Color Sensitivity Selection for Electronic Still Cameras Based on Noise Considerations in Photographic Speed Maximization

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Abstract

An analysis is presented investigating the role of spectral sensitivity in photographic speed of electronic still picture cameras. The analysis considers the tradeoff of optical speed by color filter throughput, against sensor noise amplification through color primary mixture matrices. The results indicate that spectral sensitivities corresponding to color primary mixture matrices that invoke certain noise mitigation criteria can have significantly higher speed than those that do not. An implementation of the analysis yields a spectral sensitivity set that can produce perfect color fidelity with maximum photographic speed.

Introduction

Good color reproduction in renderings of scenes captured by electronic still cameras necessarily involves transformations of the color signals detected by the camera sensor(s). Such color transforms map the detected signals of the camera to rendering signals, by accommodating the camera color performance, the rendering device color performance, and some color and tone reproduction criteria. While often a single concatenated color transform is applied to accommodate these components, the constituent accommodations can be addressed as separate problems. This paper addresses camera color performance in the context of photographic speed.

The spectral responses of the color channels in an electronic still camera fundamentally prescribe the camera constituent of the color transform. Color sensitivities selected for maximizing sensor exposure to available scene brightness generally maximize color filter spectral throughputs by detecting broad and overlapping spectral bands. Broad and overlapping spectral bands in turn require color transforms that strongly mix and amplify both the image, and the noise signals of the camera data, consequently exacerbating graininess in renderings. Color sensitivities selected for minimal color transform channel mixing and amplification generally reduce spectral throughput by necessitating narrower, less overlapping spectral bands which reduce sensor exposure, consequently increasing the proportion of noise carried with the image signal. Photographic speed is a valuable camera performance parameter and is increased only when gains in optical throughput of channel sensitivities outweigh corresponding amplification of sensor noise by color transforms.

Sensor Noise

Noise performance of electronic still picture camera sensors has been studied extensively[1]. A simple sensor noise model describing quantum exposure noise (shot noise), plus a baseline random noise is presented for this analysis.

$$\sigma = \frac{\sigma_S}{S} = \frac{\sqrt{N^2 + S}}{S} \qquad (1)$$

The standard deviation of the noise, expressed as a proportion of the signal, is shown in equation (1). This is the inverse of the incremental signal-to-noise ratio[2]. The signal variance, shown within the radical, is equal to the mean signal, plus the baseline noise squared. Increasing exposure reduces the noise proportion in the image signal. Providing increased sensor exposure for a given scene brightness, is done at the expense of exposure index.

Color Mixing

Electronic still camera color data can be expressed as estimated normalized CIE tristimuli by matrix mixing of linearized camera signals[3]. Estimates of scene colorimetry are often logically followed by application of tone and color reproduction criteria in production of color transforms for rendering electronic camera images. Three different linearized camera channels are arbitrarily described as R, G, and B in equation (2) below.

$$\begin{bmatrix} \hat{X}/X_N \\ \hat{Y}/Y_N \\ \hat{Z}/Z_N \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} R_{lin} \\ G_{lin} \\ B_{lin} \end{bmatrix} \qquad (2)$$

Non-image signal fluctuations due to noise are mixed, differenced, and amplified according to the coefficients of the camera color matrix. In the case of equal and independent camera channel noise, variances in CIE tristimuli result from camera channel variance amplified by the sums of the squared matrix coefficients of each row of matrix A

$$\sigma_X^2 = \sigma^2 \cdot \left(a_{11}^2 + a_{12}^2 + a_{13}^2\right) \qquad (3a)$$

$$\sigma_Y^2 = \sigma^2 \cdot \left(a_{21}^2 + a_{22}^2 + a_{23}^2\right) \qquad (3b)$$

$$\sigma_Z^2 = \sigma^2 \cdot \left(a_{31}^2 + a_{32}^2 + a_{33}^2\right) \qquad (3c)$$

Color Accuracy

Scene colorimetry determined from matrixed camera signals can be exact when camera spectral sensitivities are linear combinations of the spectral color matching functions for CIE primaries. One such set is shown below in figure 1.

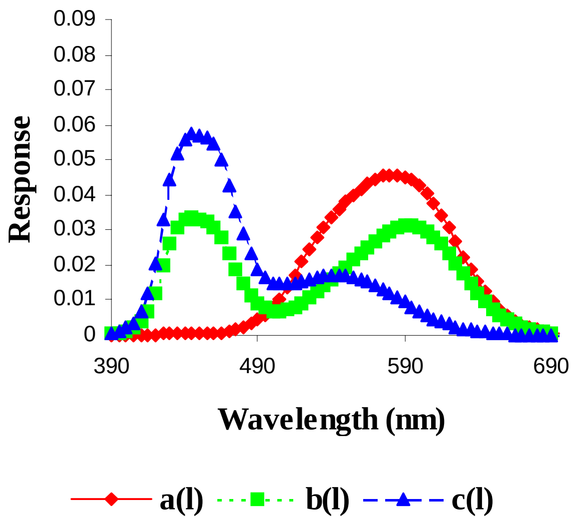

Figure 1. Spectral sensitivities formed by linear combination of color matching functions for CIE XYZ primaries.

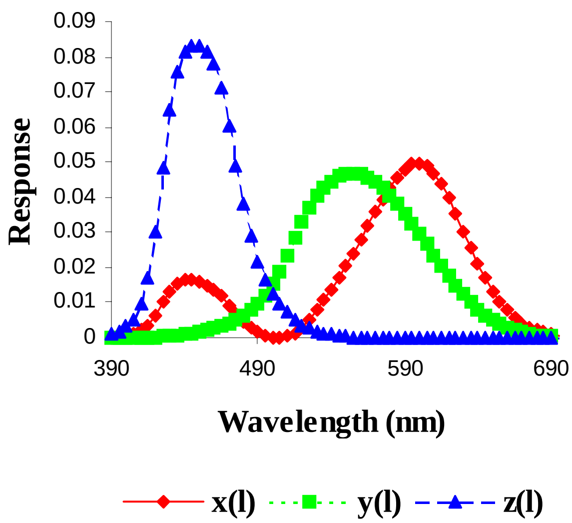

Figure 2. CIE color matching functions x(λ), y(λ), and z(λ) for primaries X, Y, and Z.

Optical Throughput

As a reference case, an electronic camera could be made with spectral sensitivities that exactly match the CIE tristimulus integration functions x(λ), y(λ), and z(λ).

The spectral sensitivities a(λ), b(λ), and c(λ) shown in figure 1., were constructed from x(λ), y(λ), and z(λ) by combination according to the matrix ABC^{-1} shown in equation (4). The functions x(λ), y(λ), and z(λ) are normalized to unit area, and the matrix ABC^{-1} is constrained to unit row sums. Therefore the sensitivities a(λ), b(λ), and c(λ) also have unit area.

$$\begin{bmatrix} a(\lambda) \\ b(\lambda) \\ c(\lambda) \end{bmatrix} = \begin{bmatrix} .5 & .6 & -.1 \\ .5 & .2 & .3 \\ -.1 & .4 & .7 \end{bmatrix} \times \begin{bmatrix} x(\lambda) \\ y(\lambda) \\ z(\lambda) \end{bmatrix} \qquad (4)$$

For the purpose of this analysis, the channel spectral sensitivities can be considered as the result of optical filtration over a baseline sensor efficiency. A simplifying assumption is made that the spectral quantum efficiency of the sensor is constant in the region of interest between 360 and 760 nanometers. As a reference, the baseline efficiency is set to the maximum value of the unit normalized x(λ), y(λ), and z(λ) set. The maximum value, 0.084, occurs at 445nm in the z(λ) function.

Spectral sensitivity sets formed by combination matrices constrained to unit row sums are likely to have peak values that differ from the reference maximum. Modeled as optical filters over a monochrome sensor, test sensitivity sets can be scaled to peak at 0.084. The departure from unit area as the sensitivity sets are scaled, indicates a change in the optical throughput of the filter set. The optical throughput benefit of a test spectral sensitivity set, relative to the reference case, is the ratio of 0.084 to the peak of the test case. For spectral sensitivity set a(λ), b(λ), and c(λ), the ratio is 1.45. Figure 3 shows the a(λ), b(λ), and c(λ) set scaled by 1.45 peaking at 0.084.

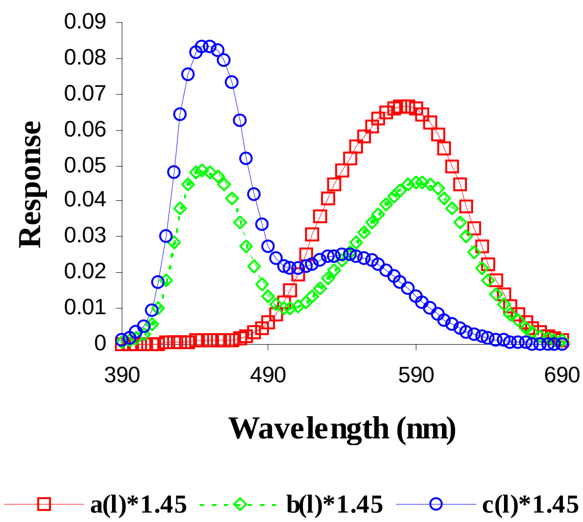

Figure 3. Spectral sensitivities a(λ), b(λ), and c(λ) scaled by 1.45 to peak at 0.084.

The increased optical throughput of the test set, relative to the reference set, suggests a potential advantage in photographic speed. Determination of an actual speed advantage requires analysis of noise amplification through color mixing.

Color Noise Experiment

An experiment was performed to investigate the image quality impairment due to addition of noise to images in the L*, a*, and b* channels. Gaussian distributed, spectrally non-selective, random noise was added to pictorial images. Psychovisual scaling of printed samples of the images produced the psychometric data shown in figure 4.

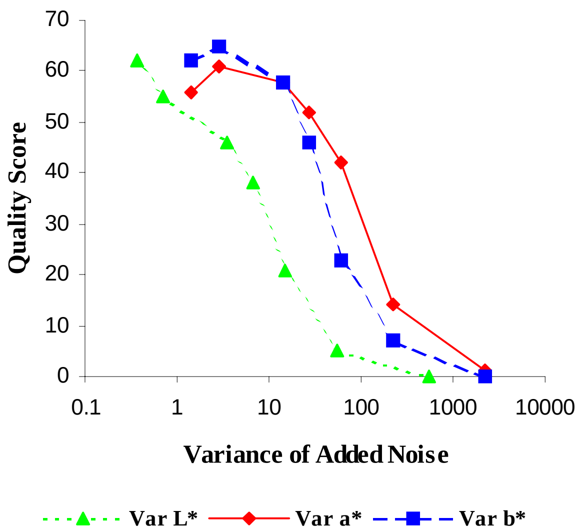

Figure 4. Image quality scores of pictorial print samples with noise added in L, a*, and b*.*

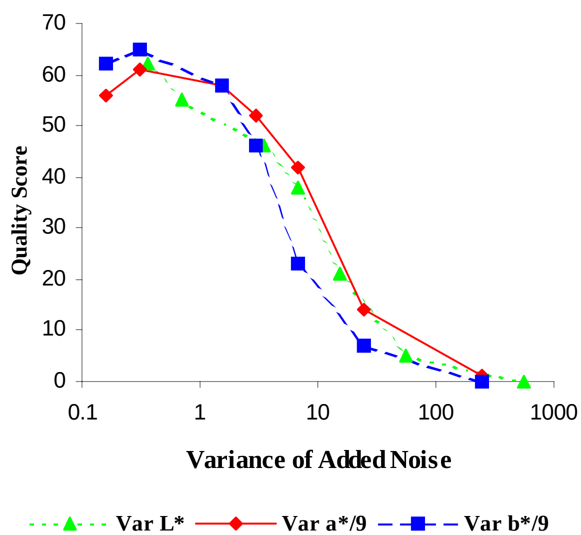

Figure 5. Data of figure 4.,with a, and b* variances scaled by 1/9.*

While L*, a*, and b* differences are generally perceptually equal for macroscopic samples with color differences of Munsell step magnitude, the data of figure 4. indicate that as a color space for small amplitude, micro-image property analysis, the L* channel is approximately nine times more sensitive per unit variance. Figure 5. shows the same data with the a* and b* variance scaled down by a factor of nine.

The data of this experiment indicate that pictorial print image quality impairment generally follows a single psychometric function of noise variance in the L* and scaled a* and b* channels.

Color Noise Analysis

The results of the experiment can be used with the color mixing noise relationship by converting the sensor channel noise proportion to L*, a*, and b* variances. Equation (5) below shows the CIE equations.

$$\begin{bmatrix} L^*-16 \\ a^* \\ b^* \end{bmatrix} \equiv \begin{bmatrix} 0 & 116 & 0 \\ 500 & -500 & 0 \\ 0 & 200 & -200 \end{bmatrix} \times \begin{bmatrix} \left(X / X_N \right)^{1/3} \\ \left(Y / Y_N \right)^{1/3} \\ \left(Z / Z_N \right)^{1/3} \end{bmatrix} \qquad (5)$$

Small signal assumptions allow the cube roots in equation (5) to be approximated by a straight-line slope. The slope of the cube root function has a value of 1.0 at ~19% reflectance which is a typical scene reflectance. This allows variances in L*, a*, and b*, at mean values around 19% scene reflectance, to be approximated by equation (6) for cases of independent noise in X, Y, and Z.

$$\begin{bmatrix} \sigma^2_{L^*} \\ \sigma^2_{a^*} \\ \sigma^2_{b^*} \end{bmatrix} \cong \begin{bmatrix} 0 & (116)^2 & 0 \\ (500)^2 & (-500)^2 & 0 \\ 0 & (200)^2 & (-200)^2 \end{bmatrix} \times \begin{bmatrix} \sigma^2_X \\ \sigma^2_Y \\ \sigma^2_Z \end{bmatrix} \qquad (6)$$

The only case where noise in X, Y, and Z channels are independent is the reference case where camera sensitivities are identical to the color matching functions for CIE X, Y, and Z primaries. For cases where camera sensitivities are formed by linear combination of the color matching functions, and cases where CIE tristimuli are estimated with a mixture matrix, X, Y, and Z variances are not independent. In these cases, to calculate the variances in a*, and b*, the XY and YZ covariances must be included. Referring to matrix A in equation (2), the covariances between X and Y, and Y and Z are shown in equations (7a,b).

$$\sigma_{XY} = \sigma^2 \cdot \left(a_{11} \cdot a_{21} + a_{12} \cdot a_{22} + a_{13} \cdot a_{23} \right) \qquad (7a)$$

$$\sigma_{YZ} = \sigma^2 \cdot \left(a_{21} \cdot a_{31} + a_{22} \cdot a_{32} + a_{23} \cdot a_{33} \right) \qquad (7b)$$

With covariances included, the L*, a*, and b* variances are calculated by equation (8).

$$\sigma_{L^*}^2 \cong 116^2 \cdot \sigma_Y^2 \quad (8a)$$

$$\sigma_{a^*}^2 \cong 500^2 \cdot \left(\sigma_X^2 + \sigma_Y^2 - 2 \cdot \sigma_{XY}\right) \quad (8b)$$

$$\sigma_{b^*}^2 \cong 200^2 \cdot \left(\sigma_Y^2 + \sigma_Z^2 - 2 \cdot \sigma_{YZ}\right) \quad (8c)$$

The CIE L*, a*, and b* scalars (116, 500, 200) in equations (8a-c), indicate that, a* noise in particular, is likely to lead picture quality impairment due to the relatively large gain on the variances in (8b) compared to (8a and c). Applying the experimental L*, a*, and b* variance factors (1:9:9), described earlier for equal noise impairment, to equations (8a-c), still indicates a* noise as the likely impairment. This points particular attention to reducing noise in the a* channel.

Examination of equations (3,7,and 8) reveals relationships which can be utilized to select spectral sensitivities with corresponding color mixture matrices that amplify sensor noise as little as possible. It is apparent from equation (3), that lower noise amplification results from lower matrix row, sums of squared coefficients. This reduces the magnitudes of the variances in X, Y, and Z. From equations (8a and 8b), it is apparent that positive covariance between X and Y, and Y and Z can be utilized to reduce noise amplification, since two times the covariance is subtracted from the variances. Equations (7a and 7b) reveal that positive covariance is a result of matrix coefficients of the same sign, column by column.

Referring back to figure 1., spectral sensitivity set a(λ), b(λ), and c(λ) were formed from x(λ), y(λ), and z(λ) according to the coefficients of matrix ABC-1 shown in equation (4). The inverse matrix, ABC is an example of the color mixture coefficients of this discussion. Matrix ABC is shown below in equation (9), where A, B, and C are the integrated signals of the spectral channels a(λ), b(λ), and c(λ).

$$\begin{bmatrix} X/X_N \\ Y/Y_N \\ Z/Z_N \end{bmatrix} \cong \begin{bmatrix} -0.08 & 1.91 & -0.83 \\ 1.58 & -1.41 & 0.83 \\ -0.91 & 1.08 & 0.83 \end{bmatrix} \times \begin{bmatrix} A \\ B \\ C \end{bmatrix} \quad (9)$$

The coefficients of the ABC matrix above, do not effectively utilize the relationships described earlier. Evaluation of the a(λ), b(λ), and c(λ) set indicates that the impairment due to color mixture amplification of the sensor noise outweighs the optical throughput advantage described earlier. This conclusion is drawn from a spreadsheet construction of the system model incorporating equations (1,2,3, 7, and 8).

Example Results

With the unit area normalized x(λ), y(λ), and z(λ) set as the reference case, the signal (S) was set to a level which produced a proportional noise (σ) of 0.02, in the context of a baseline noise (N) arbitrarily set to 60. The proportional noise level (σ) corresponds to 2% of mean, reflecting an incremental signal-to-noise ratio (SNR) of 50. At ~19% mean reflectance, an incremental SNR of 50 in the Y channel is within typical operating range of electronic cameras.[2,4] These reference conditions effectively simulate a colorimetric electronic camera.

Through the model, the reference conditions produce variances of 5.6, 208, and 33 in the L*, a*, and b* channels. The psychovisual L*, a*, b* noise experiment predicts impairment will be driven by the largest of scaled a*, scaled b*, or L* variance. Scaling the a* and b* variances down by a factor 9, to 23, and 3.4 still indicates that a* noise limits the picture quality. The ABC test conditions produce L*, a*, and b* variances of 17, 1024, and 123. These results show a net speed loss for set a(λ), b(λ), and c(λ) relative to the reference, despite reduced sensor noise from the 1.45 signal level advantage of increased optical throughput. To bring the a* variance levels to coincidence at the level of the reference set, the ABC set requires 3.7x more exposure – a loss of nearly two stops of speed.

Optimization

The model was used to determine the optimum set of spectral sensitivities constructible from linear combination of x(λ), y(λ), and z(λ). The set is optimum regarding color fidelity by construction, having been formed from CIE color matching functions, and it is optimum regarding photographic speed by trading optical throughput against noise amplification to produce the net least noise impairment for a given exposure. The set was constructed by adjusting the coefficients of the matrix A to minimize the maximum variance among L*, and scaled a* and b*. The variances were calculated based on optical throughput and signal mixing, the matrix was constrained to unit row sums, and the sensitivities were constrained to be non-negative at all wavelengths. The set, arbitrarily described as d(λ), e(λ), and f(λ) is shown in figure 6.

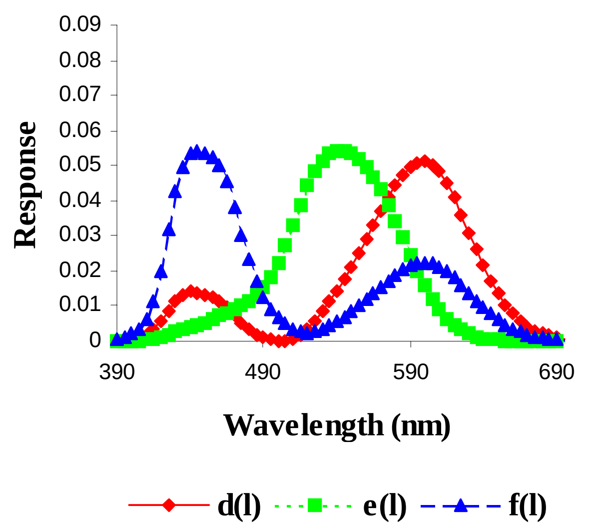

Figure 6. Optimum spectral sensitivity set: d(λ), e(λ), and f(λ).

The details of the optimization are shown in table 1. below. The relative speed advantage of set d(λ) e(λ) f(λ) is a factor of 3x. This was determined by inverting the quotient of the reference case exposure, and the exposure required to match the test case a* variance using the reference sensitivities.

	Reference Case x(λ) y(λ) z(λ)	**Optimized Case d(λ) e(λ) f(λ)**
Throughput	1.0	1.53
L* Variance	5.6	2.2
a* Variance/9	23	5.4
b* Variance/9	3.7	5.4
Speed	X	3X

Table 1. Optimization results from determination of d(λ), e(λ), and f(λ).

Conclusions

The results indicate that electronic camera channel spectral sensitivities influence photographic speed dramatically by determining the fraction of white light to contribute to each color channel, and more importantly by determining the color mixing matrix which amplifies sensor noise. Sensor noise, manifested as graininess in image renderings, can be mitigated by judicious selection of channel sensitivities. Such mitigation increases photographic speed by producing unimpaired renderings with less exposure.

Constrained to perfect color fidelity by construction from CIE color matching functions, two spectral sensitivity sets with very similar optical throughputs compared very differently with the reference case x(λ), y(λ), and z(λ). The example set a(λ), b(λ), and c(λ), while allowing 1.45x more light for sensor exposure relative to the reference, exacerbated sensor noise through channel mixing for a net speed loss of nearly 2 stops. The optimized set d(λ) e(λ) f(λ) allowed 1.53x more light for exposure relative to the same reference, and mitigated sensor noise through channel mixing for a net speed gain of more than 1.5 stops. The similar optical throughputs of the example and optimized spectral sensitivity set, in context of the large difference in noise mixing performance, indicates that the primaries for which spectral sensitivities match color, play a more significant role in electronic camera speed than does the throughput of the sensitivities. Those spectral sensitivities with corresponding color primary transform matrices that invoke the mitigation criteria described in this analysis have significantly higher speed than those that do not.

References

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