Microscopic Study of Electrostatic Forces Acting on Toner Particles on a Conductive Medium

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Abstract

The relationship between microscopic local potentials and charges in a system of toner particles placed on a conductive medium is analyzed. Expressions for electrostatic forces acting on the particles, either insulating or conductive, are derived for a few basic configurations, including linear chains of particles lying flat on the medium or standing up normal to its surface, as well as 2D monolayers.

Introduction

Typical magnetographic engines use grounded conductive media. Although the developing force acting on toner particles is magnetic by nature, auxiliary electrostatic forces are often used, either to control development or help transfer [1]. Such forces are usually created by applying a bias voltage to an external metal plate positioned in front of the medium. Macroscopically, associated electrostatic potential and field distributions depend only on medium/plate geometry and bias voltage. Microscopically, the presence on the medium surface of a system of charged toner particles generate local perturbations to those potential and field. The electrostatic force on one given particle in the system stems from the interaction of the particle charge with the local electric field, i.e., the macroscopic field modified by microscopic fields from other particles and by their interactions with the medium nearby. In the case of insulating precharged particles, all charges are preset, which makes computation of forces rather straightforward. In the case of conductive toner, the particle charges are induced charges that have first to be determined based on local electrical conditions.

Local Potentials and Charges

Let us consider a system of n particles (Figure 1). Particle P_i (i=1..n) of radius r_i is located at point (x_i,y_i,z_i). Plate-to-medium separation is d, bias voltage is U_0. Basic assumption is: $r_i \le z_i \ll d$. We further assume: (i) particle charges q_i are surface charges and therefore small, of the order of r_i^2; (ii) microscopic fields $\mathbf{E}_i$ created by those charges affect the macroscopic field $\mathbf{E}_0$ locally only; (iii) d is small compared to plate dimension L, itself smaller than medium radius R_d : $d \ll L < R_d$; therefore geometry is approximated by that of two parallel infinite planes and macroscopic field $\mathbf{E}_0$ is uniform of magnitude U_0/d ; (iv) local electrostatic potential v_i is considered constant over the small volume occupied by particle P_i; (v) medium is considered a perfect conductive plane; (vi) interactions of charges with the biased plate are neglected because of the larger distance involved, if particles stay close to medium.

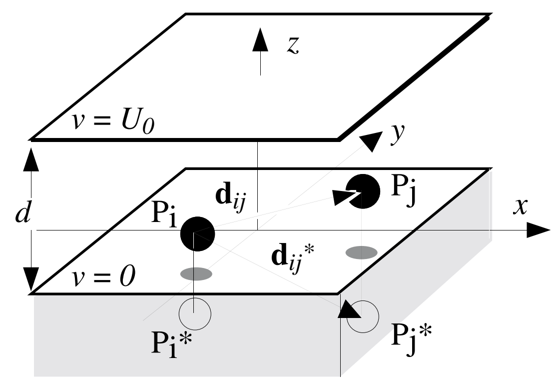

Figure 1. System of particles near surface of grounded conductive medium with parallel plate at constant potential

Condition (v) justifies the use of image theory, i.e., the medium surface at z=0 is replaced by a virtual plane located at z=-d, biased at -U_0, and by a system of virtual particles P_i^*, images of real particles P_i through plane z=0; i.e., virtual charges -q_i located at point (x_i,y_i,-z_i). This warrants plane z=0 to be equipotential v=0, as it was in the original system.

The local electrostatic potential v_i at point (x_i,y_i,z_i) occupied by particle P_i then reads:

$$v_i = \frac{U_0}{d} z_i + \frac{1}{4\pi\varepsilon_0}\left(\frac{q_i}{r_i} - \frac{q_i}{2z_i} + \sum_{j\neq i}^{n}\frac{q_j}{d_{ij}} - \sum_{j\neq i}^{n}\frac{q_j}{d_{ij}^*}\right) \tag{1}$$

where d_{ij} and d_{ij}^* are magnitudes of distance-vectors $\mathbf{d}_{ij}$ and $\mathbf{d}_{ij}$* in Figure 1.

The first term in Equation 1 is the macroscopic potential due to the biased plate. The second term is the sum of all local perturbations due to microscopic charges, either real or virtual. In this second term, the first element is the microscopic potential due to the particle charge itself, while the second element describes its interaction with the medium plane; the third element is the potential induced by all other particles, while the fourth element similarly describes their interactions with the medium plane.

Equation 1 actually produces n linear equations (i=1..n) that relate $2n$ unknowns q_i and v_i. To proceed further, we must write n other independent equations. This will be done either by assuming that the q_i are given (case of insulating precharged particles) or by setting the v_i to zero (case of conductive particles contacting the conductive medium).

Electrostatic Forces

In all cases, once the q_i are known, the force $\mathbf{F}_i$ acting on particle P_i reads:

$$\mathbf{F}_i = -q_i \frac{U_0}{d}\mathbf{k} - \frac{1}{4\pi\varepsilon_0}\left[\frac{q_i^2}{(2z_i)^2}\mathbf{k} + q_i \sum_{j\neq i} q_j \left(\frac{\mathbf{d}_{ij}}{d_{ij}^3} - \frac{\mathbf{d}_{ij}^*}{d_{ij}^{*3}}\right)\right]$$

$$Fx_i = \mathbf{F}_i \cdot \mathbf{i} \;;\qquad Fy_i = \mathbf{F}_i \cdot \mathbf{j} \;;\qquad Fz_i = \mathbf{F}_i \cdot \mathbf{k} \tag{2}$$

where **i**, **j** and **k** are the unit vectors of axes x, y and z.

For actual computation with high number of particles, it is easier to use the equivalent matrix equations:

$$\mathbf{F}x = -\frac{1}{4\pi\varepsilon_0}\mathbf{Q}\,\mathbf{X}\,\mathbf{q} \;;\qquad \mathbf{F}y = -\frac{1}{4\pi\varepsilon_0}\mathbf{Q}\,\mathbf{Y}\,\mathbf{q}$$

$$\mathbf{F}z = -\frac{U_0}{d}\mathbf{q} - \frac{1}{4\pi\varepsilon_0}\mathbf{Q}\,\mathbf{Z}\,\mathbf{q} \tag{3}$$

where $\mathbf{F}_x$, $\mathbf{F}_y$, $\mathbf{F}_z$ are column-vectors of components Fx_i, Fy_i, Fz_i, $\mathbf{q}$ is the column-vector of element q_i, $\mathbf{Q}$ is the diagonal matrix of elements $q_i\delta_{ij}$ (δ_{ij} are Kroneker symbols, i.e., δ_{ij} =1 if i=j and 0 otherwise) and **X**, **Y**, **Z** are matrices of elements:

$$x_{ij} = \frac{\mathbf{d}_{ij}\cdot\mathbf{i}}{d_{ij}^3} - \frac{\mathbf{d}_{ij}^*\cdot\mathbf{i}}{d_{ij}^{*3}} \quad (\text{if } i\neq j) \quad \text{and} \quad x_{ii} = 0$$

(same for y_{ij}, just replacing vector **i** by vector **j**

$$z_{ij} = \frac{\mathbf{d}_{ij}\cdot\mathbf{k}}{d_{ij}^3} - \frac{\mathbf{d}_{ij}^*\cdot\mathbf{k}}{d_{ij}^{*3}} \quad (\text{if } i\neq j) \quad \text{and} \quad z_{ii} = \frac{1}{(2z_i)^2} \tag{4}$$

Insulating Precharged Toner

For simplicity, we assume now that all particles have same radius r and same charge q, as determined, e.g., by precharge (corotron) conditions.

Single Precharged Particle on Medium

Here z=r and Equation 2 yields the simple result:

$$F_x = F_y = 0 \;;\qquad F_z = F_e - F_0$$

$$\text{with}\quad F_e = -q\frac{U_0}{d} \quad\text{and}\quad F_0 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2r)^2} \tag{5}$$

F_z is a quadratic function of charge q. For it to be positive, q and U_0 must be of opposite signs and $|q|$ shall not exceed a critical charge q_c :

$$|q| \;<\; q_c = 4\pi\varepsilon_0 (2r)^2 \frac{|U_0|}{d} \tag{6}$$

With typical values: U_0=-1000V; d=1mm; $2r$=10µm; we get q_c≈11fC (massic charge ≈20µC/g).

F_z is maximum if q=q_c/2, leading to $F_e = 2F_0$ and F_z=F_0. In case of above example, $F_z$max≈2.8nN with q≈5.5fC (≈10µC/g).

Horizontal Chain of n Precharged Particles

Here x_i=2(i-1)r; y_i=0; z_i=r and Equation 2 leads to:

$$Fx_i = -F_0 \sum_{j\neq i}^{n} (j-i)\left[\frac{1}{|j-i|^3} - \frac{1}{\left[1+(j-i)^2\right]^{3/2}}\right]$$

$$Fz_i = F_e - F_0 \sum_{j=1}^{n} \frac{1}{\left[1+(j-i)^2\right]^{3/2}} \tag{7}$$

with numerical values shown in Figure 2. On edges of long chains, F_x →≈0.75F_0, while image force $|F_z$-$F_e|$→≈1.5F_0; thus no dispersion will occur in absence of bias (U_0=0) as long as toner-to-medium friction coefficient remains ≥0.5. At center of long chains, $|F_z$-$F_e|$→≈2F_0; thus, with bias (U_0≠0), F_z will be positive if q<q_c/2, with maximum value being reached for q≈q_c/4.

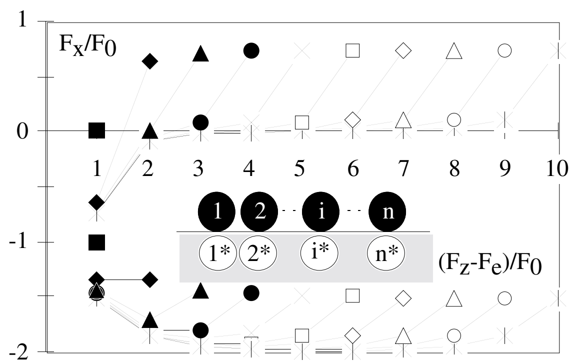

Figure 2. Normalized image force on chains of insulating precharged particles lying flat on conductive medium

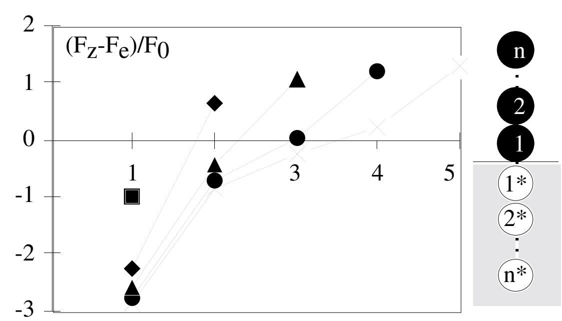

Figure 3. Normalized image force on chains of insulating precharged particles standing up normal to conductive medium

Vertical Chain of *n* Precharged Particles

Here, x_i=y_i=0; z_i=(2i-1)r, thus Equation 2 yields F_{xi}=0 and:

$$Fz_i = F_e - F_0\left[\frac{1}{(2i-1)^2} + \sum_{j\neq i}^{n}\frac{(j-i)}{|j-i|^3} + \frac{1}{(j+i-1)^2}\right] \tag{8}$$

with numerical results shown in Figure 3. For n≥2, image force always repulses the upper top particle, making that type of chain unstable in the absence of other forces (magnetic or other).

2D Monolayer of Precharged Particles

From Equation 2, force on particle P_{ij} in an *n*x*m*-array is:

$$Fx_{ij} = -F_0 \sum_{k=1}^{n}\sum_{l=1}^{m}(k-i)\left\{\frac{1}{\left[(k-i)^2+(l-j)^2\right]^{3/2}} - \frac{1}{\left[1+(k-i)^2+(l-j)^2\right]^{3/2}}\right\}$$

$$Fz_{ij} = F_e - F_0 \sum_{k=1}^{n}\sum_{l=1}^{m}\frac{1}{\left[1+(k-i)^2+(l-j)^2\right]^{3/2}} \tag{9}$$

with expression for Fy_{ij} symmetrical to Fx_{ij}. From Equation 9, it is found that maximum F_x and F_y are reached on the array corners, with such signs that tangential force F_t always pushes particles outside (dispersion), while F_z is minimum at center. At center of large arrays, $|F_z\text{-}F_e| \to \approx 6.1F_0$, thus net force is defined positive everywhere in the array only if $q < \approx q_c/6$. Table 1 gives numerical values for square arrays of increasing size.

Table 1. square arrays of insulating particles*

Array size	1x1	2x2	3x3	4x4	5x5
Ft/F0 at array corners	0	1.14	1.35	1.42	1.45
(Fz-Fe)/F0 at corners	-1	-1.90	-2.25	-2.42	-2.52
(Fz-Fe)/F0 at center	-1	-1.90	-3.18	-3.67	-4.23

Table 1. square arrays of insulating particles

Array size	1 × 1	2 × 2	3 × 3	4 × 4	5 × 5
F_t/F_0 array corner	0	1.14	1.35	1.42	1.45
$(F_z - F_e)/F_0$ corner	–1	–1.90	–2.25	–2.42	–2.52
$(F_z - F_e)/F_0$ center	–1	–1.90	–3.18	–3.67	–4.23

Inductively Charged Conductive Toner

All particles now are maintained at zero potential by contact between them and the medium. Equating all v_i to 0 in Equation 1, unknown induced charges q_i are resolved as:

$$\mathbf{q} = -4\pi\varepsilon_0 \frac{U_0}{d}\,\mathbf{C}^{-1}\,\mathbf{z} \tag{10}$$

where **q** and **z** are the column-vectors of elements q_i and z_i respectively, and **C** is the matrix:

$$\mathbf{C} = \begin{bmatrix} 1/r_1 & 1/d_{12} & \cdots & 1/d_{1n} \\ 1/d_{21} & 1/r_2 & & 1/d_{2n} \\ \vdots & & \ddots & \vdots \\ 1/d_{n1} & 1/d_{n2} & \cdots & 1/r_n \end{bmatrix} - \begin{bmatrix} 1/d_{11}^* & 1/d_{12}^* & \cdots \\ 1/d_{21}^* & 1/d_{22}^* & \\ \vdots & & \ddots \end{bmatrix} \tag{11}$$

difference of two symmetrical matrices $\mathbf{C}_1$ and $\mathbf{C}_2$, where $\mathbf{C}_1$ represents the interaction of the (real) particles, and $\mathbf{C}_2$ that of the medium plane (virtual particles). Once the induced charges q_i are determined, Equation 3 yields the force.

* Original table has been modified by author. Revised table is shown below original

Single Conductive Particle on Medium

Here, **C** reduces to the scalar $1/2r$, and induced charge is:

$$q_0 = -2\pi\varepsilon_0 (2r)^2 \frac{U_0}{d} \tag{12}$$

q_0 is used hereafter to normalize all induced charges (note that it is the optimum charge $q_c/2$ of insulating toner).

Net vertical force (field force Fe minus image force) is:

$$Fz_1 = 4\pi\varepsilon_0 r^2 \frac{U_0^2}{d^2} \tag{13}$$

It is nothing but F_0 as would yield Equation 5 with $q=q_0$ (note also that $F_e=2F_0$).

Horizontal Chain of Conductive Particles

We have $\mathbf{z}=r\mathbf{1}$, $\mathbf{1}$ being the vector with all elements equal to 1 and $\mathbf{C}_1$ and $\mathbf{C}_2$ can be written as:

$$2r\mathbf{C}_1 = \mathbf{I} + \sum_{k=1}^{n-1}\frac{1}{k+1}\left(\mathbf{J}^k + \mathbf{J}^{*k}\right)$$

$$2r\mathbf{C}_2 = \sum_{k=1}^{n-1}\frac{1}{\sqrt{1+(k-1)^2}}\left(\mathbf{J}^k + \mathbf{J}^{*k}\right) \tag{14}$$

where **I** is the unit matrix, **J** the matrix obtained from **I** by shifting the main diagonal of 1's one position to the right (i.e., j_{kl}=1 if l-k=1, zero otherwise) and **J*** is the transpose of **J**. Such structure allows for easy construction of **C**.

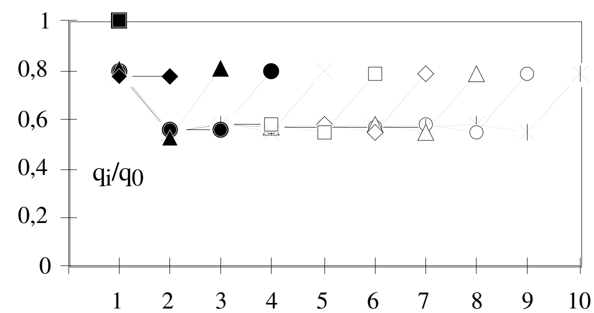

Figure 4. Normalized charges induced in chains of conductive particles lying flat on conductive medium

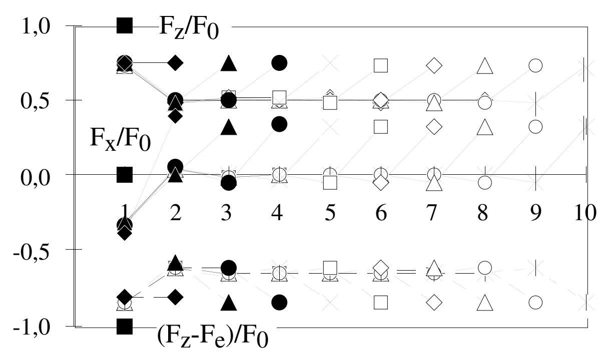

Figure 5. Normalized forces on chains of conductive particles lying flat on conductive medium (Fz:- - - image, —— net)

Figure 4 shows normalized charges q_i/q_0 for horizontal chains of various length. The charge is maximum on edges where it rapidly stabilizes around $0.79q_0$. Figure 5 shows associated normalized force. On edges of long chains $F_x \rightarrow \approx 0.33 F_0$ and $F_z \rightarrow \approx 0.85 F_0$, thus edge particle trajectory would start at ≈ 21° angle from normal.

Vertical Chain of Conductive Particles

The first particle in the chain contacts the medium. Elements of **z** now are defined as: $z_j=(2j\text{-}1)r$, and:

$$2r\mathbf{C}_1 = 2\mathbf{I} + \sum_{k=1}^{n-1} \frac{1}{k}\left(\mathbf{J}^k + \mathbf{J}^{*k}\right) \;;\; 2r\mathbf{C}_2 = \sum_{k=1}^{2n-1} \frac{1}{k}\mathbf{K}_k \qquad (15)$$

where $\mathbf{K}_p$ is the matrix of elements: $k_{ij}=1$ if $i+j=p$ and 0 otherwise. Here too, construction of **C** is easy.

Figure 6 shows normalized charges for chains of particles normal to the medium. The upper particle in the chain always takes the largest charge, always greater than q_0. The second upper particle gets a charge several times lower. Thereafter, charges of more inner particles decrease almost linearly down to a practically neglectible value for the last particle touching the medium.

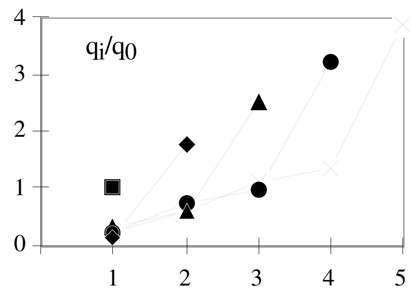

Figure 6. Normalized charges induced in chains of conductive particles standing up normal to conductive medium

Figure 7 shows associated normalized net forces. Vertical chains tend to charge strongly at the upper end, and net force, when adding field force F_e, is positive only on that last particle. The second upper particle actually is slightly pushed against the medium. In the absence of a stronger (magnetic) force tending to maintain the particles on the medium, that kind of chain is nevertheless expected to transfer completely, by successive jumps of the extreme particle, chain length getting shorter and shorter. The dynamics of such process is beyond the scope of this paper.

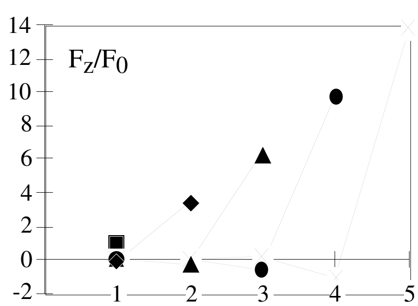

Figure 7. Normalized forces on chains of conductive particles standing up on conductive medium (Fz: - - - image, —— net)

2D Monolayer of Conductive Particles

For a $n \times n$ array, **C** takes the form of a $n^2 \times n^2$ matrix, made up of n^2 blocks, whose structures though relatively easy to figure out are not indicated here. Table 2 shows numerical results for square arrays of increasing sizes. Tangential (dispersive) force at corners F_t rapidly stabilizes at $\approx 0.3 F_0$; normal force F_z is minimum at edges.

Table 2. Square arrays of conductive particles*

Array size	1x1	2x2	3x3	4x4	5x5
Ft/F0 at array corners	0	0.388	0.330	0.329	0.323
Fz/F0 at array corners	1	1.36	1.28	1.30	1.30
Fz/F0 at array center	1	1.36	1.74	1.67	1.60

Table 2. Square arrays of conductive particles

Array size	1 × 1	2 × 2	3 × 3	4 × 4	5 × 5
F_t/F_0 array corner	0	0.388	0.329	0.329	0.323
F_z/F_0 array corner	1	0.520	0.541	0.516	0.509
F_z/F_0 array center	1	0.520	0.155	0.180	0.201
F_z/F_0 average	1	0.520	0.401	0.337	0.299

Note that Tables 1 and 2 cannot be directly compared since values of F_0 will usually be different for insulating and conductive toners. Absolute comparison requires additional assumptions as to the precharge given to the insulating toner. Two examples are shown in Fig. 8 and Fig. 9†

Insulating versus Conductive Toners

The normalized values of Tables 1 and 2 cannot be directly compared since values of F_0 will usually be different for insulating and conductive toners. Absolute comparison requires additional assumptions as to the precharge given to the insulating toner. Also the value of the coulomb electric force F_e has to be taken into account.

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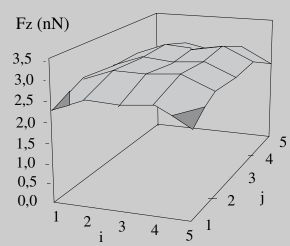

Figure 8. Vertical force on array of 5×5 conductive particles; size 10 µm; bias U0=-800V at d=1mm

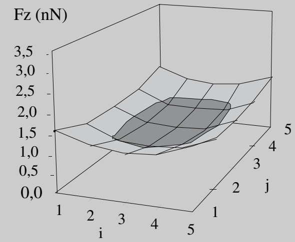

Figure 9. Vertical force on array of 5×5 insulating particles; size 10 µm; bias U0=-1200V at d=1mm; precharge q0/6≈2.2fC

* Original table has been modified by author. Revised table is shown below original
† Author has modified paragraph. Revised paragraphs follow shadded area.
‡ Original Figures 8 and 9 are now disregarded.

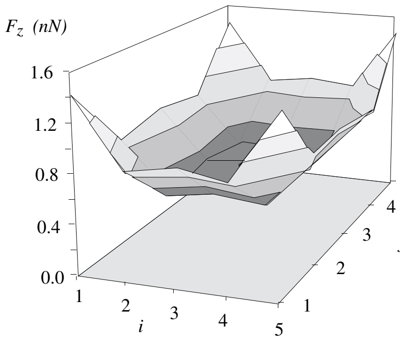

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Figure 8. Vertical force on array of 5x5 conductive particles; size 10 µm; bias U_0=-1000V at d=1mm

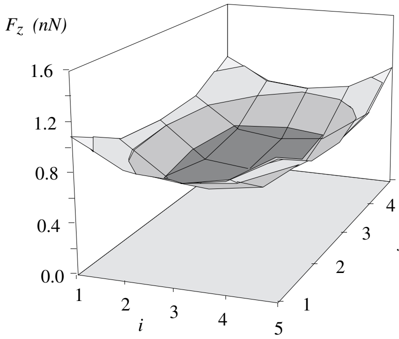

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Figure 9. Vertical force on array of 5x5 insulating particles; size 10 µm; bias U_0=-1000V at d=1mm; precharge $q_0/6≈2fC$

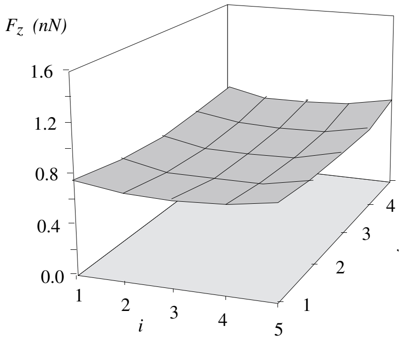

Figure 10. Vertical force on array of 5x5 insulating particles; size 10 µm; bias U_0=-1000V at d=1mm; precharge q≈1 fC

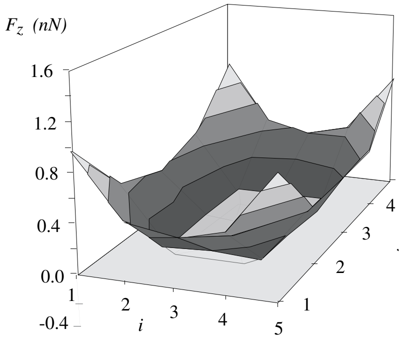

Figure 11. Vertical force on array of 5x5 insulating particles; size 10 µm; bias U_0=-1000V at d=1mm; precharge q≈3 fC

Two examples are shown in Fig. 8 and Fig. 9, where toner size is 10 µm, bias voltage U_0=1000 v and d=1mm.

* This page represent author revised/added text.
† Author has provided new Figures 8, 9, 10 and 11.
‡ Author has provided revised corresponding text with new figures.

Precharge q of the insulating toner is close to its optimal value 2fC≈q_c/6. Although the average force over the array happens to be of the same order for both types of toner, a stronger edge effect is exhibited by the conductive one (larger charges induced at corners).

Fig. 10 and Fig. 11 show how the transfer force distribution is modified for charges q respectivelly lower (q=1fC) and larger (q=3fC) than the optimal charge ≈q_0/6. Lower values yield weaker force but more uniform distribution, while higher values tend to transfer the edges but not the center of the array.

Conclusion

Electrostatic forces appear on conductive toner only where and when an electric field exists (e.g., when entering an electrostatic transfer nip). In contrast, insulating precharged particles can benefit from image electrostatic forces as soon as they get charged (i.e., long before entering transfer nip).

When electrostatic forces build up on conductive toner, vertical components always immediately tend to remove particles from the medium: such premature transfer may cause image fuzziness. With precharged insulating toner, preexisting image forces tend to hold particles onto the medium: disturbances such as pneumatic forces (due to high speed and/or airtight papers) can be overcome [2].‡

Transfer force on edges of 2D particle array is minimum for conductive toners and maximum for precharged insulating toners (Figures 8 and 9): precharged insulating toners generally yield sharper edges than conductive toners.

The self-maintaining image force of precharged insulating toners directly opposes any subsequent transfer forces: charges should not exceed a critical value depending on the bias voltage (this point experimentally well verified).

As a result, maximum optical density attainable with insulating toners is usually lower than that of conductive toners, but it is more stable as regards variations of the electric field (e.g., with humidity), since dependence of vertical force on bias voltage is just linear instead of being quadratic for conductive toners (Equations 2 and 10).

Tangential repulsive forces on corners of 2D particle arrays are usually higher for conductive toners than for precharged insulating toners: precharged insulating toners generally yield sharper edge prints than conductive toners.

The self-maintaining image force of precharged insulating toners directly opposes any subsequent transfer forces. To be transferred by a bias electric field, charges should not exceed a critical value depending on the bias voltage (this point experimentally well verified).

Maximum optical density attainable with insulating toners may be lower than that of conductive toners, but it is more stable as regards variations of the electric transfer field (e.g., with paper water content), since dependence of vertical force on bias voltage is just linear instead of being quadratic for conductive toners (Equ. 2 and 10).

References

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2. J. J. Eltgen "Magnetographic Printing", *Proceedings SPIE Hard Copy Output Technologies*, Los Angeles, 1987, Vol. **759**, ISBN 0-89252-794-3.