g as follows for an ideal system.

$$T_i = \sqrt{R_k \cdot R_g} \tag{12}$$

Application of equations (12), (8), (9), (10), (11), and (2), with the constant w adjusted to provide a best fit, has been shown to model not only R versus F but also R_i and R_p versus F.[16,17]

A Modification for Scattering Inks

Non-ideal behavior can be introduced into the probability model as shown previously by assuming the ink scatters some of the light before it passes through the dot.[16,17] This produces a reflectance factor, R_{ii}, that would be observed if the ink were printed on top of a perfectly black substrate (R_g = 0). We define k as the ratio $k = R_{ii}/R_k$ and use k as a second arbitrary for fitting the model to experimental data. However, we restrict the value, $0 < k < 1$. Then from k and measured values of R_k we obtain R_{ii}, Kubelka-Munk theory can be solved for the transmittance of the scattering ink, T_{io}. As a simplification, it can be shown numerically that T_{io} from Kubelka-Munk theory can be approximated closely by equation (13) for inks with $R_k < 0.3$.

$$T_{io} = \sqrt{\frac{R_k - R_{ii}}{R_g}} \tag{13}$$

Then T_{io} replaces T_i in equations (10) and (11). By adjusting k and w as empirical parameters, equations (13), (8), (9), (10), (11), and (2) may be applied in order to model R, R_i and R_p versus F from measured values of R_k and R_g. Moreover, ink penetration into the substrate has been shown to behave is if the scattering of the ink increased.[16] Thus, the empirical parameter k may be used to model either an ink with a significant scattering coefficient or an ink that penetrates into the substrate.

A Modification for Edge Effects

Application of the k-modified probability model can fit the data in Figures 1 and 2 moderately well except for R_i versus F in the lower regions of F. An additional modification of the probability model to describe the sharpness of the edge of the halftone dots can be included in the model to account for R_i at low F. The modification, as shown previously, is an empirical description of the variation of T_i across the dots, and fill-in of colorant between the dots.[10,17] Equations (14) and (15) incorporate a third arbitrary fitting parameter, v, associated with dot edge sharpness.

$$T_i = 1 - (1 - T_{io}) \cdot F^v \tag{14}$$

$$T_p = 1 - (1 - T_{io}) \cdot \left[1 - (1 - F)^v\right] \tag{15}$$

Equation (14) describes the decrease in T_i of the dot typically observed at small values of F, and equation (15) describes colorant buildup over the paper between the dots as F becomes large. Both effects are associated with non-sharp edges of dots.

The probability model with both dot opacity and dot edge sharpness involves three arbitrary fit parameters, w, k, and v. By minimizing RMS error simultaneously over the three sets of data (R_i, R_p, and R versus F), the solid lines shown in Figure 1 are obtained. The corresponding solid line in ΔR vs F is shown in Figure 2.

Summary of n Factors

Physical dot gain and the lateral scatter of light (optical dot gain) have both been known for many decades to influence the n factor which best fits the empirical Yule-Nielsen equation. Physical dot gain easily accounts for n values from 1 to infinity when correlating data with the dot fraction, F_s, commanded by the printer. However, if F in the Yule-Nielsen equation is the actual printed dot size, measured by microdensitometry, then optical dot gain and the Yule-Nielsen effect is able to rationalize n values only up to 2.0 for ideal halftone systems. However, by including edge effects and the scattering of light within the colorant of the dots, n values can be rationalized from 1 to infinity.

By varying the parameters w, k, and v in the model, several interesting insights emerge for the system with $R_g = 0.89$ and $R_k = 0.14$, as summarized in Table II. For example, if w = 0 (no lateral light scatter), k = 0 (no scatter in the dots) but v = 1 (complete edge spreading), the model becomes numerically indistinguishable from the Yule-Nielsen equation at n = 2. Similarly, for w = 1, but v = k = 0 (maximum optical dot gain), the model is indistinguishable numerically from Yule-Nielsen at n = 2, and for w = v = k = 0, the model is numerically indistinguishable from Yule-Nielsen at n = 1. From the examining the model at these and other limits, as summarized in Table II, it is evident that values of n > 0 can be rationalized by the probability model only by assuming some degree of light scatter within the ink dots.

Table II: Summary of Correlation Between The Yule-Nielsen and Probability Models. The ≡ indicates the models are numerically identical in R vs F.

w	*v*	*k*	*n*
0	0	0	≡ 1
1	0	0	≡ 2
0	1	0	≡ 2
0	0	1	≡ 1
1	1	0	≡ 2
0.6	0	1	∞
0	0.4	1	∞
1	0	0.9	∞

References

1. J. A. Yule and W. J. Nielsen, *TAGA Proceedings*, p.65 (1951)
2. F. R. Clapper and J. A. Yule, *J. Opt. Soc. Am*, **43**: 600 (1953)
3. J. A. Yule, D. J. Howe, and J. H. Altman, *TAPPI*, **50**: 337 (1967)
4. A. Murray, *J. Franklin Inst.*, **221**: 721 (1936)

5. I. Pobboravsky and M. Pearson, Report 150, Info. Service, Graphic Arts Research Center, Rochester Inst. of Tech., Rochester, NY (1972).
6. J. A. Stefen Viggiano, *TAGA Proceedings*, p. 647 (1985) and p. 44 (1990).
7. Y. Shiraiwa and T. Mizuno, *J. Imag. Sci. Technol*.,**37**: 385 (1993).
8. For a current review of optical dot gain see S. Gustavson, *J. Imag. Sci. Technol.*,**41**: 283 (1997).
9. Peter G. Engeldrum, *J. Imag. Sci. & Tech*., **38**, 545 (1994)
10. J. S. Arney, P. G. Engeldrum, and H. Zeng, *J. Imag. Sci. Technol.*, **39**, 502 (1995).
11. G. L. Rogers, *J. Imag. Sci. Technol*., **42**: 643 (1997).
12. B. Kruse and M. Wedin, *TAGA Proceedings*. p. 329 (1995).
13. Peter G. Engeldrum, *J. Imag. Sci. & Tech*., **40**, 240 (1996)
14. F. Ruckdeschel and O.G. Hauser, *Applied Optics*, **17**, 3376 (1978)
15. J. S. Arney and M. Katsube, *J. Imag. Sci. Technol.*, **41**, 637 (1997).
16. J. S. Arney, et. al, *J. Imag. Sci. Technol*., **41**: 633 (1997). **41**: 331, 335 (1998).
17. J. S. Arney and S. Yamaguchi, submitted *J. Imag. Sci. Technol*. Fall,1998.