Transmission Deviation Caused by Eccentricity in Multi Gear Systems

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Abstract

Tandem-type color printers have four photoreceptor drums which respectively transfer four colors, black, yellow, cyan, magenta, to fed papers. A major factor of the color registration error in that printer is the rotational deviation of drums. If the drums are driven by gears, the transmission deviation caused by eccentricities of gears yields the rotational deviation. Therefore, to achieve fine printing quality, it is important to clarify the transmission behavior of gears with eccentricity.

In this paper, we formulate the transmission deviation in multi gear systems in terms of the magnitudes and phases of eccentricities of gears. We describe the ratio of rotation velocities of a pair of eccentric gears by the inverse ratio of the rotation radii, and solve it as a differential equation of rotation angles. The obtained solution gives an expression of the rotational deviation. Experimental results under practical conditions ensure validity of the proposed formulation. Then, we extend the formulation to general multi gear systems. For the optimum design, we can utilize it to calculate the minimum transmission deviation achieved by appropriate adjustment of phases of eccentricities.

1. Introduction

In tandem-type color printers, rotational deviations of drums induce registration errors of four colors. A major factor of the rotational deviation is the transmission deviation caused by the eccentricities of gears in driving systems. While a number of analyses for a pair of eccentric gears have been reported[1,2,3], there is no study on the transmission deviation in general multi gear systems as far as the authors know.

In this paper, from the viewpoint of the practical printer design, we consider the position deviation on the pitch circle as the transmission deviation, and clarify the effect of eccentricity on the transmission deviation in multi gear systems. We assume that the ratio of rotation velocities of a pair of gears is equal to the inverse ratio of rotation radii. We write this relation as a differential equation of rotation angles, and solve it to compute the rotational deviation in angle and the transmission deviation on the pitch circle. The solution is verified by experiments. In this way, we obtain a formula of the transmission deviation in terms of the magnitude of eccentricity of each gear and relative eccentricity positions, namely, phases of meshing gears. We can use this result to reduce the transmission deviation by means of the phase adjustment.

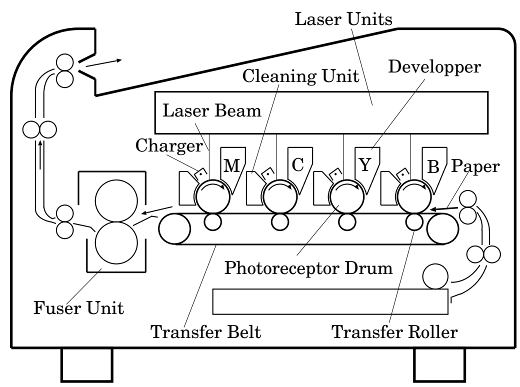

Figure 1. A tandem-type color printer

2. Multi Gear Systems for Driving Drums

The tandem-type color printer is composed of four drums as shown in Fig. 1, where the image of each color (black, yellow, cyan, magenta) is developed on the surface of a corresponding drum. After development, the drums transfer the images to the fed paper sequentially to realize the full color image. Generally, the drums are driven by one common motor through four gear systems which transmit the rotation to each drum as shown in Fig. 2. If the rotational deviations exist on the drums, the images on the drums are not developed identically in space, even though the scanning intervals of the laser units are identical in time.

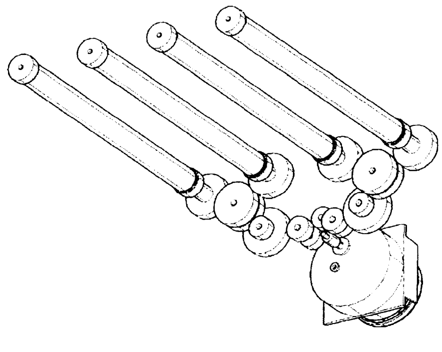

Figure 2. A multi gear system driving four drums

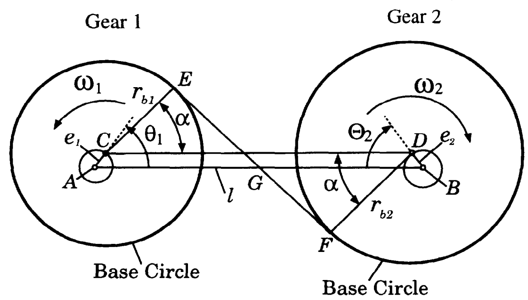

Figure 3. Meshing two gears modeled by Asano[2]

3. Formulation of Transmission Deviation

To compute the transmission deviation caused by eccentricities of gears in multi gear systems, we first formulate the transmission deviation of meshing two gears. We then compare the computed result with an experimental result. Based on this experimental verification, we extend the formula to the case of meshing three gears. We assume that the pitch and base circles are perfect in the following analysis.

3.1. Transmission Deviation of Meshing Two Gears

Before we present results of this paper, we review a result by Asano[2] for meshing two gears. In Fig. 3, Gears 1, 2 are the driving and driven gears, respectively, which are eccentric. Let C, D be the centers of the base circles of Gears 1, 2, and r_{b1}, r_{b2} be their radii. We denote the pressure angle by α, the line of action by EF, and the centers of rotation of Gears 1, 2 by A, B, respectively. The counterclockwise angle $\angle CAB$ and the clockwise angle $\angle DBA$ are denoted by θ_1 and Θ_2. The rotation velocities of Gears 1, 2 are ω_1 and ω_2. Then, the ratio of the rotation velocities $\omega_1/\omega_2 = d\Theta_2/d\theta_1$ is given by $\overline{AG}\,/\,\overline{BG}$ where G is the intersection of the lines AB and EF. Thus, the following equation is obtained, where ℓ, e_1, e_2 are the distances between A and B, A and C, B and D, respectively.

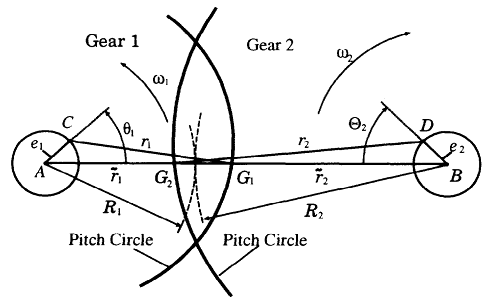

Figure 4. Proposed model of meshing two gears

$$
\frac{d\Theta_2}{d\theta_1} = \frac{\begin{bmatrix} \begin{aligned} & r_{b1}\{(\ell - e_2\cos\Theta_2 - e_1\cos\theta_1)^2 \\ & \quad +(e_1\sin\theta_1 - e_2\sin\Theta_2)^2\} \\ & +e_1(r_{b1}+r_{b2}) \\ & \quad \cdot\{(\ell - e_2\cos\Theta_2 - e_1\cos\theta_1)\cos\theta_1 \\ & \quad +(e_2\sin\Theta_2 - e_1\sin\theta_1)\sin\theta_1\} \\ & +e_1\{(e_2\cos\Theta_2 - \ell)\sin\theta_1 \\ & \quad +e_2\sin\Theta_2\cos\theta_1\} \\ & \quad \cdot\{(\ell - e_2\cos\Theta_2 - e_1\cos\theta_1)^2 \\ & \quad +(e_1\sin\theta_1 - e_2\sin\Theta_2)^2 \\ & \quad -(r_{b1}+r_{b2})^2\}^{\frac{1}{2}} \end{aligned} \end{bmatrix}}{\begin{bmatrix} \begin{aligned} & r_{b2}\{(\ell - e_1\cos\theta_1 - e_2\cos\theta_1)^2 \\ & \quad +(e_2\sin\Theta_2 - e_1\sin\theta_1)^2\} \\ & +e_2(r_{b1}+r_{b2}) \\ & \quad \cdot\{(\ell - e_1\cos\theta_1 - e_2\cos\Theta_2)\cos\Theta_2 \\ & \quad +(e_1\sin\theta_1 - e_2\sin\Theta_2)\sin\Theta_2\} \\ & -e_2\{(e_1\cos\theta_1 - \ell)\sin\Theta_2 \\ & \quad +e_1\sin\theta_1\cos\Theta_2\} \\ & \quad \cdot\{(\ell - e_1\cos\theta_1 - e_2\cos\Theta_2)^2 \\ & \quad +(e_2\sin\Theta_2 - e_1\sin\theta_1)^2 \\ & \quad -(r_{b1}+r_{b2})^2\}^{\frac{1}{2}} \end{aligned} \end{bmatrix}} \qquad (1)
$$

By solving this differential equation numerically, we can obtain the relationship between the rotation angles θ_1 and Θ_2 of Gears 1, 2. However, since the angle of the driven gear is not described by that of the driving gear explicitly, this result can not be extended to more general multi gear systems.

For this reason, we propose an explicit description of the rotation angle of the driven gear in terms of the rotation angle of the driving gear. We consider that gears rotate as depicted in Fig. 4, where Gear 1 is the driving gear and Gear 2 is the driven gear. In the figure, C, D are the centers of pitch circles, and r_1, r_2 are their radii. The centers of rotation are A and B. The distances of A and B, A and C, B and D are ℓ, e_1, e_2. We denote the intersections of the line AB and the pitch circles of Gears 1, 2 by G_1 and G_2, respectively. While the pitch circles are drawn as intersecting each other in Fig. 4, they may be apart at some rotation angle because of eccentricities of gears. The distance between A and G_1, B and G_2 are $\tilde{r}_1$ and $\tilde{r}_2$. The counterclockwise angle $\angle CAG_1$ and the clockwise angle $\angle DBG_2$ are denoted by θ_1 and Θ_2.

which satisfy

$$R_1 + R_2 = \ell \tag{2}$$

$$R_1 \cdot d\,\theta_1 = R_2 \cdot d\,\Theta_2 \tag{3}$$

where $d\,\theta_1$ and $d\,\Theta_2$ are infinitesimal changes of θ_1 and Θ_2. We assume that R_1 and R_2 are proportional to $\tilde{r}_1$ and $\tilde{r}_2$ as

$$R_1 = \frac{\tilde{r}_1}{\tilde{r}_1 + \tilde{r}_2}\,\ell \tag{4}$$

$$R_2 = \frac{\tilde{r}_2}{\tilde{r}_1 + \tilde{r}_2}\ell. \tag{5}$$

Hence, Eq. (3) is reduced to

$$\tilde{r}_1 \cdot d\,\theta_1 = \tilde{r}_2 \cdot d\,\Theta_2. \tag{6}$$

According to the cosine theorem, $\tilde{r}_1$ and $\tilde{r}_2$ satisfy

$$r_1^2 = \tilde{r}_1^2 + {e_1}^2 - 2\tilde{r}_1 e_1 \cos\theta_1$$

$$r_2^2 = \tilde{r}_2^2 + {e_2}^2 - 2\tilde{r}_2 e_2 \cos\Theta_2,$$

which imply

$$\tilde{r}_1 = \frac{2e_1\cos\theta_1 + \sqrt{4({r_1}^2 - {e_1}^2) + 4{e_1}^2\cos^2\theta_1}}{2}$$

$$\tilde{r}_2 = \frac{2e_2\cos\Theta_2 + \sqrt{4({r_2}^2 - {e_2}^2) + 4{e_2}^2\cos^2\Theta_2}}{2}.$$

Then, since $e_1^2 \ll r_1^2$, $e_2^2 \ll r_2^2$, we obtain

$$\tilde{r}_1 \cong r_1 + e_1\cos\theta_1 \tag{7}$$

$$\tilde{r}_2 \cong r_2 + e_2\cos\Theta_2. \tag{8}$$

Now, we introduce θ_2 as a new rotation angle of Gear 2. The angle θ_2 is defined to be 0 when $\theta_1 = 0$. Using θ_2, we write

$$\Theta_2 = \theta_2 + P_2, \tag{9}$$

where P_2 is the angle Θ_2 at $\theta_1 = 0$ and $\theta_2 = 0$, which is the phase of Gear 2. Then, Eq. (6) is written as

$$\begin{aligned}&(\,r_1 + e_1\cos\theta_1\,)\,d\,\theta_1\\ &\quad = \{\,r_2 + e_2\cos(\,\theta_2 + P_2\,)\,\}\,d\,\theta_2.\end{aligned} \tag{10}$$

We integrate both sides to obtain

$$\begin{aligned}&r_1\theta_1 + e_1\sin\theta_1\\ &\quad = r_2\theta_2 + e_2\sin(\theta_2 + P_2) - C,\end{aligned} \tag{11}$$

where C is an integration constant. Since $\theta_2 = 0$ when $\theta_1 = 0$,

$$C = e_2\sin P_2. \tag{12}$$

We define $\Delta\theta_2$ such that

$$\theta_2 = \frac{r_1}{r_2}\theta_1 + \Delta\theta_2 \tag{13}$$

which represents the deviation of the rotation angle of Gear 2 from the ideal case that Gears 1, 2 have no eccentricity. By Eqs. (11), (12), and (13), $\Delta\theta_2$ is computed as follows. We first apply the addition theorem.

$$\begin{aligned}&r_1\theta_1 + e_1\sin\theta_1\\ &\quad = r_2\theta_2 + e_2\sin(\theta_2 + P_2) - e_2\sin P_2\\ &\quad = r_2\left(\frac{r_1}{r_2}\theta_1 + \Delta\theta_2\right)\\ &\qquad + e_2\sin\left(\frac{r_1}{r_2}\theta_1 + \Delta\theta_2 + P_2\right)\\ &\qquad - e_2\sin P_2\\ &\quad = r_2\left(\frac{r_1}{r_2}\theta_1 + \Delta\theta_2\right)\\ &\qquad + e_2\left\{\sin\left(\frac{r_1}{r_2}\theta_1 + P_2\right)\cos\Delta\theta_2\right.\\ &\qquad\qquad \left. + \cos\left(\frac{r_1}{r_2}\theta_1 + P_2\right)\sin\Delta\theta_2\right\}\\ &\qquad - e_2\sin P_2\end{aligned} \tag{14}$$

Since $|\Delta\theta_2| \ll 1$, we use the approximations $\cos\Delta\theta_2 \cong 1$ and $\sin\Delta\theta_2 \cong \Delta\theta_2$, which reduce the right side of Eq. (14) to

$$\begin{aligned}&\Delta\theta_2\left\{r_2 + e_2\cos\left(\frac{r_1}{r_2}\theta_1 + P_2\right)\right\}\\ &\quad + r_1\theta_1 + e_2\sin\left(\frac{r_1}{r_2}\theta_1 + P_2\right) - e_2\sin P_2.\end{aligned}$$

Then, we obtain

$$\Delta\theta_2 = \frac{e_1\sin\theta_1 - e_2\sin\left(\frac{r_1}{r_2}\theta_1 + P_2\right) + e_2\sin P_2}{r_2 + e_2\cos\left(\frac{r_1}{r_2}\theta_1 + P_2\right)}. \tag{15}$$

Since $|e_2| \ll |r_2|$, we approximate $\Delta\theta_2$ as

$$\Delta\theta_2 \cong \frac{e_1\sin\theta_1 - e_2\sin\left(\frac{r_1}{r_2}\theta_1 + P_2\right) + e_2\sin P_2}{r_2} \tag{16}$$

Multiplying Eq. (16) by r_2, we get the position deviation h_2 on the pitch circle of Gear 2, which is considered as the transmission deviation.

$$h_2 = e_1\sin\theta_1 - e_2\sin\left(\frac{r_1}{r_2}\theta_1 + P_2\right) + e_2\sin P_2 \tag{17}$$

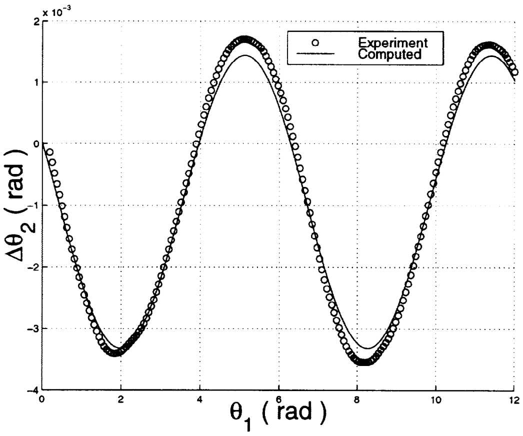

Figure 5. Experimental and computed results

In order to verify the expression of Eq. (16), we compare the computed result of $\Delta\theta_2$ with an experimental result under the conditions

$$r_1 = 32\,(\text{mm}), \quad r_2 = 32\,(\text{mm})$$
$$e_1 = 0.07\,(\text{mm}), \quad e_2 = 0.03\,(\text{mm})$$
$$P_2 = 4.71\,(\text{rad}),$$

which are shown in Fig. 5. We see that the computed result well fits the experimental result. The authors have enough experimental data which guarantee that Eq. (16) represents the deviation of the rotation angle with sufficient accuracy.

3.2. Transmission Deviation of Meshing Three Gears

We extend the result of the previous subsection for meshing two gears to the case of meshing three gears. The meshing relation of a three-gears system is shown in Fig. 6, where Gears 1, 2, and 3 are the driving, idler, and driven gears, respectively. The symbol "$*$" indicates the eccentricity position of each gear. We denote the center of rotation of Gear 3 by E. The positions H_2 and H_3 are the intersections of the line BE with the pitch circles of Gears 2, 3. Let Θ_3 be the counterclockwise angle of the eccentricity position from the line EH_3, and $\hat{P}_3$ be the angle $\angle H_2BG_2$.

Now, referring to Eq. (10), we write the relation of the rotation angles Θ_2, Θ_3 of Gears 2, 3 as

$$\begin{aligned}\{r_2+e_2\cos(\Theta_2+\hat{P}_3)\}\,d\Theta_2 \\ = \{r_3+e_3\cos\Theta_3\}\,d\Theta_3.\end{aligned} \tag{18}$$

We introduce θ_3 as a new rotation angle of Gear 3, which is defined to be 0 when $\theta_1 = 0$ and $\theta_2 = 0$. Using this definition, we write

$$\Theta_3 = \theta_3 + P_3 \tag{19}$$

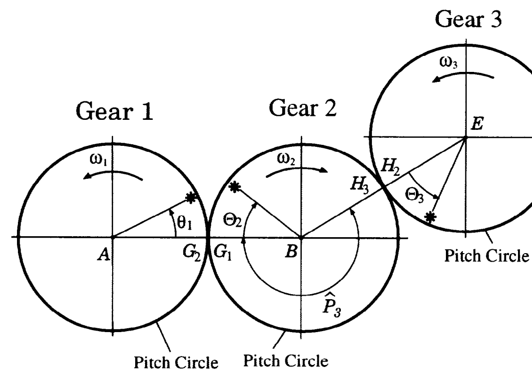

Figure 6. Meshing three gears

where P_3 is the angle Θ_3 at $\theta_1 = 0$, $\theta_2 = 0$ and $\theta_3 = 0$, which is the phase of Gear 3. Thus, from Eqs. (9), (18), and (19), we obtain

$$\begin{aligned}\{r_2+e_2\cos(\theta_2+P_2+\hat{P}_3)\}\,d\theta_2 \\ = \{r_3+e_3\cos(\theta_3+P_3)\}\,d\theta_3.\end{aligned} \tag{20}$$

By integrating this equation, we get

$$\begin{aligned}r_2\theta_2 + e_2\sin(\theta_2+P_2+\hat{P}_3) \\ = r_3\theta_3 + e_3\sin(\theta_3+P_3) \\ + e_2\sin(P_2+\hat{P}_3) - e_3\sin P_3.\end{aligned} \tag{21}$$

We define $\Delta\theta_3$ such that

$$\theta_3 = \frac{r_2}{r_3}\theta_2 + \Delta\theta_3, \tag{22}$$

which represents the angle deviation of Gear 3 from the ideal case that Gears 2, 3 have no eccentricity. In a similar way to obtain Eq. (16), we can compute $\Delta\theta_3$ as

$$\begin{aligned}\Delta\theta_3 \cong \frac{1}{r_3}\{e_2\sin(\theta_2+P_2+\hat{P}_3) \\ - e_3\sin\left(\frac{r_2}{r_3}\theta_2+P_3\right) \\ - e_2\sin(P_2+\hat{P}_3) + e_3\sin P_3\}\end{aligned} \tag{23}$$

Using Eqs. (13) and (22), we combine deviations of the rotation angles caused between Gears 1, 2 and between Gears 2, 3 to see that θ_3 is represented as

$$\begin{aligned}\theta_3 &= \frac{r_1}{r_3}\theta_1 + \frac{r_2}{r_3}\Delta\theta_2 + \Delta\theta_3 \\ &= \frac{r_1}{r_3}\theta_1 + \frac{1}{r_3}(r_2\Delta\theta_2 + r_3\Delta\theta_3).\end{aligned} \tag{24}$$

The transmission deviation on Gear 3, that is, the position deviation on the pitch circle is computed from the second

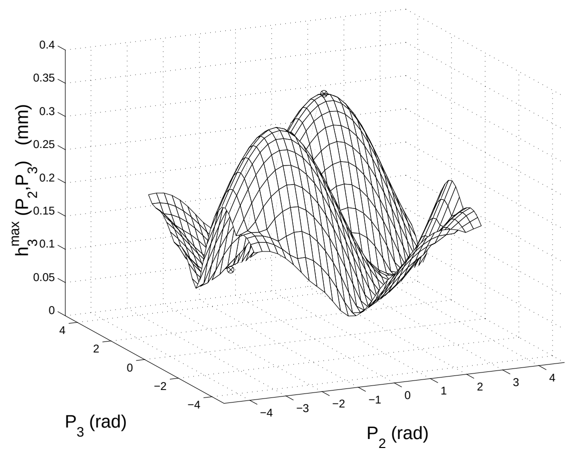

Figure 7. $h_3^{max}\,(P_2, P_3)$

of right side of Eq. (24) as

$$\begin{aligned} h_3 &= r_2\Delta\theta_2 + r_3\Delta\theta_3 \\ &= e_1\sin\theta_1 - e_2\sin\left(\frac{r_1}{r_2}\theta_1 + P_2\right) \\ &\quad + e_2\sin\left(\frac{r_1}{r_2}\theta_1 + P_2 + \hat{P}_3\right) \\ &\quad - e_3\sin\left(\frac{r_1}{r_3}\theta_1 + P_3\right) + e_2\sin P_2 \\ &\quad - e_2\sin(P_2 + \hat{P}_3) + e_3\sin P_3 \qquad (25) \end{aligned}$$

where we used Eq. (16).

It is obvious that we can readily extend this formula to the cases of meshing four or more gears, and also to the case of reduction gears.

4. Phase Adjustment

Using the expression (25), we can easily compute the maximum transmission deviation on Gear 3 with respect to the rotation angle θ_1 of Gear 1 for each pair of phases $(P_2,\,P_3)$, when the magnitudes $e_1,\ e_2,\ e_3$ of eccentricities are given. That is, we compute

$$h_3^{\max}\,(P_2,\,P_3) = \max_{\theta_1\in\boldsymbol{\theta_1}} |h_3\,(\theta_1,\,P_2,\,P_3)| \qquad (26)$$

$$-\pi\ \le\ P_2 < \pi,\quad -\pi\ \le\ P_3 < \pi$$

where the range of θ_1 is

$$\boldsymbol{\theta_1} = \{\,\theta_1 \mid 0 \le \theta_1 \le 2\pi\cdot LCM(\hat{r}_1,\,\hat{r}_2,\,\hat{r}_3)\,\},$$

$LCM(\hat{r}_1, \hat{r}_2, \hat{r}_3)$ is the least common multiple of $\hat{r}_1,\ \hat{r}_2,\ \hat{r}_3$, and $\hat{r}_1,\ \hat{r}_2,\ \hat{r}_3$ are the integer ratio of the radii $r_1,\ r_2,\ r_3$ of the pitch circles. From the result, we can obtain the following information:

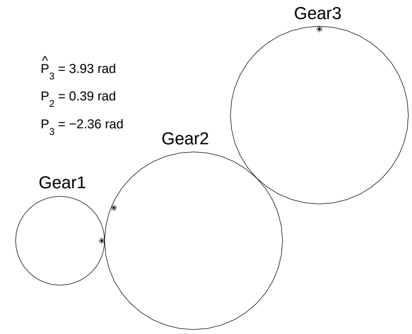

Figure 8. Location of eccentricities attaining $\min_{P_2,\,P_3} h_3^{max}\,(P_2, P_3)$

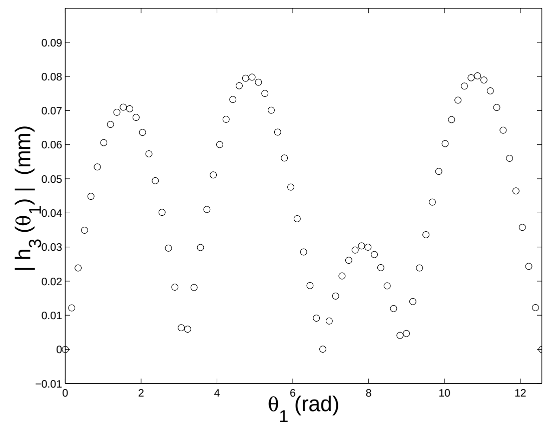

Figure 9. $|h_3(\theta_1)|\ \ [P_2 = 0.39\ (rad), P_3 = -2.36\ (rad)]$

(1) Phase adjustment of meshing gears for minimization of the transmission deviation, when the positions and magnitudes of eccentricities of gears are known.

(2) Sensitivity of the transmission deviation to the phases, which implies how accurately we need to adjust phases to reduce the transmission deviation.

For example, we compute the transmission deviation in the following case:

$$r_1 = 16\,\text{(mm)},\quad r_2 = 32\,\text{(mm)},\quad r_3 = 32\,\text{(mm)}$$

$$e_1 = 0.065\,\text{(mm)},\quad e_2 = 0.035\,\text{(mm)}$$

$$e_3 = 0.085\,\text{(mm)},\quad \hat{P}_3 = 3.93\,\text{(rad)}$$

The computed result is illustrated in Fig. 7. The minimum of $h_3^{max}\,(P_2, P_3)$, that is,

$$\min_{P_2,\,P_3} h_3^{max}\,(P_2, P_3)$$

is 0.080 (mm), which is attained at $P_2 = 0.39$ (rad) and $P_3 = -2.36$ (rad). Fig. 8 shows this phase adjustment, that is, the

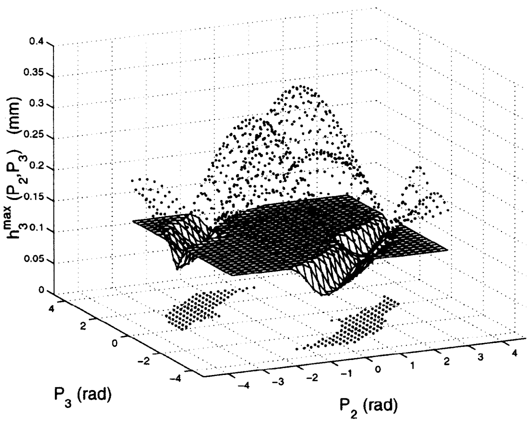

Figure 10. Region of (P_2, P_3) *such that* $h_3^{max}(P_2, P_3) < 0.13$

is, the relation of positions of eccentricities of the gears, denoted by "$*$". In this case, $|h_3(\theta_1)|$ behaves as Fig. 9. From Fig. 7, we can also see the sensitivity of the transmission deviation to the phases P_2 and P_3.

Fig. 10 shows the regions of (P_2, P_3) such that

$$h_3^{max}(P_2, P_3) < 0.13,$$

which are shaded on the (P_2, P_3) plane. This result implies that if the transmission deviation less than 0.13 (mm) is required in the three-gears system, we need to adjust (P_2, P_3) in the shaded region. In the present case, the region is not so small, and we are allowed to adjust the phases imprecisely to some extent.

On the other hand, the maximum of $h_3^{max}(P_2, P_3)$, that is,

$$\max_{P_2,\, P_3} h_3^{max}(P_2, P_3)$$

is 0.347 (mm) attained at $P_2 = 0.87$ (rad) and $P_3 = 1.27$ (rad). Fig. 11 illustrates the phases of the gears. In this case, the behavior $|h_3(\theta_1)|$ is shown by Fig. 12. If we can consider that this $\max_{P_2,\, P_3} h_3^{max}(P_2, P_3)$ is small enough, we don't need to take the phase adjustment into account.

5. Conclusions

We have considered the transmission deviation caused by the eccentricity in multi gear systems from the practical viewpoint. We presented a formula of the transmission deviation in terms of magnitudes and phases of eccentricities of gears. We showed that the computed and the experimental results of the transmission deviation are sufficiently coincident, and confirmed the validity of the formulation of this paper within the practical range (e.g., JIS B 1702-1976 class 6)[4]. We illustrated that the proposed formula

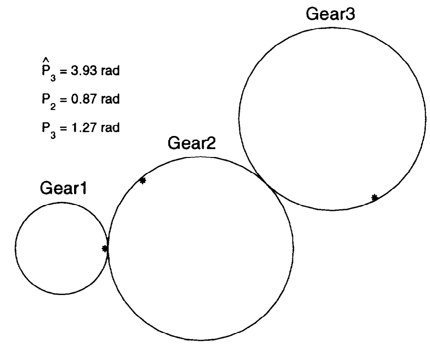

Figure 11. Location of eccentricities attaining $\max_{P_2,\,P_3} h_3^{max}(P_2, P_3)$

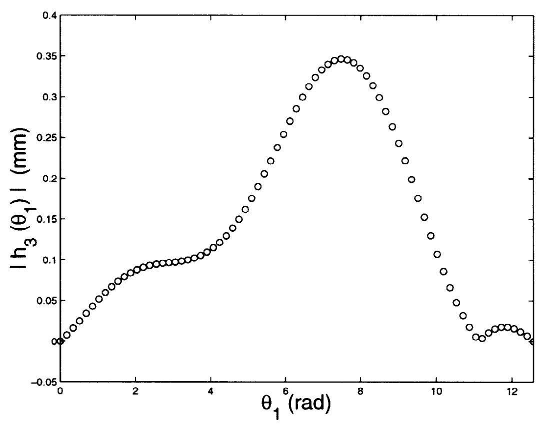

Figure 12. $|h_3(\theta_1)|$ $[P_2 = 0.87\,(rad), P_3 = 1.27\,(rad)]$

can be used for appropriate phase adjustment to reduce the transmission deviation.

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