

# Tradeoff Between Aliasing Artifacts and Sharpness in Assessing Image Quality

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## Abstract

All Digital Still Cameras employ some form of CCD imaging arrays to record images. These sampled images are prone to aliasing artifacts. This work systematically develops a method to calculate the probability of a given Digital Still Camera to produce aliasing artifacts. The Potential for Aliasing is defined as the ratio of the total power of the aliased spectrum to that of the total power of the image spectrum between the Nyquist frequencies of the system. The results show that the Potential for Aliasing is inversely proportional to the number of sampling sites, thus demonstrating that CCDs with Color Filter Arrays will be much more prone to aliasing artifacts than monochrome sensors. The results also clearly indicate that the Interline Transfer devices are much more prone to aliasing than their Frame Transfer counterparts. The Potential for Aliasing increases rapidly as the ratio of the pixel dimension to pixel pitch decreases. The model developed allows one to study the trade-offs between sharpness (as measured by CMT Acutance) and the Potential for Aliasing as a function of all system parameters. It can be shown that one can greatly reduce the Potential for Aliasing by using an appropriate optical pre-filter without greatly reducing the sharpness of the image. However, to completely remove aliasing artifacts, image sharpness will be significantly reduced.

## Introduction-Sampled Images

Images from Digital Still Cameras (DSCs) are sampled by the active pixel areas of Charged Coupled Devices (CCD). The pixels are normally rectangles or squares with the same pitch in both the x-direction and y-direction; "square pixels". For the purposes of this paper, the pixel's active area will be a square of linear dimension  $d$  with a pitch of  $a$ ; see Figure 1.

When an image is sampled with a sensor like that shown in Figure 1, a "potential for aliasing" takes place. Aliasing is the conversion of high (spatial) frequency

information being sub-sampled and appearing as low frequency information in the final image.<sup>1,2</sup> Figure 2 shows a simple example of a high frequency line pattern that has been sub-sampled, resulting in low frequency bands. To understand this phenomena one must consider the nature of the spectrum of a sampled image. If an image is sampled, the resulting spectrum is made up of an infinite series of replicated spectra that are displaced by multiples of the sampling frequency,  $f_s = 1/a$ . If  $\Phi(f)$  is the spectrum of a one dimensional signal (image), then the spectrum of the sampled image is shown in Figure 3. The spectrum of the final image is found between the two Nyquist frequencies,  $f_N = \pm f_s/2$ . Comparing the regions between the two Nyquist frequencies in the original spectrum and the sampled spectrum shows that any information above the Nyquist frequency is "folded" back into the lower frequency range. Thus a single high frequency line pattern that is above the Nyquist frequency will appear as a much lower frequency signal. Figure 4. shows this effect in an image taken from a Digital Still Camera.

## Potential For Aliasing

It is very desirable to have a metric that measures the amount of aliasing that will take place in a DSC. However, it is impossible to predict the exact amount of aliasing in any given image, since aliasing is very scene dependent. For example, the test pattern shown in Figure 4 is very prone to aliasing due to its high frequency content. On the other hand, a uniform wall will never produce an aliased image since there are no high frequency components. The image of a striped blouse or oxford shirt will often show the effects of aliasing, while the portrait of a face will not (except in the hair).

Instead of "calculating" the exact amount of aliasing that will take place in any given image, it is more reasonable to "calculate" the Potential for Aliasing, PA, for given DSC system.<sup>3,4,5</sup> The Potential for Aliasing is able to "rank" DSC system as to their propensity to introduce

aliasing into the final image. The concept of Potential for Aliasing is best understood within the context of a simple linear CCD array. Consider the top row of the CCD array shown in Figure 1. The important parameters are the dimension of the square pixel,  $d$ , the pitch of the pixels in the linear array, defined by  $a$ , and their ratio,  $\beta = d/a$ . Assume that the input spectra is flat; the effect of the taking lens and any optical pre-filter will be discussed in a later section. Figure 5 shows the spectrum of a pixel of dimension  $d$  and the first two replicas for the spacing shown in Figure 1,  $d = a/2$ , as well as for the case where the pixels touch,  $d = a$ . The spectrum is given by the absolute value of the Modulation Transfer Function of the pixel;

$$MTF_{pixel}(f) = \text{sin}(\pi df) / \pi df \dots\dots\dots(1)$$

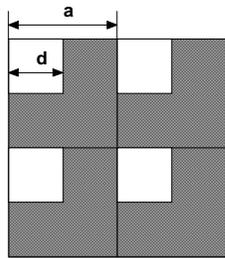


Figure 1. The white area represents the active pixel and the shaded area does not collect light. The active pixel area is a square of linear dimension  $d$  and pitch  $a$  in both directions.

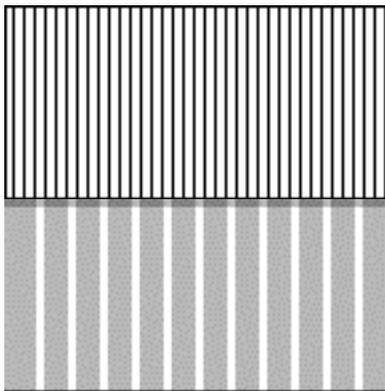


Figure 2. The top line pattern is sampled by a "pixel" that is three times the size of the pixels making it up. The resulting image on the bottom demonstrates an averaging over the larger pixel and the results of the three-fold sub-sampling.

The portion of the spectrum beyond the two Nyquist frequencies will be folded back to form the aliased signal. It can be shown that the aliased signal is equivalent to the sum of the portions of the replicated spectra that fall inside the two Nyquist frequencies. The Potential for Aliasing is defined as the ratio of the aliased power to that

of the non-aliased power between the two Nyquist frequencies. Using a dummy variable,

$$u = \pi df \dots\dots\dots(2)$$

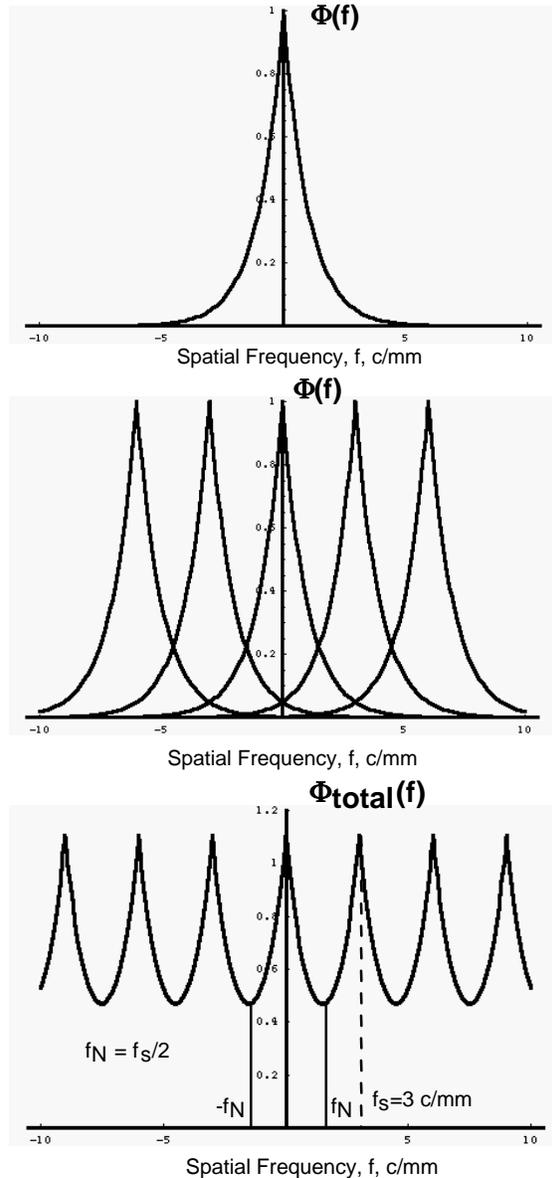


Figure 3. The top plot show spectrum of the original image. When it is sample by a linear CCD array with a pitch of  $a = 0.333$  mm an infinite set of replicas are formed separated by the sampling frequency  $f_s = 1/a = 3$  c/mm. The middle plot shows the original spectrum and four of the replicas. The bottom plot shows the resulting spectrum (the sum of the infinite set of replicas). The portion of the spectrum between the two Nyquist frequencies,  $f_N$  is the spectrum that is seen in the final image.

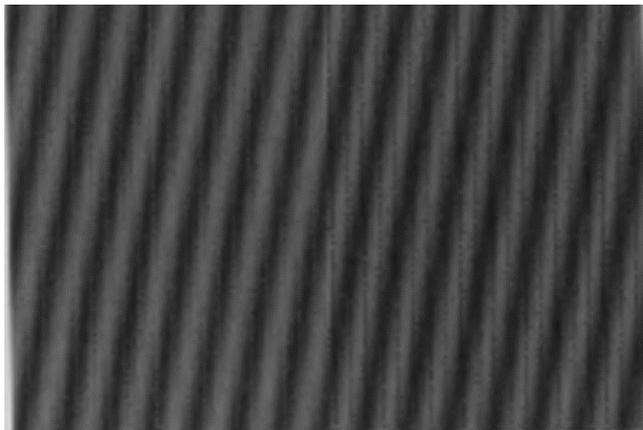


Figure 4. This is the aliased image of a test chart taken with a consumer Digital Still Camera. The test chart has a frequency of 80 line pairs per target width. As can be seen from the above image, there are approximately 15 line pairs, showing the strong aliasing that has taken place.

the non-aliased signal between the two Nyquist frequencies is given by

$$\Psi = \left( \frac{1}{\pi d} \right) \int_{-\beta\pi/2}^{\beta\pi/2} \frac{\sin^2(u)}{u^2} du \dots\dots\dots(3)$$

where  $\beta = d/a$  and gives a measure of how much of the sensor's area is dedicated to capturing light. The power of the aliased signal is just the difference between the total power in the spectra,  $1/d$ , and the non-aliased power,  $\Psi$ . Hence the Potential for Aliasing is given by

$$PA = \frac{1/d - \Psi}{\Psi} \dots\dots\dots(4)$$

Equation (4) can be simplified by using denoting

$$\Gamma(\beta) = \int_{-\beta\pi/2}^{\beta\pi/2} \frac{\sin^2(u)}{u^2} du \dots\dots\dots(5)$$

giving

$$PA = \frac{\pi - \Gamma(\beta)}{\Gamma(\beta)} \dots\dots\dots(6)$$

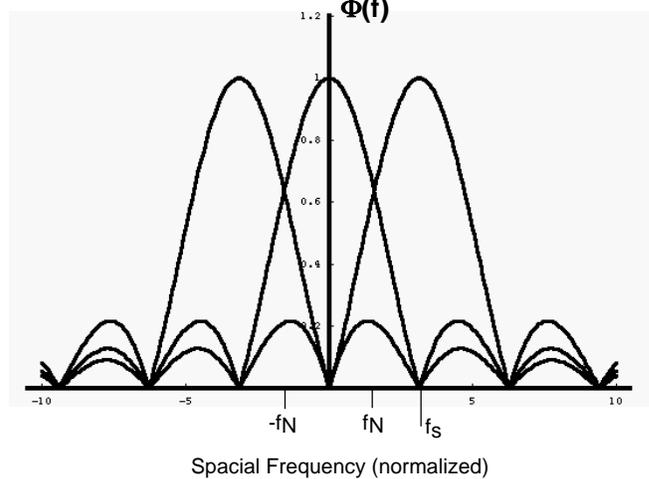
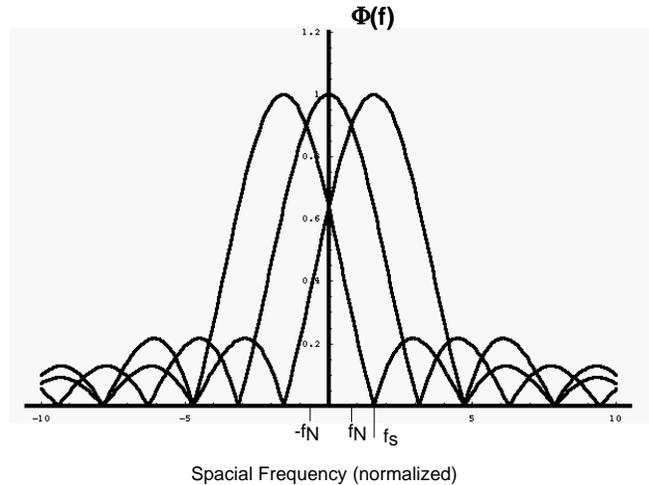


Figure 5. The top plot show the overlapping of the spectra for the case shown in Figure 1,  $\beta = 0.5$ . The lower plot show the case for pixels that are touching,  $d = a$  or  $\beta = 1$ . In both cases there is considerable overlapping, but the case for  $\beta = 0.5$  will demonstrate much more aliasing. The PA for  $\beta = 0.5$  is 1.14 and for  $\beta = 1.0$  PA = 0.29.

Figure 6. show a plot of PA as a function  $\beta$ . Note the sharp rise in PA as  $\beta$  becomes smaller. This reflects the general rule that Interline Transfer Devices tend to alias more than Frame Transfer Devices due to the smaller area allocated to the active pixel. As the effective pixel area decreases, the MTF associated with the pixel has greater high frequency response, thus providing a greater potential for aliasing. It is also important to note that PA is only a function of  $\beta$  and not of the pixel edge size,  $d$ , or the number of pixels. When considering the sensor alone, only the ratio between the pixel edge and the pitch is important.

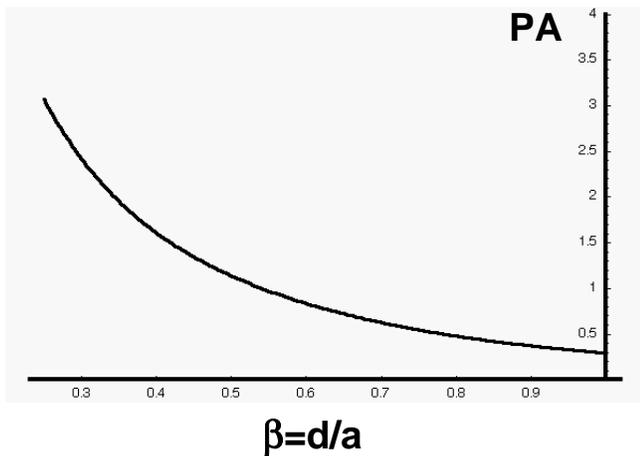


Figure 6. The Potential for Aliasing, PA, as a function of  $\beta = d/a$ . As the linear dimension,  $d$ , of the active area decreases for a fixed pitch,  $a$ , the Potential for Aliasing increases rapidly.

### Aliasing in Two-Dimensional Arrays

The analysis above can be extended to two-dimensional arrays. Care must be taken to insure that one properly calculates the power of the spectrum that is not aliased. As in the one-dimensional case, the linear dimension of the active, square pixel area will be denoted by  $d$ . The pitch,  $a$ , will be assumed to be equal in the x-direction and y-direction. Two cases will be considered. The first case corresponds to a monochrome sensor that could be a Frame Transfer Device or an Interline Transfer Device. Figure 7 shows the general structure of the array and the placement of the first set of replica spectra in the spatial frequency domain. Only the white area in the sensor collects light. Note that closest set of replicas (indicated by very small diamonds on the grid) form a diamond structure around the central spectrum. The region inside the “Nyquist frequency range” is given by the diamond shaped, shaded area in the center. The Nyquist frequency along the diagonal is less than that along the  $f_x$  or  $f_y$  directions, thus leading to the diamond shaped “Nyquist range”. Only power from the central spectrum in this shaded area will be un-aliased. The total power associated with the square pixel is  $1/d^2$  which is equal to the integral of the  $MTF_{pixel}(f_x, f_y)^2$  of the square pixel:

$$MTF_{pixel}(f_x, f_y) = \frac{\text{Sin}(\pi d f_x)}{\pi d f_x} \cdot \frac{\text{Sin}(\pi d f_y)}{\pi d f_y} \dots\dots\dots(7)$$

The non-aliased signal is given by

$$\Psi = \frac{2}{(\pi d)^2} \int_0^{\beta\pi/2} \frac{\text{Sin}^2(v)}{v^2} \int_{v-\beta\pi/2}^{\text{Sin}^2(u)} \frac{\text{Sin}^2(u)}{u^2} dudv \dots\dots\dots(8)$$

where  $u$  and  $v$  are dummy variables similar to the one given in Equation (2).

Simplifying in the same manner as above for the one-dimensional case, the Potential for Aliasing is then given by

$$PA = \frac{\pi^2 - \Gamma_2(\beta)}{\Gamma_2(\beta)} \dots\dots\dots(9)$$

where

$$\Gamma_2(\beta) = 2 \int_0^{\beta\pi/2} \frac{\text{Sin}^2(v)}{v^2} \int_{v-\beta\pi/2}^{\text{Sin}^2(u)} \frac{\text{Sin}^2(u)}{u^2} dudv \dots\dots\dots(10)$$

When Bayer Color Filter Array (CFA) is used to encode color on a Frame Transfer Device or an Interline Transfer Device, the sampling structure of all three colors introduce more complex aliasing patterns. The Bayer CFA is shown in Figure 8. First consider the green sampling structure. Figure 8 shows the “Nyquist region” around the central spectrum. The “Nyquist region” forms a square about the central spectrum with the limits half of those of the monochrome sensor. This sparse sampling results in a much lower sampling frequency and hence more aliasing. As with the case above, the non-aliased power is found by integrating the  $MTF_{pixel}(f_x, f_y)^2$  in this shaded area. The non-aliased power is given by

$$\Psi_G = \frac{1}{(\pi d)^2} \int_{-\beta\pi/4}^{\beta\pi/4} \frac{\text{Sin}^2(v)}{v^2} \int_{-\beta\pi/4}^{\beta\pi/4} \frac{\text{Sin}^2(u)}{u^2} dudv \dots\dots\dots(11)$$

and again simplifying, the Potential for Aliasing is given by

$$PA = \frac{\pi^2 - \Gamma_G(\beta)}{\Gamma_G(\beta)} \dots\dots\dots(12)$$

where

$$\Gamma_G(\beta) = \int_{-\beta\pi/4}^{\beta\pi/4} \frac{\text{Sin}^2(v)}{v^2} \int_{-\beta\pi/4}^{\beta\pi/4} \frac{\text{Sin}^2(u)}{u^2} dudv \dots\dots\dots(13)$$

Figure 9 shows plots for PA as a function of  $\beta$  for the expressions given by Equations (10) and (12). What is very clear is that as one moves from one-dimensional arrays to two-dimensional arrays the amount of aliasing increases many fold. Furthermore, in the case of a sensor with a Bayer Color Filter Array, the very sparse sampling of the green image (half as many samples) doubles the Potential for Aliasing. The sampling pattern of the red or blue image is more like that of the monochrome sensor but with just half the resolution. The Potential for Aliasing can be computed using Equations (8),(9) and (10) but with the limits of integration adjusted to  $\beta\pi/4$ . Figure 10 shows the

results for either the red or blue sampling structure for the Bayer CFA. Besides the increased Potential for Aliasing for two-dimensional arrays, the Potential for Aliasing is inversely proportional to the number of sampling points. The green layer of a Bayer CFA has one-half the sampling sites as a monochrome sensor of the same dimensions and the red and blue have one-fourth as many. Figures 9 and 10 clearly show this relationship.

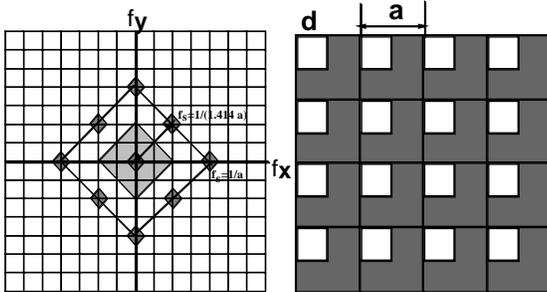


Figure 7. A typical two-dimensional imaging array is shown on the right. The white areas collect light. For a Frame Transfer device  $d = a$  and the full area is used to collect light. On the left is the region around the central spectrum in the spatial frequency domain. The small diamonds represent the replica positions in the grid. The diamond shaped, shaded area in the center represents the region inside the "Nyquist range" of the sensor. All the signal from the central spectrum inside this area is not aliased, but all the signal falling outside this region is aliased. This pattern also corresponds to the red or blue sampling patterns of a Bayer CFA, but the dimension must be adjusted in both the spatial domain and the spatial frequency domain.

As in the case of the of the one-dimensional arrays, the Potential for Aliasing is only a function of  $\beta$  for monochrome sensors and a function of  $\beta$  and the exact nature of the CFA for color arrays.

### System Potential for Aliasing

In the previous section only the effects of the imaging array were considered. Since an observer only "sees" the image on a print (or soft display) one must take into account the impact of all the system components. Figure 11 shows a simplified version of the Digital Imaging Chain (DIC). The DIC consists of the taking lens, an optical pre-filter, the CCD sensor, any image processing (interpolation algorithms), the laser printer and the photographic paper used to form the final image. Each of these components can be characterized by a well defined MTF. For the sake of exposition and brevity, only a one-dimensional model will be considered, but as shown above the extension to two-dimensions is straight forward and will always show an increase in the Potential for Aliasing.

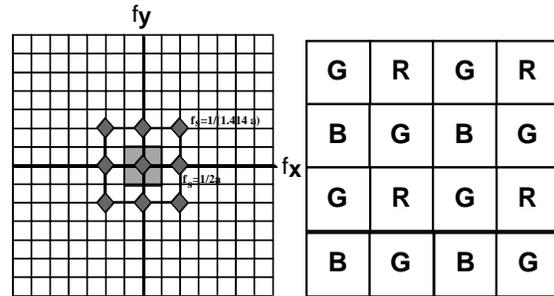


Figure 8. The image array on the right represents the Bayer Color Filter Array used to encode color. On the left, the nearest set of replica spectra for the green sampled image are shown relative to the central spectrum. Note how much closer they are to the central spectrum than in Figure 7 for the monochrome sensor. The central, shaded square denotes the un-aliased "Nyquist region". It is clear that this arrangement will introduce much more aliasing than the pattern shown in Figure 7. The red and blue replicas are similar to that in Figure 7 but with the boundaries adjusted to reflect the lower sampling frequencies.

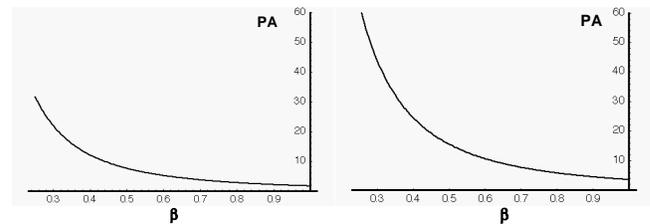


Figure 9. The plot on the left is for a monochrome sensor. As  $\beta$  decreases (moving from a Frame Transfer to an Interline Transfer Device) the Potential for Aliasing increases greatly. Also, when compared to the values for a one-dimensional array ( see Figure 6), it is clear the Potential for Aliasing greatly increases for two-dimensional arrays. The plot on the right is for the green sampled image for a sensor using the Bayer Color Filter Array. Due to the sparse sampling, the Potential for Aliasing doubles.

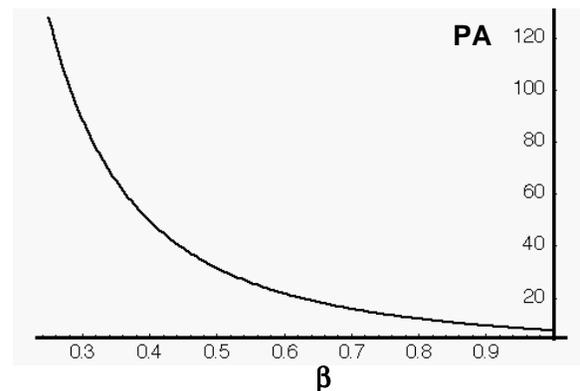


Figure 10. The Potential for Aliasing for the red or blue sampling structure of the Bayer CFA. Note that the Potential for Aliasing is twice that of the green sampling structure and four times that of a monochrome sensor; see Figure 9.

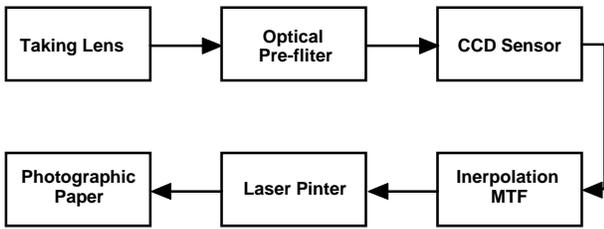


Figure 11. The Digital Imaging Chain

This DIC model will assume diffraction-limited lens MTF curves since they form a very nice family of curves that are only a function of the wavelength of light (550 nanometers in all cases) and the F-Number of the lens, N. The lens MTF are given by

$$MTF_{Lens}(f) = \frac{2}{\pi} \left[ \frac{f}{f_c} \sqrt{1 - \left(\frac{f}{f_c}\right)^2} \right] \dots\dots\dots(14)$$

for  $f \leq f_c$ , and

$$MTF_{Lens}(f) = 0, f > f_c$$

where  $f_c = (1/N\lambda)$

The optical pre-filters used in most DSCs is a thin sheet of birefringent material that creates two images of equal intensity by displaced by some distance  $\delta$ . The MTF associated with the optical pre-filter is

$$MTF_{pre-filter}(f) = \text{Cos}(\pi\delta f) \dots\dots\dots(15)$$

When color filter arrays are used to encode color, the sampling is sparse and it is necessary to uses some form of interpolation to fill in the missing pixels. If the sampling distance between pixels is a, then the interpolation MTF is given by

$$MTF_{int}(f) = \text{Cos}^2(\pi a f) \dots\dots\dots(16)$$

The above MTFs are all referred to the image plane of the sensor. Since the next two components are in the plane of the paper in is necessary to adjust Equations (14), (15) and (16) (along with the MTF of the sensor) to their equivalent values in the plane of the paper. In what follows, this adjustment is assumed. The laser printer MTF is defined by the nature of the gaussian light beam. If  $\sigma$  characterizes the light gaussian light beam, the laser printer MTF is given by

$$MTF_{Laser}(f) = e^{-2(\pi\sigma f)^2} \dots\dots\dots(17)$$

To insure that no scan lines are visible it is best to make  $\sigma = 0.4 a$  where  $a$  is the equivalent sensor pixel pitch in the plane of the paper. Furthermore, to make the system as sharp as possible, it is best to double the printing resolution and use an appropriate interpolation algorithm. In this case,  $\sigma = 0.2 a$ .

The final component of the system is the photographic paper. The MTF of paper can be nicely modeled by

$$MTF_{Paper}(f) = \frac{1}{1 + \left(\frac{f}{f_o}\right)^2} \dots\dots\dots(18)$$

where  $f_o$  is the spatial frequency at which the MTF is equal to 50%. A value of  $f_o = 5.0$  is reasonable for most photographic papers.

The above component MTFs allow one to calculate the effect they have on the aliased and non-aliased parts of the image spectrum. As before, a flat image spectrum is assumed. Also, only the non-aliased and aliased spectra between the Nyquist frequencies need be considered, for they are what is seen on the final print. The flat input image spectrum is first modified (low passed filtered) by MTFs of the lens and the optical pre-filter. These modifications greatly reduce Potential for Aliasing and also reduces the system sharpness. The resulting spectrum is then sampled by the sensor and the aliasing takes place. In the case of a DSC with a CFA, both the aliased and non-aliased spectra are modified (low pass filtered) by the interpolation process. During the printing process, both the laser and paper MTFs low pass filter the spectra. The final spectra are then used to calculate the Potential for Aliasing as defined in the previous sections.

Figure 12 shows the effect of a F/5.6 lens and an optical pre-filter for an Interline Transfer device where  $d = 0.5a$  and  $\delta = d$  and the resulting aliased spectra. The effect of the lens and optical pre-filter is to greatly reduce the signal that extends beyond the Nyquist frequency. The reduction in the aliased spectrum is seen at the bottom of Figure 12.

Figure 13 shows the results for the complete system using the same conditions as cited above for Figure 12. The spectra shown here are in the plane of the paper. Note that while the non-aliased signal spectrum shows only slight changes due to the introduction of an optical pre-filter, the aliased spectrum is greatly reduced as is the value for PA. Thus it is clear that the introduction of an optical-prefilter can have a dramatic effect based on the values of PA. This apparent improvement must be correlated to what is seen in images before an definite conclusions can be drawn.

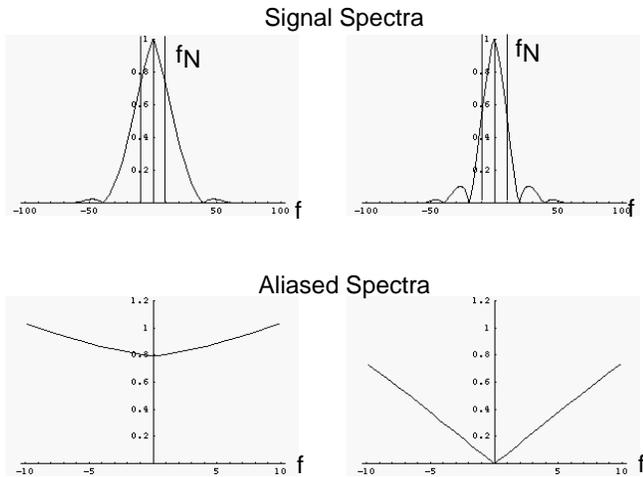


Figure 12. The signal spectra at the top-left is for the case of no optical-pre-filter. Note that the signal extends far beyond the Nyquist frequency. When an optical pre-filter is introduced, the signal beyond the Nyquist frequency is greatly reduced, thus reducing the aliased spectrum, as seen at the bottom-right, compared with the case of no optical pre-filter as shown on the bottom-left.

**System Sharpness**

The sharpness of an image is related to the cascading effect of the system MTFs. The metric to be used to quantify sharpness will be the CMT Acutance.<sup>6</sup> The system MTF is given by

$$MTF_{System}(f) = \prod_{i=1}^n MTF_i(m_i, f) \dots \dots \dots (19)$$

where  $MTF_i$  is the MTF of the  $i$ th component and  $m_i$  is the magnification required to bring each component into the plane of the paper. The spatial frequency,  $f$ , is in the plane of the paper. The human visual response to the system MTF is the measure of sharpness incorporated into the CMT Acutance. The human visual response is approximated by an eye MTF given by<sup>1,7</sup>

$$MTF_{eye}(f) = 2.6[0.0192 + (6.53Lf)]e^{-(6.53Lf)^{1.1}} \dots (20)$$

where  $L$  is the viewing distance in millimeters and  $f$  is the spatial frequency in cycles per millimeter in the paper plane. The visual response,  $VR$ , is then given by

$$VR = \frac{\int_0^{f_N} MTF_{System}(f) \cdot MTF_{eye}(f) df}{\int_0^{\infty} MTF_{eye}(f) df} \dots \dots \dots (21)$$

where the integration in the numerator stops at  $f_N$  since there is no information beyond the Nyquist frequency due to the sampling. The CMT Acutance is then given by

$$CMT = 100 + 66 \text{Log}[VR] \dots \dots \dots (22)$$

An observer can usually detect a sharpness change in an image corresponding to a unit CMT difference. Table provides a useful classification of image sharpness as a function of CMT values.

For all examples in this paper, it is assumed that a four inch by six inch print is produced and that the observer is viewing the print from 16 inches.

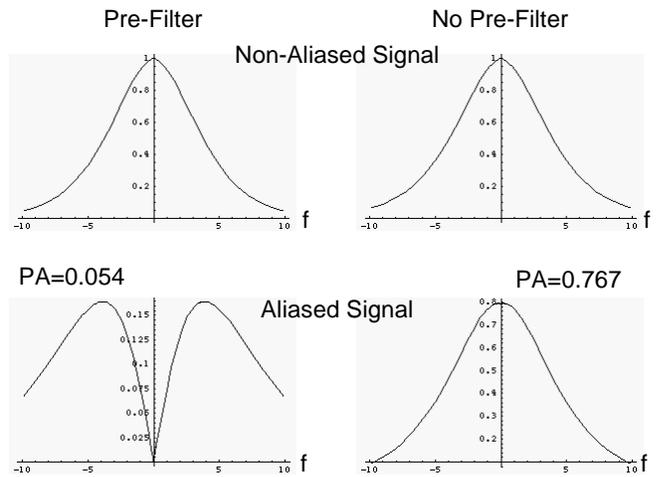


Figure 13. The above spectra are for the total system, including paper and laser printer and are based on the spectra shown in Figure 12. There is a dramatic reduction in the Potential for Aliasing when an optical pre-filter is introduced.

**Examples of DSC Systems**

Using methods outlined above it is now possible to systematically study the Potential for Aliasing and CMT Acutance for a range of systems as a function of the number of lines per picture height, the pixel spacing, the pixel size, the MTF of the lens and the nature of the optical pre-filter. All the examples given below are based on a one-dimensional treatment of a DSC. This one-dimensional analysis, while understating the Potential for Aliasing, will provide a clear view of the trade-offs between sharpness and artifacts due to aliasing. For the examples given below the camera lens has a MTF given by a diffraction limited F/5.6 lens.

CMT Range	Sharpness Classification
92 and above	Excellent
86 to 91	Good
81 to 85	Fair
75 to 80	Acceptable
74 and below	Unacceptable

Table 1. Sharpness classification based on CMT Acutance values.

Figure 14 shows the case of monochrome Frame Transfer device with the no optical pre-filter. As the lines per picture height increases from 500 to 2000, the CMT Acutance increases and the Potential for Aliasing decreases. It is clear that such monochrome imaging systems will have the least potential for introducing artifacts due to aliasing.

Figure 15 shows the results for a monochrome Interline Transfer DSC with no optical pre-filter. The fact that this system has smaller pixels results in greatly increased Potential for Aliasing. The rise in PA varies from a six-fold increase at low resolution to a 15-fold increase at high resolution. However, due to the smaller pixels, the CMT Acutance is higher. For example, a 750 line image will have a CMT value of about 85 (a good image) in the Interline Transfer format while it would take over 800 lines in the Frame Transfer format. However, this small increase in sharpness costs a seven fold increase in the Potential for Aliasing.

Figure 16 show the case of the Interline Transfer device illustrated in Figure 15 but with the introduction of an optical pre-filter with  $\delta=0.005$  mm. The use of the optical pre-filter reduces the Potential for Aliasing to the level of the Frame Transfer device shown in Figure 14. The optical pre-filter lowers the CMT values by two units at low resolution systems and about one unit a higher resolution systems. Since the values for PA are lowered by an average factor of ten, the loss of one or two CMT values is acceptable.

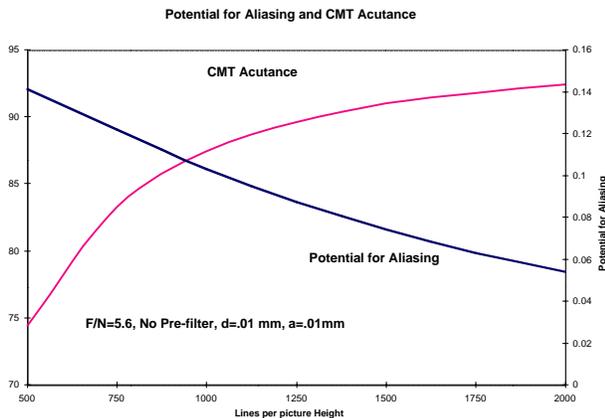


Figure 14. CMT Acutance and Potential for Aliasing for a monochrome Frame Transfer DSC as a function of the number of imaging lines per picture height.

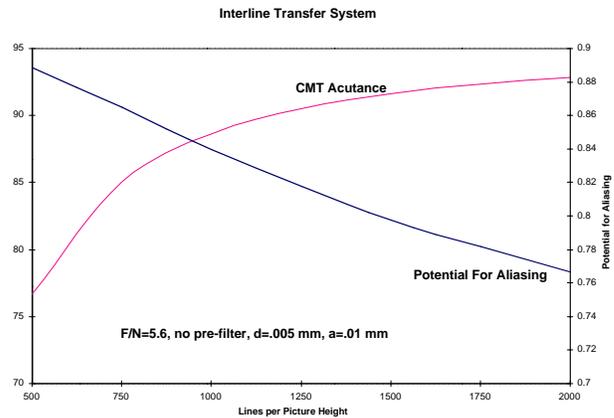


Figure 15. CMT Acutance and Potential for Aliasing as a function of lines per picture height for an Interline Transfer DSC.

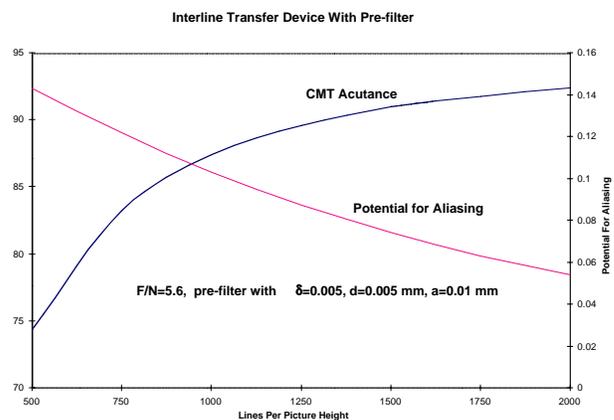


Figure 16. The CMT Acutance and Potential for Aliasing for an Interline Transfer device that has an optical pre-filter.

Figure 17 shows the sharpness and the Potential for Aliasing for a color system that uses a CFA with alternating color filters. In this case each color image is sampled 1000 times and interpolated up to 2000 samples during image reconstruction. The introduction of the CFA greatly increases the Potential for Aliasing. An optical pre-filter is used reduce the aliasing. The abscissa is the shift introduced by the pre-filter. The plot shows the ideal case is when the shift  $\delta = a/2$  where the Potential for Aliasing reaches a minimum. However, the CMT Acutance continues to decrease with increasing  $\delta$ . One can plot the same information as a parametric function of  $\delta$  and this is shown in Figure 18. This plot enables one to clearly see the system trade-off between sharpness and aliasing. To minimize the aliasing in such a system one will have to give up at least one full CMT value. The PA drops to about 0.35 which is still very high compared to monochrome, Frame Transfer DCTs. These results are fairly representative of many of the DCTs on the market that use Frame Transfer devices. However, most of them have much lower resolution and hence have much higher PA's and much lower CMT values.

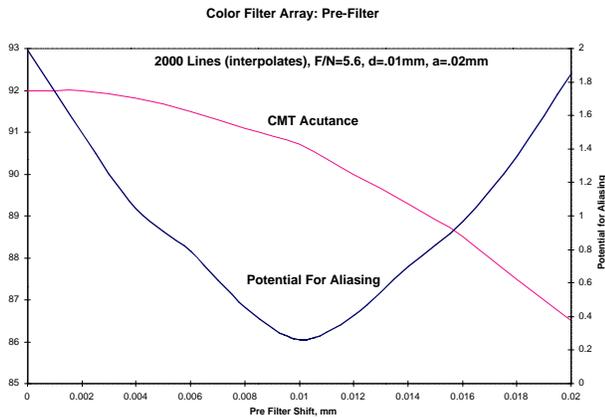


Figure 17. CMT Acutance and the Potential for Aliasing for a 2000 line, color, Frame Transfer DSC using a CFA plotted as a function of the image shift introduced by an optical pre-filter.

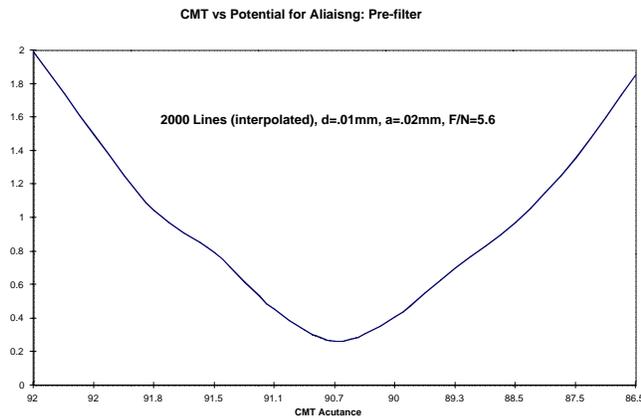


Figure 18. The CMT and PA values Figure 16 re-plotted using the image shift,  $\delta$ , as the a common parameter.

Figure 19. show the case of a DSC that uses a Frame Transfer Device and a CFA, but no optical pre-filter. The 1500 and 2000 line DSCs are typical of some of the most expensive DSCs on the market. The introduction of the CFA has greatly increased the Potential for Aliasing, even though at higher line resolutions the images are reasonably sharp at normal magnification and viewing distances (four inch by six inch prints viewed from 16 inches).

### Summary

The Potential for Aliasing is inversely proportional to the number of sampling points. A monochrome Frame Transfer device DSC will have the lowest potential to aliasing, while an Interline Transfer device with a Color Filter Array will have the greatest Potential for Aliasing. The use of a properly designed optical pre-filter can greatly reduce the Potential for Aliasing while introducing only marginal losses in sharpness as measured by CMT Acutance. If one

wished to completely eliminate aliasing the image sharpness will greatly deteriorate.

If one were to design a DSC to maximize sharpness and keep aliasing to a minimum the following process can be used. The first step is to decide the print size desired and what the viewing distance will be. A normal print of four inches by six inches is viewed from about 16 inches. Based on the eye MTF for this viewing distance, one can resolve no more than eight cycles per millimeter. This means that the pixel size on the print is 0.0625 mm and that the array must be about 1625 pixels by 2438 pixels. This will insure that the eye will see all the information up to the Nyquist frequency in the plane of the print; no information beyond the Nyquist frequency can be reproduced in sampled systems. For a monochrome or three Frame Transfer CCD color system, this will produce images with CMT values in excess of 92 (excellent quality) and the Potential for Aliasing will be below 0.07. If a single Frame Transfer CCD DSC is used with a CFA, then the CMT value drops to about 90 and the Potential for Aliasing climbs to about 2.0. This example clearly demonstrates the



Figure 19. The CMT Acutance and Potential for Aliasing for a DSC with a Color Filter Array.

impact of sub-sampling due to the use of a CFA. To reduce the aliasing, one must introduce an optical pre-filter. Using the optimum design for a birefringent optical pre-filter one can reduce the Potential for Aliasing to about 0.3 with a CMT value of about 87. By using a less sharp camera lens, one can further reduce the Potential for Aliasing to under 0.05 and have a CMT value of 85 (a good print). Simply doubling the resolution of the sensor with a CFA and using an optimum birefringent optical pre-filter will give a PA = 0.18 and a CMT = 93.

While this study clearly shows the differences between the Potential for Aliasing for different system configurations, it does not provide an absolute calibration of the artifacts seen in images from Digital Still Cameras. Non-systematic experiments have shown that all Digital

Still Cameras that have with CFAs produce significant color banding due to aliasing. The major differences are at what spatial frequency the aliasing takes place. Lower end cameras (500-by-750 pixels) with optical prefilters show color banding a moderate spatial frequencies and produce images considerably poorer than those from inexpensive 35 mm film cameras. High resolution DSCs (2000-by-3000 pixels) with CFAs and without optical pre-filters and very good lenses can produce images that compare relatively well with 35 mm film images at low magnification (on a four inch by six inch print) but not at higher magnification. However, these cameras display very strong aliasing at higher frequencies and lead to significant color banding and local artifacts in the fine details of the image.

The next stage of research is to use the theoretical work outlined in this paper to guide a systematic calibration of the Potential for Aliasing results with the aliasing artifacts produced in real and simulated DSC systems.

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