Modulation Transfer Functions and Aliasing Patterns of CFA Interpolation Algorithms

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Introduction

Video cameras and most electronic still cameras employ a single image sensor covered with a mosaic of suitably chosen color filters, a so-called color filter array (CFA), to capture real-time color images. Each pixel only records one of the three colors (e.g. red, green, blue or cyan, magenta, yellow; some sensors employ an additional 4th color) that are required to produce a full three-channel color image. The missing pixels in each channel are obtained by interpolation from the neighboring existing pixels to reconstruct the full three-color image. Examples of commercially used CFA patterns and interpolation schemes can be found in the literature and in patents.[1-3] This paper focuses on two artifacts that are associated with this type of image capture and reconstruction: [1,4,5]

(1) Loss of sharpness is caused by the interpolation of the missing pixels in each channel from neighboring pixels and can be described by a modulation transfer function (MTF).

(2) Aliasing occurs because image sensors, like CCD arrays, record image information at regularly spaced, discrete locations, i.e. the image is sampled. Whittaker-Shannons sampling theorem implies that only those scenes that are band-limited to within one half of the sampling frequency can be fully reconstructed from the sampled image. Spatial frequency content beyond this is mapped or aliased to lower spatial frequencies. If the image sensor is covered with a CFA, each of the color channels is sampled at a frequency below that of the full sensor. Also, the regular array of sampling points in one color channel is always shifted by a known number of pixels compared with that of another channel. Especially the latter property of CFAs gives rise to the appearance of colored texture and colored fringes at edges, i.e. the chromatic nature of aliasing.

System optimization in video and electronic still capture involves, among other factors, a trade-off between aliasing and sharpness. Therefore, it is important to understand how to quantify both phenomena.

In this paper, various interpolation schemes pertaining to the Bayer pattern, shown in Figure 1, will be analyzed. While the calculation of MTFs for simple linear interpolation schemes is a straightforward application of the theory of Fourier transforms,[1,4,5] this is not the case for the more sophisticated CFA interpolation algorithms that have recently been developed for the Bayer pattern.[3] These algorithms utilize the correlation between the different color channels to obtain more accurate estimates of the missing pixels, and edge detection to decide if interpolation in the horizontal or vertical direction is preferable. We will explore whether any predictions can be made for those highly nonlinear algorithms.

GRGRGRGR
BGBGBGBG
GRGRGRGR
BGBGBGBG

Figure 1: Bayer pattern

Theory

Application of Theory of Fourier Transforms

Sharpness degradations and aliasing patterns associated with simple linear CFA interpolation schemes can be fully explained by the theory of Fourier transforms (FT). This involves the application of several known properties of FTs, i.e. additivity theorem, convolution theorem, shift theorem, similarity theorem and preservation of symmetries.[5] Figure 2 illustrates the application of Fourier theory to the interpolation of the green channel of the Bayer pattern. The input image is multiplied with the lattice function, convolved with the interpolation kernel [0.5 1 0.5] and finally convolved with a rectangle that corresponds to the pixel size. In the frequency domain, the first operation is equivalent to a convolution of the FT of the image and the lattice function. This gives rise to aliasing as the FT of the image is replicated at each individual lattice point. The two convolutions in the spatial domain translate to multiplications in the frequency domain and explain the perceptual loss of sharpness associated with CFAs. According to the additivity theorem and listings[5] of pairs of functions and their FTs, the convolution kernel [0.5 1 0.5] corresponds to the expression (1+cos (2πf)) in frequency space (f - spatial frequency in [cycles/sample]). The MTF, M, is calculated by normalizing this expression to unity at zero spatial frequency:

$$M(f) = 0.5 \cdot (1 + \cos(2\pi f)) \quad (1)$$

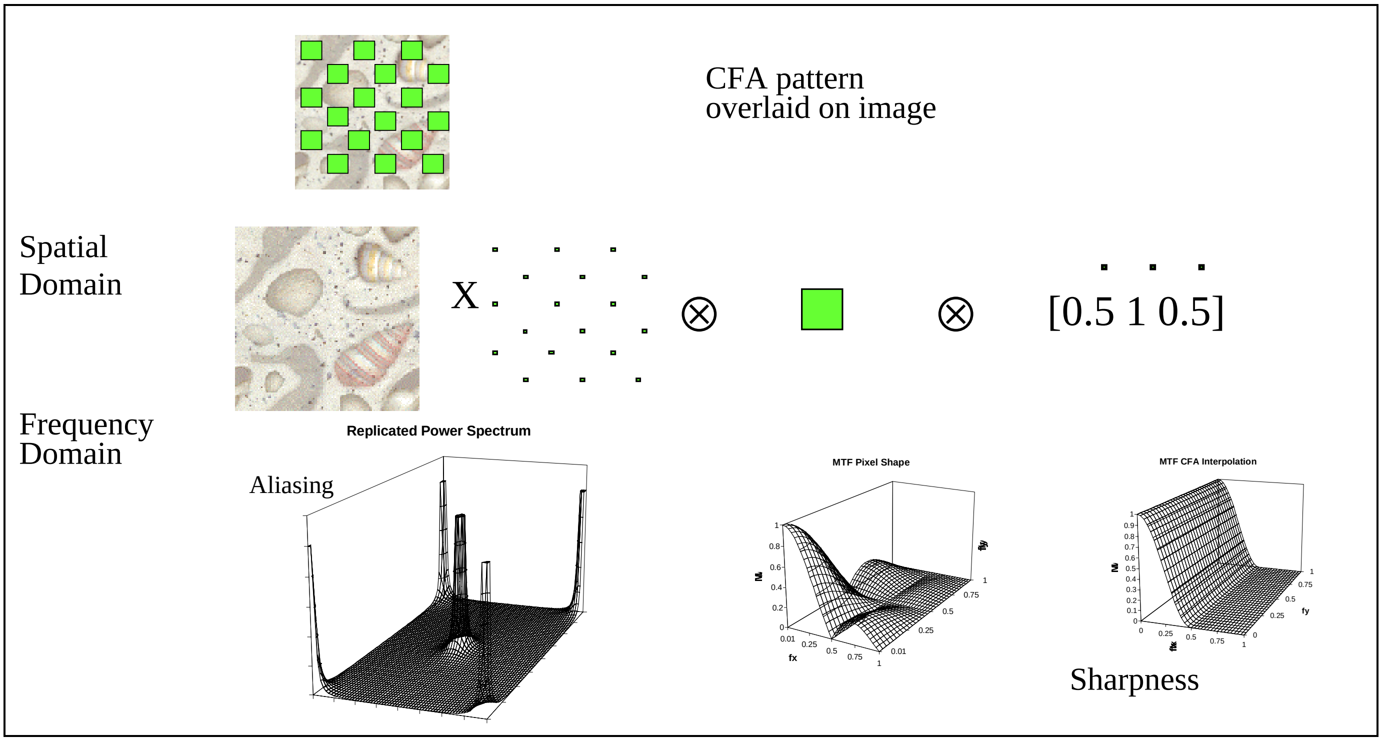

Figure 2: Illustration of CFA interpolation in the spatial and frequency domain, Bayer pattern, green channel

Finally the result is multiplied with the "sinc" function corresponding to the rectangular pixel aperture in the spatial domain.[5] This operation occurs in any pixelated system and is not part of our analysis of CFA interpolation.

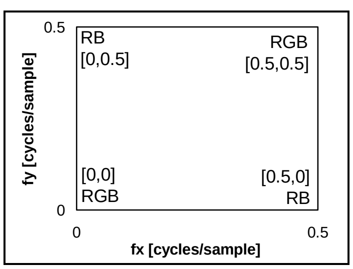

Figure 3: Replication points of spectra in the frequency plane, spatial frequency in cycles/sample of the full color plane, Bayer pattern, all 3 colors.

The aliasing pattern observed for the simple linear interpolation scheme associated with the Bayer pattern reflects differences in the symmetry of the red, green, and blue channels, as well as a lateral shift between the sampling lattices in all three channels. The green channel is sampled at twice the frequency of the red and blue channels and possesses diagonal symmetry, whereas the red and blue channels have quadratic symmetry. Figure 3 illustrates how these properties affect the replication points of the spectra in the frequency domain in each color. Also, the red sampling lattice is horizontally shifted by one pixel distance compared with the green lattice, while the blue lattice is shifted by the same distance in the vertical direction. According to the shift theorem, this means that their FTs are in anti-phase at the replication points labeled [0,0.5] and [0.5,0] in Figure 3. This shift gives rise to the orange/blue fringes and textures frequently observed with the Bayer pattern.

Correlation between Color Channels

In preparation for the analysis of more sophisticated CFA interpolation algorithms, we analyzed the correlation between the red, green, and blue records of 12 images stored in digital format as RGB code values representing relative logarithmic exposures. The cross correlation, $\rho_{j,k,}$ between the individual channels j,k was calculated as follows: [5]

$$\rho_{j,k} = \frac{\overline{\Delta D_j \Delta D_k}}{\overline{\Delta D_j^2}\ \xi \overline{\Delta D_k^2}} \tag{2}$$

(ΔD_j - density difference from the mean for record j; a bar denotes the average over all pixels of the image).

The calculated values $\rho_{j,k}$ fell between 0.25 and 0.99 with averages of 0.86 for red/green, 0.79 for red/blue and 0.92 for green /blue cross correlation.

Verification Targets

Several computer-generated targets were employed to verify the results of the theoretical analysis of sharpness and aliasing. CFA interpolation was applied to these targets in the form of image simulations

- *7 degree slanted edge target with 35% exposure modulation:* The analysis of the spectral frequency response, in our terms MTF, was carried out according to the ISO TC42/WG18 working draft procedure.[6] Curve 1 in Figure 4 represents the result of this procedure applied to an edge, that was sampled by the Bayer pattern and underwent linear interpolation (symbols), together with the prediction from Equation 1 (solid line).

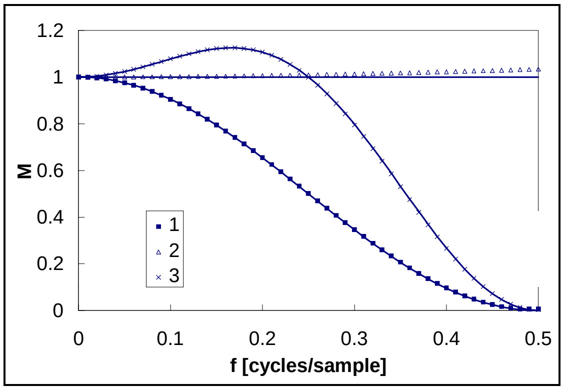

Figure 4: Solid lines: MTFs predicted from Equation 1 (curve 1) and 2c (curve 3). Curve 2, representing a unit MTF, applies to adaptive schemes and the green channel, vertical direction). Symbols: Results from slanted edge analysis.

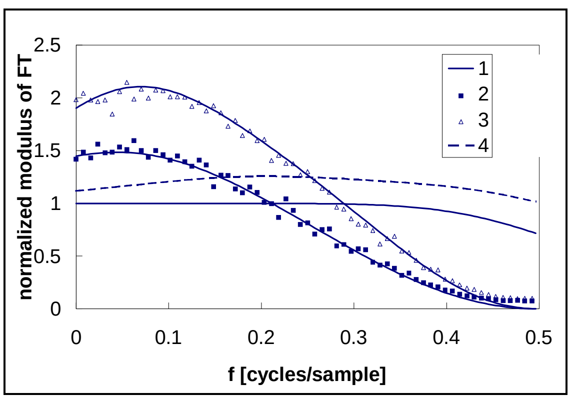

Figure 5: Modulus of Fourier transform of color-correlated noise before (curve 1) and after CFA interpolation. Lines: predicted according to Figure 2, curve 2: linear interpolation, curve 3: chroma-luma, curve 4: adaptive, chroma-luma and color correlation. Symbols: from calculated NPS after image processing.

- *Band-limited noise targets with zero and full correlation between the color channels:* Noise power spectra (NPS)[7] of these targets were calculated after CFA interpolation and compared with the input spectrum (the latter corresponds to curve 1 in Figure 5). Noise targets were employed to analyze MTFs of interpolation schemes that use correlation between the color channels, and to verify aliasing calculations. Curve 2 in Figure 5 shows the calculated NPS of the noise pattern after linear interpolation in the red and blue channels of the Bayer pattern (symbols) together with the predictions according to the theory outlined in Figure 2.

- *Sinusoidal zone plate containing frequencies up to one half of the sampling frequency of the full color plane:* This target helps to illustrate the symmetry and the phase relations of the aliasing pattern (Figure 3). Bright colored fringes may be observed near the replication points of the spectra.

Analysis of Advanced CFA Interpolation Schemes

This section explains the principle of each interpolation scheme and lists the advantages and disadvantages in terms of sharpness and aliasing compared with the simple linear interpolation discussed in the theory section.

Chroma/Luma Interpolation

These interpolation schemes make use of the fact that the human visual system can resolve much finer detail in the luminance channel than in the two chrominance channels (red-green and yellow-blue color difference signals).[8] For example, the Bayer pattern has twice as many pixels in the green channel compared with the red and blue channels to make use of the fact that the green channel most closely resembles achromatic visual sensitivity. Because of the excess number of pixels, the interpolation errors in the green, or so-called luma channel will be relatively small, which is equivalent to a higher MTF and a smaller sharpness penalty.

In a luma/chroma type interpolation, the green channel is reconstructed first.[3] Subsequently, red-green and blue-green color difference signals, so-called chroma signals, are calculated at the locations of the red and blue pixels. Because of the high color correlation between the channels discussed above, these signals are relatively slowly varying. After reconstruction of the chroma signals with the appropriate convolution kernels (kernel [0.5 1 0.5] applied in the horizontal and vertical direction), the green channel is added back in, resulting in the fully reconstructed red and blue channels.

This type of interpolation leaves the green channel unmodified in terms of MTF and aliasing compared with the linear scheme described in the theory section. However, the MTF of the red and blue channels is increased in both directions resulting in moderate improvements of sharpness. Given the asymmetry in the interpolation of the green channel (interpolation in one direction only), the red and blue channels inherit the unit MTF of the green channel in the vertical direction (curve 2 in Fig. 4, instead of Equation 1 for the simple linear scheme, curve 1 in Fi. 4), and a low-frequency boost is observed in the other direction (curve 3 in Figure 4). This boost manifests itself in bright orange/blue fringes at edges. The analysis of fully color-correlated noise indicates increased aliasing compared with the simple linear scheme (curves 2 and 3 in Fig. 5 representing the horizontal direction). The zone plate shows that aliasing increased in the direction in which the green pixels were interpolated, whereas no chromatic aliasing was observed in the vertical direction (location [0 0.5] in Fig. 3).

Correlation between Color Channels

Because of the high correlation between the three color channels, an advantage in terms of sharpness can be gained by using red and blue pixels to interpolate the green channel. This approach has been patented.[3] If we consider a red/green row of the Bayer pattern, the missing green value at the position, i, of a red pixel can be obtained as follows:

$$G_i = 0.5 \cdot (G_{i-1} + G_{i+1}) + 0.25 \cdot (-R_{i-2} + 2R_i + R_{i+2}) \qquad (2a)$$

Assuming full correlation between the records, this equation is equivalent to the convolution kernel

$$[0.5\ 0\ 0.5] + [-0.25\ 0\ 0.5\ 0\ \ -0.25] \qquad (2b)$$

Taking into account that one half of the green pixels are reconstructed according to Equation 2a/b while the other half that was captured needs no reconstruction, the overall MTF of the green channel is (this curve is identical to curve 3 in Figure 4):

$$M = 0.5 \cdot (1+0.5+\cos(2\pi f) - 0.5\cos(4\pi f)) \qquad (2c)$$

In a chroma/luma type interpolation, the red and the blue channels mainly inherit the MTF of the green channel with small deviations from Equation 2c in the horizontal direction and a unit MTF in the vertical direction. Because of the additional improvement in the green MTF, sharpness is expected to improve significantly compared with the chroma/ luma interpolation discussed in the previous paragraph. The aliasing pattern is very similar to the chroma/luma scheme discussed above in magnitude and its asymmetric nature.

Adaptive Algorithms

Adaptive algorithms are mainly employed in connection with the Bayer pattern where the interpolation could be done in two directions. If we imagine a perfect edge in the vertical direction and the interpolation is done along the edge rather than across it, the missing green pixels can be reconstructed without error. Adaptive algorithms employ classifiers to identify the best direction for the interpolation at each location, e.g. the direction of smaller changes in the code values.[3] If the algorithm identified the correct direction in which to interpolate in 50 percent of the cases the MTF associated with this interpolation scheme in both directions could be expressed as

$$M = 0.5 \cdot (1 + M_{directional}) \qquad (3)$$

where $M_{\mathrm{directional}}$ is given by Equation 1 or 2c, depending on the interpolation scheme.[3,9] This equation was indeed validated for color correlated noise targets. Adaptive schemes for the Bayer pattern are usually combined with chroma/luma interpolation. As expected for highly nonlinear adaptive interpolation algorithms, the MTF depends on the choice of target. A unit MTF in both directions and in all three colors is obtained for the 7 degree slanted edge target and other sine and square wave targets (curve 2 in Figure 4). Assuming that sharpness in images is mainly assessed by the rendition of edges, this adaptive scheme represents a significant improvement over all other algorithms discussed above, in particular at higher spatial frequencies. Likewise, aliasing is reduced compared with all algorithms discussed previously (curve 4 in Figure 5). Because of the perfect reconstruction of the green channel in the horizontal and vertical direction and the high correlation between color channels, MTF and aliasing on axis are determined by the resolution of the sensor (full color plane) and not by the resolution of each individual plane. Off axis, with increasing magnitude towards the diagonal direction, the aliasing pattern is similar to the simple linear interpolation scheme in terms of the replication points of the spectra and colored appearance (Figure 3). A MTF that varied as a function of orientation was employed in the calculation of curve 4 in Figure 5. Adaptive algorithms maintain quadratic symmetry in the red and blue channels and do not favor or disadvantage one direction as the non-adaptive two chroma/luma schemes discussed above.

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