Fluorescent Ink Halftone with Optical Dot Gain

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Abstract

A model of a fluorescent ink halftone print is presented. The model includes the effects of optical dot gain – the scattering and diffusion of light within the paper – and includes the effects of multiple reflections between the paper substrate and the ink-layer. A formula for the halftone spectral reflectance is derived, expressed in terms of the ink fluorescence and absorption spectra, and the probability functions that characterize diffusion within the paper.

Introduction

Recently a model of the reflectance of fluorescent ink on paper was presented[1, 2]. The current work extends this theory to the case of the halftone print, and includes the effects of optical dot gain[6, 7]. The theory uses a modified Clapper-Yule model[4, 3, 5] to account for multiple internal reflections between the ink-layer and the paper substrate. In the first section, the model is presented as a pair of coupled first order differential equations with boundary conditions. In the second section, the equations are solved for the primary photon streams – the photons that are absorbed and create the fluorescent photons. In the third section, the equations are solved for the fluorescent photon streams. In the last section, the reflected streams are combined to give the halftone spectral reflectance.

Development of the Model

The model consists of an ink-layer with thickness t on a paper substrate. Light is incident on the top surface of the ink-layer, is transmitted through the layer, is multiply reflected between the layer and the paper substrate, and is transmitted through the layer on reflection. The model assumptions are:
1) The layer is *thin*: i.e. if a transmitted photon enters the layer at the point x, y, it exits the layer at the same x, y point.
2) The ink-layer is defined by the dot array functions $\mathcal{C}_0(x, y)$ and $\mathcal{C}_1(x, y)$[6]. Areas of the ink-layer that are covered with ink are specified by $\mathcal{C}_1(x, y)$: $\mathcal{C}_1(x, y)$ is equal to 1 at points x, y covered by ink, and is zero if it is void of ink. Regions void of ink are specified by $\mathcal{C}_0(x, y) = 1 - \mathcal{C}_1(x, y)$. The fractional area of ink is $\mu_1 = \int \mathcal{C}_1(x, y)\mathrm{d}x\,\mathrm{d}y$, and $\mu_0 = 1 - \mu_1$ is the fractional area void of ink.
3) Since the paper and ink have approximately equal indices of refraction, the reflectance at the paper-ink boundary is negligible, and the reflectance at the paper-air and ink-air boundaries have the same value r.
4) As incident light passes through the ink-layer, it is selectively absorbed by the ink to create fluorescent photons. The absorption and fluorescence spectra do not overlap, so the fluoresced photons are not absorbed by the ink.
5) One chooses $+z$ downward, with $z = 0$ the top surface of the ink-layer and $z = t$ the bottom surface. The incident light is diffuse, and the paper is assumed to be Lambertian reflecter.

One defines $I_+(x, y, z; \lambda)$ to be the downward $(+z)$ photon stream, and $I_-(x, y, z; \lambda)$ is the upward stream $(-z)$. Within the ink layer, I_+ and I_- satisfy the following coupled equations, as derived in Ref. [1]:

$$\frac{\mathrm{d}}{\mathrm{d}z} I_+(z; \lambda) = -\Sigma(\lambda) I_+(z; \lambda) \\ \frac{Q}{2} f(\lambda) \int \Sigma(\lambda') \Big[I_+(z; \lambda') + I_-(z; \lambda') \Big] \mathrm{d}\lambda', \quad (1)$$

$$-\frac{\mathrm{d}}{\mathrm{d}z} I_-(z; \lambda) = -\Sigma(\lambda) I_-(z; \lambda) \\ + \frac{Q}{2} f(\lambda) \int \Sigma(\lambda') \Big[I_+(z; \lambda') + I_-(z; \lambda') \Big] \mathrm{d}\lambda', \quad (2)$$

with

$$\Sigma(x, y; \lambda) = \mathcal{C}_1(x, y)\, \varepsilon(\lambda)$$

and $\varepsilon(\lambda)$ the extinction coefficient. Points x, y at which there is ink, $\Sigma = \varepsilon$; points void of ink, $\Sigma = 0$. The normalized fluoresence spectrum is $f(\lambda)$, and Q is the quantum efficiency. The absorption, $\varepsilon(\lambda)$, and fluorescence spectra do not significantly overlap.

The boundary condition for the top surface is:

$$I_+(x,y,z=0) = I_0 + r_t(x,y)\, I_-(x,y,z=0), \quad (3)$$

where $I_0(\lambda)$ is the spectral power distribution of the incident light, assumed spatially uniform, and the second term represents the internally reflected upward stream, with r_t the internal reflection from the top of the ink-layer:

$$r_t(x,y) = r\, \mathcal{C}_1(x,y)$$

where r is the internal reflectance at the ink-air boundary. At those points x, y with ink, the reflectance from the top of the ink-layer is r, at those points of the layer void of ink, the reflectance is 0.

The boundary condition for the upward stream at the lower surface includes the multiple internal reflections between the paper and the bottom surface of the ink-layer:

$$\begin{aligned} I_-(x,y,t) &= (1-r_b) \\ &\times \Big\{ h * I_+(x,y,t) + h * \Big[r_b\ h * I_+(x,y,t) \Big] \\ &+ h * \Big[r_b\ h * \Big(r_b\ h * I_+(x,y,t) \Big) \Big] + \ldots \Big\} \quad (4) \end{aligned}$$

with $*$ indicating a convolution. The paper's point spread function (PSF)[6, 7] is $h(x,y)$ with normalization:

$$\int\!\!\int h(x,y)\, \mathrm{d}x\, \mathrm{d}y = r_g$$

with r_g the reflectance of the the paper substrate. The downward reflectance from the lower surface of the ink-layer is:

$$r_b(x,y) = r\, \mathcal{C}_0(x,y)$$

The bottom surface of the ink-layer reflects downward only at those points void of ink; at points covered by ink, there is no downward reflection from the lower ink-layer surface.

The downward stream passes from the ink-layer into the paper, is scattered and reflected upwards. Each term of the series represents a scatter/reflection process: the upwardly scattered light is internally reflected back into the paper from the bottom surface of the ink-layer, to be scattered upwards, etc. The different terms are shown in Figure 1.

The object is to solve for the upward stream $I_-(x,y,0;\lambda)$ and then integrate over the surface to obtain the average reflected intensity:

$$\langle I_-(\lambda) \rangle = \frac{1}{\text{area}} \int_{(\text{area})} \Big[1 - r_t(x,y) \Big]\, I_-(x,y,0;\lambda)\, \mathrm{d}x\, \mathrm{d}y. \quad (5)$$

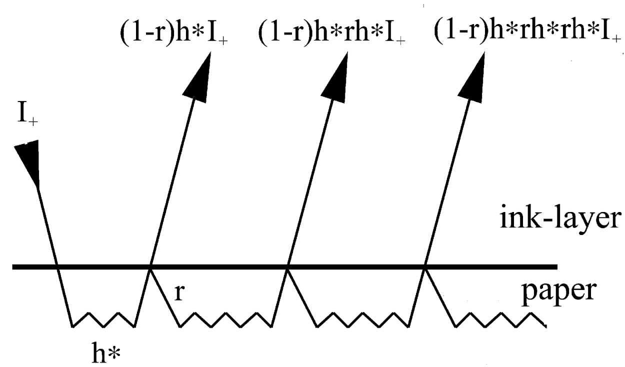

Figure 1. The multiple scattering/reflection processes that contribute to the upward primary stream at the ink-layer lower boundary.

The upward and downward streams, I_- and I_+, can each be separated into two parts: a part consisting of the fluorescent photons – upwards and downwards fluorescent streams: $u_\pm$ – and a part consisting of the originally incident photons – upward and downward primary streams: $v_\pm$.

$$I_\pm(z) = v_\pm(z) + u_\pm(z).$$

Since the absorption and fluorescent spectra do not significantly overlap, the equations for the photon streams, Eqs. (1) and (2), can also be separated:

$$\pm \frac{\mathrm{d}}{\mathrm{d}z} v_\pm(z) = -\Sigma\, v_\pm(z) \quad (6)$$

and

$$\pm \frac{\mathrm{d}}{\mathrm{d}z} u_\pm(z) = \mathrm{F}_\lambda\, \Sigma \Big[v_+(z) + v_-(z) \Big] \quad (7)$$

where to simplify the notation the integral operator F_λ is defined as:

$$\mathrm{F}_\lambda\, g = \frac{Q}{2} f(\lambda) \int g(\lambda')\, \mathrm{d}\lambda'.$$

Primary Streams

The equations Eq. (6) can be integrated to get:

$$\begin{aligned} v_+(z) &= \exp[-\Sigma z]\, v_+(0) \\ v_-(z) &= \exp[-\Sigma(t-z)]\, v_-(t) \quad (8) \end{aligned}$$

with $v_+(0)$ and $v_-(t)$ obtained from the boundary conditions:

$$v_+(0) = I_0 + r_t\, \exp(-\Sigma t)\, v_-(t) \quad (9)$$

$$\begin{aligned} v_-(t) = (1-r_b) \times \Big(& h * \Big[\exp(-\Sigma t)\, v_+(0) \Big] \\ & + h * \Big\{ r_b\, h * \Big[\exp(-\Sigma t)\, v_+(0) \Big] \Big\} + \ldots \Big) \quad (10) \end{aligned}$$

Inserting the expression for $v_+(0)$, Eq. (9) into Eq. (10) one can solve for $v_-(t)$ iteratively as a power series in r, to get:

$$v_-(t) = I_0\,(1-r_b) \times \Bigg[h * \exp(-\Sigma t) + h * \Big\{ R\,h * \exp(-\Sigma t) \Big\} + h * \bigg(R\,h * \Big\{ R\,h * \exp(-\Sigma t) \Big\} \bigg) + \ldots \Bigg] \quad (11)$$

with R defined as:

$$R = r_b + r_t \exp(-2\varepsilon t).$$

This is the total reflectance back into the paper from the ink-layer. The ink transmittance is $\tau = \exp(-\varepsilon t)$, so that the reflectance R back into the paper from the ink-layer is equal to r at points void of ink, and equal to $r\tau^2$ at points with ink, as shown in Figure 2.

The expression for $v_-(t)$, Eq. (11), is essentially the same as that used in the modified Clapper-Yule model of halftone reflectance[5]. Using the results of Ref. [5], one can write $v_-(t)$ as:

$$v_-(t) = I_0(1-r_b)\Big\{ \mathcal{C}_0'\Big[\mathbf{P}_{00}+\mathbf{P}_{01}\,\tau\Big] + \mathcal{C}_1'\Big[\mathbf{P}_{10}+\mathbf{P}_{11}\,\tau\Big] \Big\} \quad (12)$$

where $\mathcal{C}_1' = \mu_n^{-1}\,\mathcal{C}_n$. The elements of the matrix $\mathbf{P}$ are the diffusion probabilities: the joint probabilities that a photon enter and exit the paper through the different regions of the halftone microstructure. For example, the element $\mathbf{P}_{10}$ is the probability that a photon enter the paper through a region void of ink, diffuse and internally reflect within the paper, and and then exit through a region with ink. In Reference [5], it is shown that $\mathbf{P}$ is given by:

$$\mathbf{P} = \left[\mathbf{1} + \sum_{n=1}^{\infty} \Big(\mathbf{h}'\,\mathbf{R} \Big)^n \right] \mathbf{h}$$

where $\mathbf{1}$ is the identity matrix, and

$$\mathbf{R} = \begin{bmatrix} r & 0 \\ 0 & r\tau^2 \end{bmatrix}.$$

The elements of the matrix $\mathbf{h}$ are bracket integrals[8]:

$$\mathbf{h}_{nm} = \frac{1}{\text{area}} \int\!\!\int \mathcal{C}_n(x,y)\, h * \mathcal{C}_m(x,y)\, \mathrm{d}x\, \mathrm{d}y$$

and

$$\mathbf{h}'_{nm} = \frac{\mathbf{h}_{nm}}{\mu_m}$$

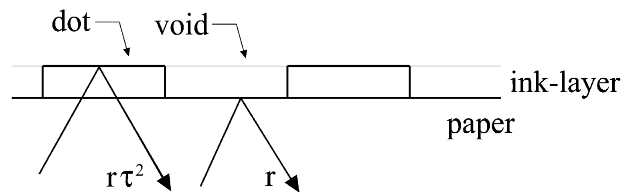

Figure 2. The ink-layer showing region of ink – the dots – and regions void of ink. The ink-layer internal reflectance is r from points void of ink, and $r\tau^2$ from points with ink.

The downward stream, $v_+(z)$ is given by Eqs. (8) and (9):

$$v_+(z) = \exp(-\Sigma z)\, \Big[I_0 + r_t\, \exp(-\Sigma t)\, v_-(t) \Big] \quad (13)$$

Fluorescent Streams

The fluorescent streams, $u_\pm(z)$ are the solutions to Eq. (7), with the following boundary conditions, from Eqs. (3) and (4):

$$u_+(0) = r_t\, u_-(0) \quad (14)$$

$$u_-(t) = (1-r_b) \Bigg\{ h * u_+(t) + h * \Big[r_b\; h * u_+(t) \Big] + h * \bigg(r_b\; h * \Big[r_b\; h * u_+(t) \Big] \bigg) + \ldots \Bigg\}. \quad (15)$$

Equations (7) can be immediately integrated using Eqs. (8) and (13) to get:

$$u_+(t) = u_+(0) + S, \quad u_-(0) = u_-(t) + S \quad (16)$$

where the source term for the flourescent light, S, is given by:

$$S = \mathrm{F}_\lambda \Big[1 - \exp(-\Sigma t) \Big] \Big[v_+(0) + v_-(t) \Big].$$

Fluorescent photons are created as the the downward stream passes through the ink-layer and as the upward stream passes through the ink-layer. There are two parts contributing to the downward stream: the incident light and the part of the upward stream reflected downwards from the top of the ink-layer.

The fluorescent photons are multiply reflected between the ink-layer and the paper, with no difference in reflection between dots and voids since the fluorescent photons are not absorbed by the ink. Therefore, using Eqs. (14) and (16) the multiple reflections can be summed exactly to give the total reflectance $R_p = r_g(1-r)/(1-r_g r)$. One then finds that the spatially averaged $u_-(0)$ is proportional to

$$\Big[(1-r)+(1+r)\,R_p\Big]\,\langle S\rangle.$$

There are three streams of multiply reflected fluorescent photons, corresponding to the three terms in $u_-(0)$: as shown in Figure 3, (a) is $(1-r)\langle S\rangle$, (b) is $r\,R_p\,\langle S\rangle$, and (c) is $R_p\,\langle S\rangle$, with the fluorescent source:

$$\langle S\rangle = \mathrm{F}_\lambda I_0\,(1-\tau)\Big[\mu_1+\Big(1+r\tau\Big)\Big(\mathbf{P}_{10}+\tau\,\mathbf{P}_{11}\Big)\Big]$$

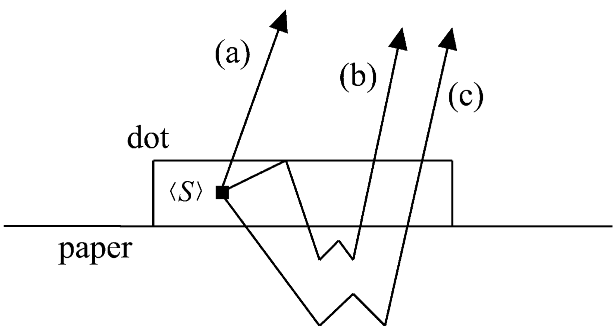

Figure 3. The three contributions to the fluorescent light: a) $(1-r)\langle S\rangle$*, b)* $rR_p\langle S\rangle$*, and c)* $R_p\langle S\rangle$

Halftone Reflectance

The average reflected radiation is given by Eq. (5), and the halftone reflectance is:

$$R_H = \langle I_-\rangle / I_0$$

The contribution to the halftone reflectance from the upward primary stream, $\langle(1-r_t)v_-(0)\rangle$, is:

$$\langle(1-r_t)v_-(0)\rangle = I_0(1-r)\Big\{\mathbf{P}_{00}+\tau\,\mathbf{P}_{01}+\tau\,\mathbf{P}_{10}+\tau^2\,\mathbf{P}_{11}\Big\}$$

The contribution to the halftone reflectance from the upward fluorescent stream, $\langle(1-r_t)\,u_-(0)\rangle$, is:

$$\langle(1-r_t)u_-(0)\rangle = \Big[(1+R_p)-r\,(1-R_p)\Big]\,\langle S\rangle$$

Combining these in Eq. (5) one obtains the halftone reflectance:

$$R_H = \Big\{(1-r)\Big[\mathbf{P}_{00}+\tau\,\mathbf{P}_{01}+\tau\,\mathbf{P}_{10}+\tau^2\,\mathbf{P}_{11}\Big] + \Big[(1+R_p)-r(1-R_p)\Big]I_0^{-1}\,\mathrm{F}_\lambda I_0\,(1-\tau) \Big[\mu_1+\Big(1+r\tau\Big)\Big(\mathbf{P}_{10}+\tau\,\mathbf{P}_{11}\Big)\Big]\Big\} \qquad (17)$$

Conclusion

The formula for the spectral reflectance of a fluorescent ink halftone has been presented. The reflectance is expressed in terms of the absorption and fluorescence spectra of the ink, and the diffusion probability functions which characterize the diffusion of photons within the paper, and the multiple reflections between the ink-layer and substrate.

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