Further Applications of the ATD Model for Color Vision

S. Lee Guth
Department of Psychology or School of Optometry
Indiana University, Bloomington, Indiana 47405

Abstract

Previous and recent revisions of the ATD model for color perception and visual adaptation are incorporated into the version that is fully described in this paper.

Keywords: color models, chromatic adaptation, color appearances, color discriminations

2. Introduction

The "ATD95"' model that is outlined in this paper consolidates all prior improvements[1,2] that have been made to the model for color perception and visual adaptation, whose very preliminary version[3] was later published in what can be considered as its initial form.[4] At the present time, ATD95 promises to be the nearly final, if not final, version of the model that will be published in a mainstream journal as a sequel to its initial form.

3. Brief Description of the Model

Figure 1 shows a schematic diagram of ATD95. The initial L and M cone relative absorption curves are exactly as derived by Smith and Pokorny,[5,6] but their S receptor has been modified to have somewhat enhanced sensitivity in longer wavelengths, along the lines suggested by the "bleaching hypothesis" of Wyszecki and Stiles[7] (p. 632). The Smith and Pokorny receptors (as well as the here-modified S) are expressed as transformations of Judd's[8] revised color matching functions, as approximated from CIE functions by Vos.[9]

The initial weighting factors that are applied to the receptors are 0.66 for L 1.0 for M and 0.43 for S. Note that these weighting factors should *not* be taken as representing the relative frequencies of the three classes of cones in the retina. Rather, the initial weights are chosen to provide the model with certain desirable features, such as, for example, the approximate intensity invariance of unique blue and yellow. (If receptor noise and the early exponential and gain control nonlinearities are eliminated from the model, and if the initial weighting factors are then factored into the first opponent stage receptor weights, then it can be seen that the model's cone distributions are reasonable; for example, the S receptors are very scarce.)

The receptor responses are then raised to the 0.7 power, and constant "noise", or "dark light" signals are added in order to yield the pre-gain control receptor responses. Noise signals in the L and M channels (0.024 and 0.036, respectively) are proportional to the initial weighting factors for those receptors, but noise in the S channel (0.31) is 20 times greater than its initial weighting factor. (The hypothesized high noise in the S channel allows prediction of the low-level intensity-dependence of unique green. Other implications of that hypothesis have not yet been fully explored).

After noise is introduced, receptor signals are subjected to nonlinear gain control, which causes almost no signal attenuation at low levels, and very extreme attenuations at high levels. This stage of the model has been sometimes identified as the "adaptation" stage, but it is now preferred that the word "adaptation be used to describe a *procedure* rather than a neural process. One reason is the tendency to assume that "adaptation" effects obtain mainly under conditions of prolonged exposure to light; For example, one might hear, "Adaptation did not occur, because lights were exposed only briefly." But gain control begins almost immediately (within milliseconds) and is almost always associated with the normal visual response. It is preferable to use "adaptation" for a procedure, and "gain control" for a neural process.

In addition to "self attenuation," which is associated with the gain control of receptor activity associated with, say, a "target" light viewed in the dark, gain control is also under spatial or temporal influences. That is, simultaneously or previously presented adapting lights (with the term, "adapting" being used to describe a procedure) can have very strong effects on gain control for the receptors that are associated with a target light, itself.

The attenuated (more or less) L, M and S signals then feed the "initial" mechanisms of the first opponent stage of the model. After compressive transformations, neural outputs from these mechanisms define the A_1, T_1 and D_1 signals for a particular target light. These signals mediate, (i) "luminances", (ii) brightnesses, (iii) discriminations, and (iv) detections (which, within the model, are actually discriminations from noise). In particular, A, is defined as the "luminance" channel, the vector sum of A_1, T_1 and D_1 correlates with apparent brightness, and distances between tips of $A_1T_1D_1$ vectors for two lights correlate with "small step" color discriminations between them.

Neural outputs from the initial mechanisms of the first opponent stage also interact and feed the initial mechanisms of the second opponent stage of the model. After compressive transformations, outputs from the second opponent stage initial mechanisms define the A_2, T_2 and D_2 signals for the target light. These signals are like the classic "white" (W), "red vs. green" (R/G), and blue vs. yellow (B/Y) signals, and they mediate the apparent hue and saturation of the target light. Large-step perceptual distances (as opposed to small-step discriminations) are related to distances between relevant $A_2T_2D_2$ vectors.

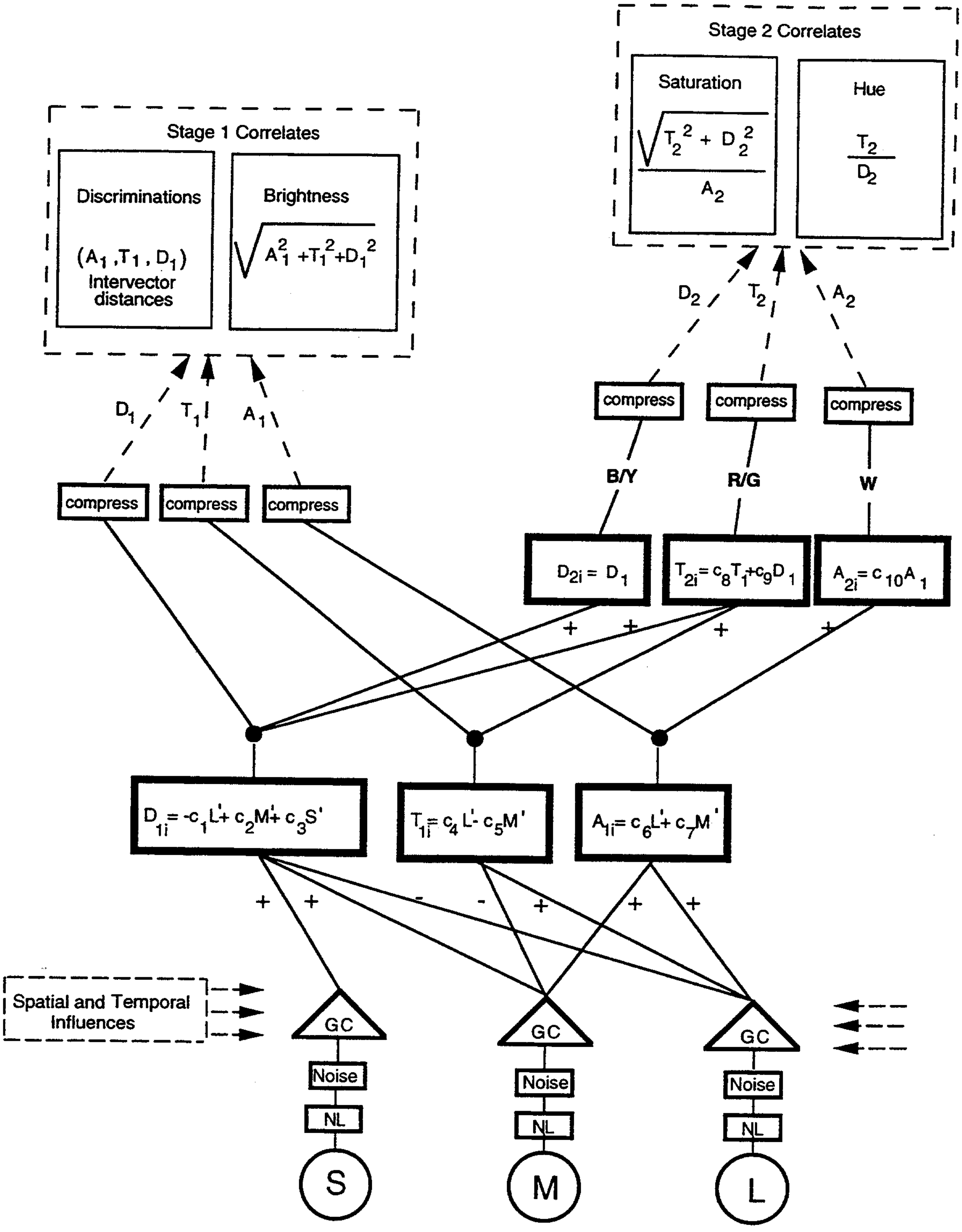

Figure 1. Schematic diagram of ATD95.

4. Calculations

4.A. "Self Adaptation"

It is assumed that CIE *x,y,z* chromadcity coordinates, and *Y*, in photopic trolands are specified for a given "target" light, and the problem is to derive the final (after compression) values for A_1,T_1,D_1, and A_2,T_2 and D_2. It is assumed that, strictly speaking, only "self adaptation" is involved, as would be the case if a light were viewed in a dark surround. ("Adaptation" is the appropriate term here, because regular steady-state viewing can indeed be thought of as a procedure that allows gain control to occur.) However, many predictions made by the model using only self adaptation are similar to those that would be made if gain control were due to "normal" viewing conditions, where it can be assumed that average adaptation due to the wandering gaze is approximately equal to adaptation due to achromatic light of about the same luminance as the target light. Also, the size and duration of a target light itself almost certainly affects its own gain control, but such spatial and temporal aspects of the model have not yet been fully explored.

1. Transform the *xyz* chromaticity coordinates to Judd's[8] *x'y'z'* values using equations from Vos[9]:

$$x' = \frac{1.027x - 0.00008y - 0.0009}{0.03845x + 0.01496y + 1} \tag{1a}$$

$$y' = \frac{0.00376x + 1.0072y + 0.00764}{0.03845x + 0.0149y + 1} \tag{1b}$$

$$z' = 1 - x' - y' \tag{1c}$$

2. Rescale the *x'y'z'* chromaticity coordinates to **X'Y'Z'** tristimulus values, with *Y'* = *Y* in photopic trolands. That is, multiply *x'*, *y'* and *z'* each by *Y/y'* (Issues regarding the strict validity of transforming CIE chromaticity coordinates or tristimulus values to Judd's primed values are here ignored). As noted elsewhere,[2] log trolands are linearly related to log luminance, and an equation for converting luminance (in cd/m^2) to trolands (td) is,

$$\text{td} = 18\ (\text{cd/m}^2)^{0.8}$$

3. Calculate the responses in the L and M channels by determining the cone absorptions[5,6] (with peak absorptions normalized to unity) multiplying by the initial receptor weighting factors, raising the results to the 0.70 power, and adding the appropriate "dark light" or constant noise signals. The same operations define S, but, as mentioned above, it has been made to have enhanced sensitivity in longer wavelengths, by adding 0.04 *Y'* to it.

$$L = [0.66(0.2435\ X' + 0.8524\ Y' - 0.0516\ Z')]^{0.70} + 0.024, \tag{2a}$$

$$M = [1.0(-0.3954\ X' + 1.1642\ Y' + 0.0837\ Z')]^{0.70} + 0.036, \tag{2b}$$

$$S = [0.43(0.04\ Y' + 0.6225\ Z')]^{0.70} + 0.31. \tag{2c}$$

4. Calculate L_g, M_g and S_g, which are the receptor channel responses after gain control due to "self attenuation". In *ATD95*, the gain control rule is that the receptor signals, L, M or S, as defined in equations 2a - 2c, are multiplied by their respective attenuation factors, $\sigma/(\sigma + L)$ or $\sigma/(\sigma + M)$ or $\sigma/(\sigma + S)$ where $\sigma = 300$.

$$L_g = L\ [\sigma/(\sigma + L)] \tag{3a}$$

$$M_g = M\ [\sigma/(\sigma + M)] \tag{3b}$$

$$S_g = S\ [\sigma/(\sigma + S)] \tag{3c}$$

5. Calculate what are now called the initial A_{1i}, T_{1i} and D_{1i} responses for the first stage.

$$A_{1i} = 3.57L_g + 2.64\ M_g, \tag{4a}$$

$$T_{1i} = 7.18\ L_g - 6.21\ M_g, \tag{4b}$$

$$D_{1i} = -0.70\ L_g + 0.085\ M_g + 1.00\ S_g. \tag{4c}$$

6. Calculate the initial A_{2i}, T_{2i}, and D_{2i}, responses for the second stage.

$$A_{2i} = 0.09\ A_{1i}, \tag{5a}$$

$$T_{2i} = 0.43\ T_{1i} + 0.76\ D_{1i} \tag{5b}$$

$$D_{2i} = D_{1i}. \tag{5c}$$

7. Calculate the final (compressed) responses for $A_1T_1D_1$ and for $A_2T_2D_2$

$$A_1 = A_{1i},/(200 + |A_{1i}|); \qquad A_2 = A_{2i},/(200 + |A_{2i}|), \tag{6a; 6b}$$

$$T_1 = T_{1i},/(200 + |T_{1i}|); \qquad T_2 = T_{2i},/(200 + |T_{2i}|), \tag{6c; 6d}$$

$$D_1 = D_{1i},/(200 + |D_{1i}|); \qquad D_2 = D_{2i},/(200 + |D_{2i}|) \tag{6e; 6f}$$

These values define vectors in the three-space fix the first stage or in the three-space for the second stage. As mentioned above, the first stage mediates apparent bdghtnesses, as well as small color differences (*i.e.*, discriminations and detections). (Actually, detections are simply discriminations from backgrounds, with backgrounds being just noise in the case of "dark" thresholds.) The second stage mediates color appearances and judgments of large step color differences.

In particular, the apparent brightness of a light (*Br*) is proportional to,

$$Br = (A_1^2 + T_1^2 + D_1^2)^{0.50}, \tag{7}$$

and if A_1, T_1, and D_1 values are calculated for two lights, then their discriminability is inferred by calculating the *small* color difference,

$$\Delta E_s = (\Delta A_1^2 + \Delta T_1^2 + \Delta D_1^2)^{0.50}. \tag{8}$$

The small color difference required for a discrimination depends, of course, on many factors. Here, $\Delta E_s = 0.002$ is appropriate for predicting MacAdam's[12] results, whereas $\Delta E_s = 0.005$ is more appropriate for predicting a wide variety of other discrimination and detection results.

Judgments of *large* color differences are directly related to,

$$\Delta E_L = (\Delta A_2^2 + \Delta T_2^2 + \Delta D_2^2)^{0.50}, \qquad (9)$$

and judgments of hue and saturation (or chroma) are directly related to,

$$H = T_2/D_2, \text{ and} \qquad (10)$$

$$C = (T^2 + D^2)^{0.50}/A. \qquad (11)$$

4.B. Chromatic or Achromatic Adaptation

As emphasized earlier, within the framework of the model, even normal viewing conditions involve adaptation, because gain control is thought to operate almost instantaneously, and gain control is accordingly an integral part of the normal visual response. However, it is also necessary to describe how to apply the model in situations where previously or simultaneously presented lights affect the perception of a target light.

In the original version of the model, chromatic adaptation effects were modelled by changing the gain control parameters in order to exaggerate the attenuations of the receptor responses. The revised procedure,[2] which is intuitively more appealing, is to weight the adapting stimuli by factors that relate to display characteristics such as the size and duration of adapting lights. (This change also produces a major conceptual improvement in the model, because the gain control system remains always fixed, which means that the entire model is now completely "hard wired".) For example, in many cases, an adapting light covers a relatively large portion of the visual field, whereas a "target" light, whose appearance is of concern, is quite small. Within the model, it is then assumed that, *in regard to its effect on gain control for the receptors associated with the target light*, the luminance of the adaptation light must be multiplied by a factor, now called α, that can be at least as high as, say, 50. It is as if there is a retinal flooding of neural information from surrounding areas that affects gain control in center areas. Similarly, in cases of successive contrast, a long duration and intensely fixated adapting light might also be associated with large α's. It should be noted that, within the model, no conceptual distinction is made between successive and simultaneous (spatial) contrast effects on a target light, in the sense that both phenomena are produced by gain control changes in the receptor signals that are associated with the target light.

Consider now the general procedure for making predictions about adaptation effects on a target light. The most straightforward case is when it can be assumed that gain control for the target light is determined solely by the adapting light. This is the assumption for all predictions made in the present paper.) The assumption is probably reasonable for situations in which there are large adapting fields and small target lights (unless the target light is intensely fixated) and/or long duration adaptation lights and briefly viewed target lights.

First, to evaluate the color appearance of the target light as it would appear in the dark, the X'Y'Z' values (in trolands) for the light are put through the model in the usual way in order to obtain the light's $A_1T_1D_1$ or $A_2T_2D_2$ values. (It is probably true that XYZ's, rather than X'Y'Z's, can be used in most situations.)

Second, the X'Y'Z' values of the adapting light (call them $X'_aY'_aZ'_a$) are each multiplied by α (α has ranged from 15 to 50 in earlier work[2]) and the associated L_a, M_a & S_a receptor responses are determined using equations 2a, 2b and 2c. Third, the attenuation factors, $(\sigma/\sigma + L_a)$ and $(\sigma/\sigma + M_a)$ and $(\sigma/\sigma + S_a)$ are determined, and they are substituted for the LMS attenuation factors that were previously determined for the target light, itself. (i.e., they are substituted for the attenuation factors associated with "self attenuation" of the target light.) After this substitution, new L_g, M_g and S_g values are used in equations 4a, 4b and 4c, and the rest of the model is then applied in the usual fashion to yield the $A_1T_1D_1$ or $A_2T_2D_2$ values for the target light viewed under the adapting light. Alternatively stated, as shown below, L_a, M_a & S_a (for $\alpha X_a'$, $\alpha Y_a'$ and $\alpha Z_a'$) are substituted for the L, M and S values *inside* the square brackets of equations 3a, 3b and 3c, and the model is then applied as usual to obtain the $A_1T_1D_1$ or $A_2T_2D_2$ values for the target light viewed under adaptation conditions.

$$L_g = L\,[\sigma/(\sigma + L_a)] \qquad (12a)$$

$$M_g = M\,[\sigma/(\sigma + M_a)] \qquad (12b)$$

$$S_g = S\,[\sigma/(\sigma + S_a)] \qquad (12c)$$

If it is of interest to know the X'Y'Z' values of an unadapted light that would have the same ATD values as the adapted target light, then the model's equations (of course, without any substituted attenuation factors) can be reversed to solve for the X'Y'Z's given those ATD's.

If it is assumed that the target light itself, as well as the adapting light, contributes to gain control for the target light, then the X'Y'Z's for the two lights must be appropriately weighted, summed, and then used to determine L_a, M_a & S_a. (For example, if α is assumed to be 5.0 and the test light is given its "self attenuation" weight of 1.0, then X'Y'Z's should be determined for a light mixture of 5.0 times the adaptation lipt plus 1.0 of the target light, and the resulting X'Y'Z's should be used to calculate the L_a, M_a & S_a values that will determine the adaptation attenuation factors.)

It might also be of interest to determine the X'Y'Z's for a light which, when viewed under a particular adaptation light, will have some specified $A_1T_1D_1$ or $A_2T_2D_2$ value. For example, in some of the predictions that appear elsewhere,[2] it is necessary to solve for the xy chromaticity coordinates for a light of a particular luminance that will appear achromatic (ie., that will have some specified A_2, and $T_2 = D_2 = 0$). If gain control is determined completely by the adapting light, then a solution is straightforward and exact; however, if gain control is partially from the target light and partially from the adapting light, then it is necessary to search X'Y'Z' space in order to approximate the answer. (As mentioned above, the simplifying assumption that gain is mainly from the adapting light is often quite reasonable.)

5. Applications

5.A. Large-step Percentual Distances

Figure 2 shows x,y chromaticity coordinates of Munsell value 5 colors as given by Wyszecki and Stiles.[7] Figure 3 shows XYZ's of these colors (constant $Y = 300$ td) transformed into corresponding $A_2T_2D_2$'s, and then plotted in the T_2,D_2 space (constant A_2) of ATD95. (A constant Y plane is not exactly constant A, but the variation in A is negligible.) It is assumed that only "self attenuation" is involved, as if the Munsell colors were each viewed in isolation with a dark background. Figure 4 shows the reverse kind of prediction. That is, if the model were perfect, and if the Munsell data perfectly represented human color perception, then, in the T,D plane of the model, straight lines radiating from the origin would represent loci of constant hue and circles around the origin would represent loci of constant saturation or chroma. Figure 4, which compares favorably to Fig. 2, shows such straight lines and circles transformed into x,y space.

It was mentioned above that many predictions involving only "self attenuation" are very similar to those that assume general whitish adaptation. This can be seen by comparing Fig. 2 with Fig. 5 the latter of which, which shows Munsell 5 colors in TD space, as calculated under the assumption that gain control is determined only by illuminant C at 300 td. (Although it is clear that this, rather than with each chip viewed with a dark background, is actually closer to the situation under which the relevant Munsell judgments were made, it seems that the absolute luminances reflected by the Munsell chips judged during the "renotadon" sessions are nowhere specified.)

In examining Figs. 2 or 5, note that the angle between Munsell 5B and 5PB (which are the dotted hue loci heading northwest and almost north, respectively) is relatively small, and, correspondingly, the angle between 5PB and 5P (which heads northeast) is relatively large. Also, the peripheral saturation (chroma) loci are seriously compressed. However, it appears that these are problems with the data—not with the model: "In studies in the author's laboratory, colors on 5B and 5PB always appear, according to global metrics, too close together and shift toward 5B as a whole so that the angle between 5PB and 5P reaches 70 deg in contrast to 36 deg in the Munsell notation''.[10] Also, "....all the color practitioners with whom I have discussions are unanimous in pointing out that Munsell colors in the region from 5B to 5PB look too much alike one another," and, in regard to saturation loci, "...contours at higher levels...tend to be jammed together..."[11] These comments suggest that ATD95 (as well as earlier versions) provides a rather exceptional account of large-step color differences. (Perhaps, "medium-step" differences would be a better term, because the color judgments associated with the Munsell colors are not "global" and are made within relatively circumscribed regions of color space.)

Sets of Figs. 6-9 and 10-13 are exactly comparable to Figs. 2-5, except that they are for Munsell 2 (at 47 trolands) and Munsell 5 (at 900 trolands) respectively. (Note that similar figures in an earlier paper[2] had the troland values too high by a factor of 3.0).

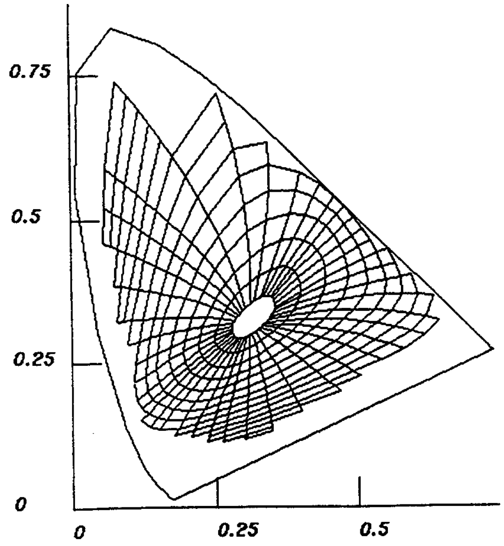

Figure 2. Munsell color (value 5) in x,y space.

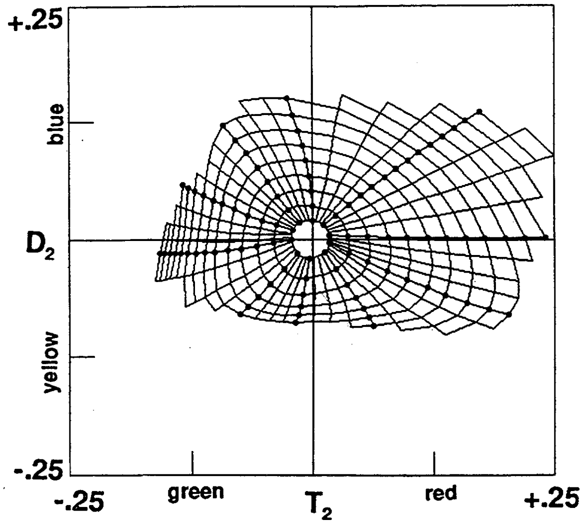

Figure 3. Munsell value 5 colors (assuming consent luminance of 300 td) transformcd into T_2D_2 space.

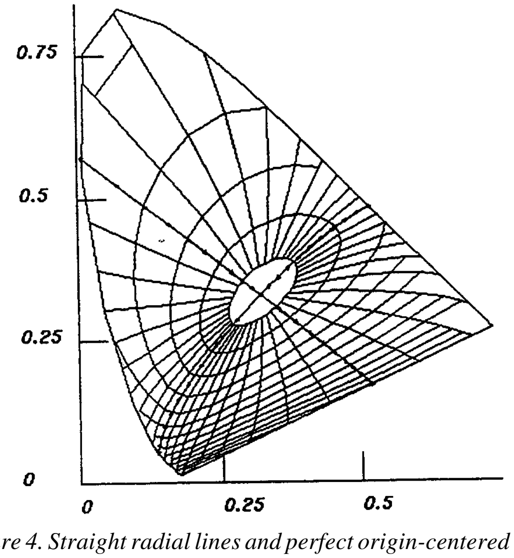

Figure 4. Straight radial lines and perfect origin-centered circles from Fig. 3 transformed to x,y space.

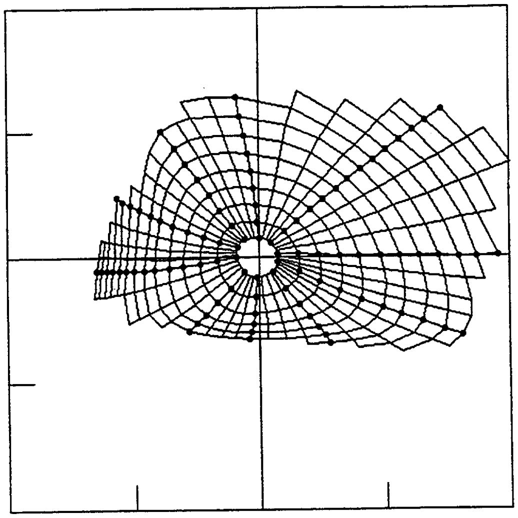

Figure 5. Same as fig. 3, except gain control is assumed to be determined only by Illuminant C at 300 td.

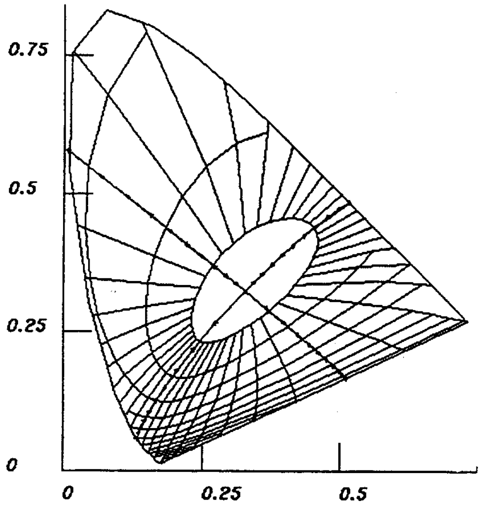

Figure 8. Straight radial lines and perfect origin-centered circles from fig. 7 transformed to x,y space.

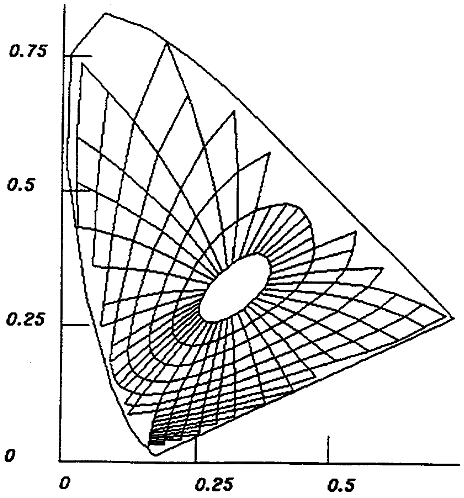

Figure 6. Munsell colors (value 2) in x,y space.

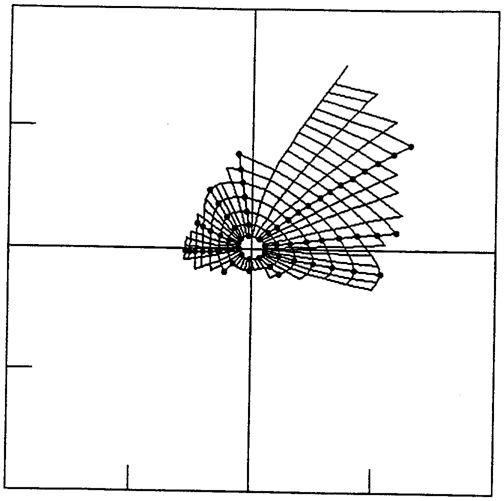

Figure 9. Same as fig. 7, except gain control is assumed to be determined only by Illuminant C at 47 td.

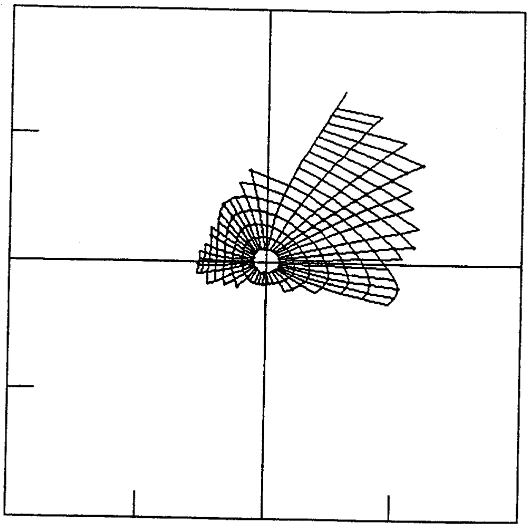

Figure 7. Munsell value 2 colors (assuming constant luminance of 47 td) transformed into T_2D_2 space.

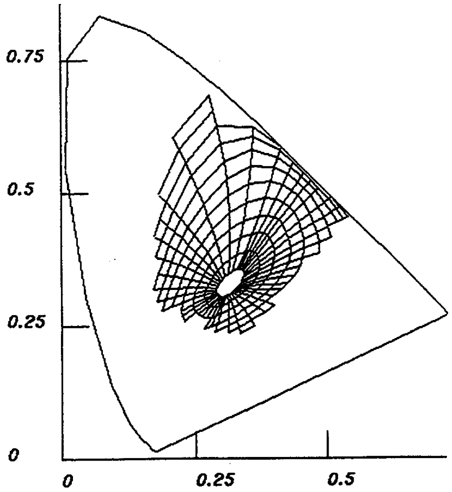

Figure 10. Munsell colors (value 8) in x,y space.

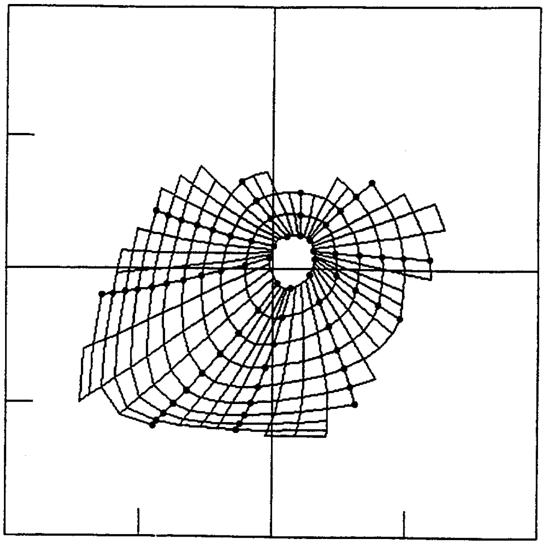

Figure 11. Munsell value 8 colors (assuming constant luminance of 900 td) transformed into T_2D_2 space.

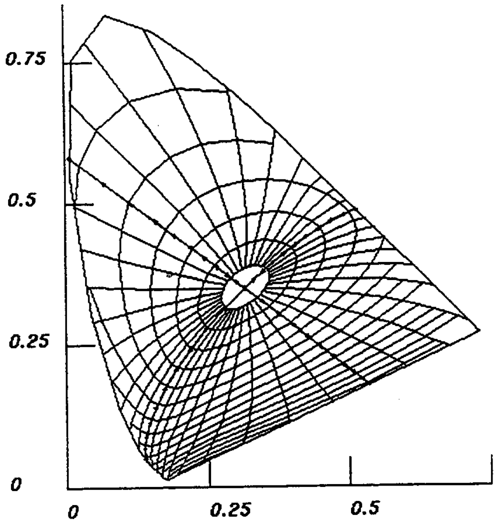

Figure 12. Straight radial lines and perfect origin centered-circles from fig. 11 transformed to x,y space.

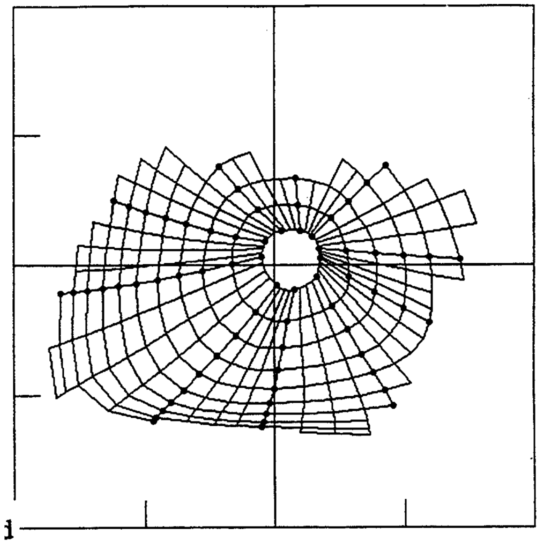

Figure 13. Same as fig. 11, except gain control is assumed to be determined only by Illuminant C at 900 td.

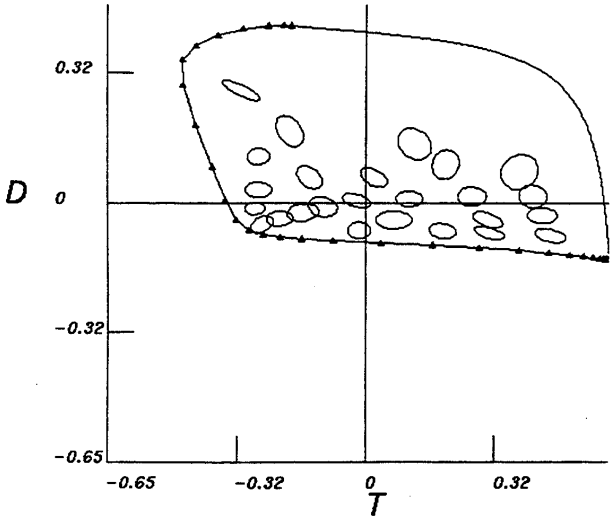

Figure 14. MacAdam's ellipses (magnified by 15) in 400 troland T_1D_1 space.

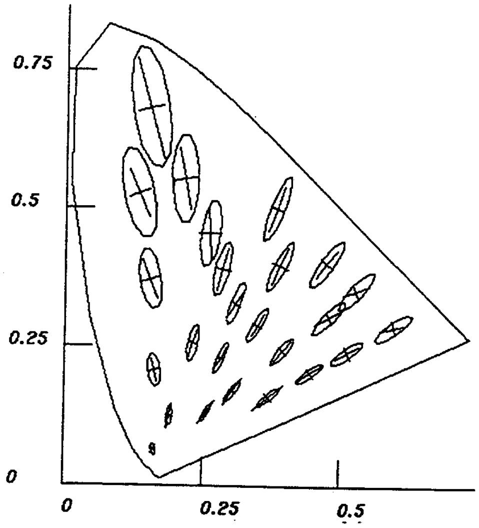

Figure 15. Predicted x,y ellipses, based upon transformations of perfect and identical circles (drawn around the center points of the shapes in fig. 14). "Crosses" are the major and minor axes obtained by MacAdam. Predictions and data are magnified by 10.

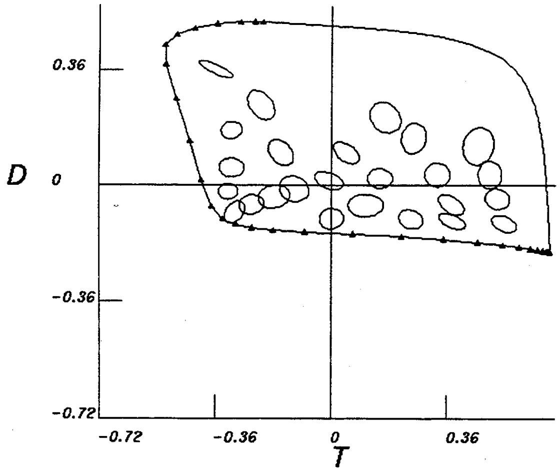

Figure 16. Same as fig. 14, except based upon a slightly modified model.

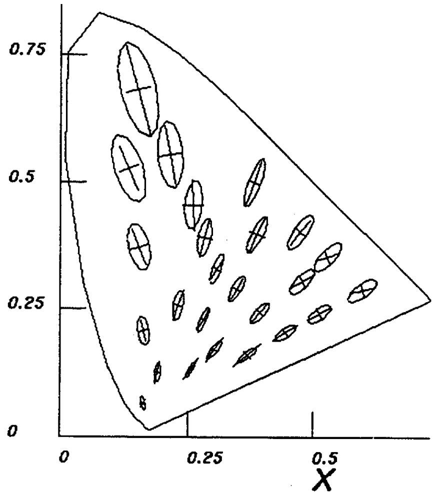

Figure 17. Same as fig. 15, except based upon a modified model.

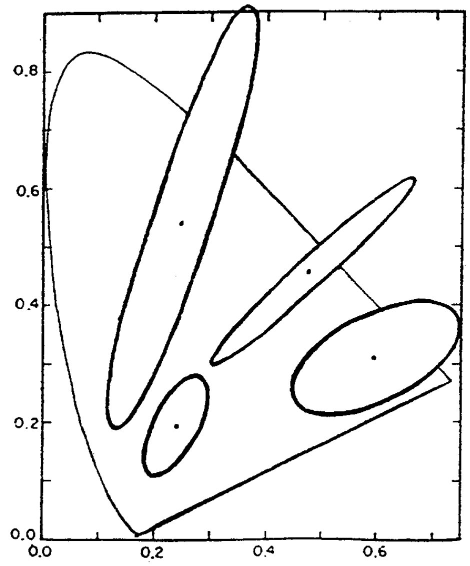

Figure 18. A subset of low-level discrimination ellipses obtained by Brown.

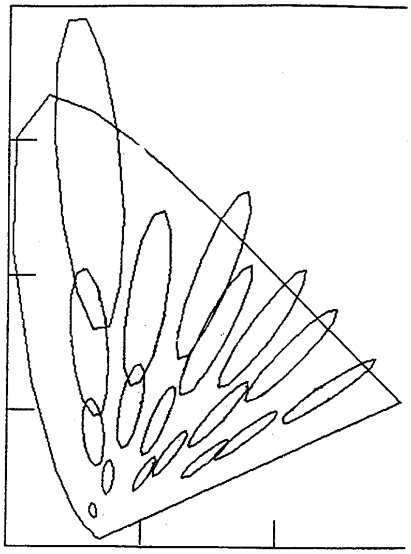

Figure 19. Predictions of ellipses that MacAdam would have obtained if he examined some at a low level (80 trolands) as well as at 400 trolands (as shown in fig. 15).

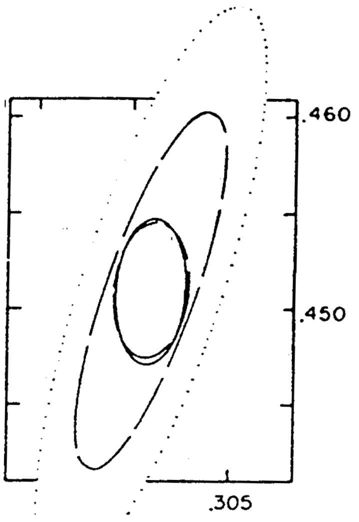

Figure 20. Ellipses from Brown illustrating not only size, but also orientation, changes that accompany luminance variations.

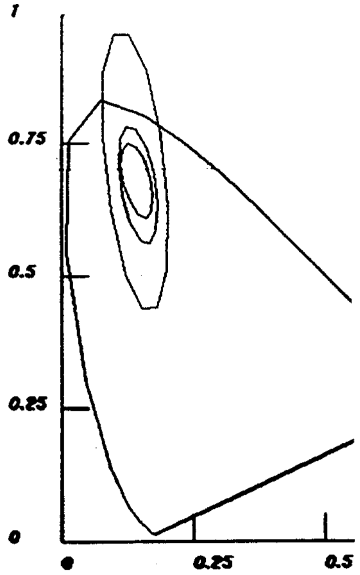

Figure 21. Demonstration that ellipses predicted by ATD95 show appropriate size and orientation changes as luminance varies.

5. B. Small-Step Color Discriminations

Figure 14 shows T_1D_1 values (equal A_1's) derived from XYZ's (equal Y = 48 cd/m^2 = 400 td) from the circumferences of MacAdam's[12] actual (not magnified) discrimination ellipses (The shapes in TD space are magnified by 15.) If the data perfectly represented discriminations from each of MacAdam's center points, and if equal distances in T_1D_1 space represent just noticeable differences (JND's) then the shapes in Fig. 14 would all be identical perfect circles. Figure 15 shows the reverse prediction. That is, it shows projections of identical perfect circles, drawn around the center points in Fig. 14, into xy space. (The radius of that perfect circle is 0.002, which is the mean center-to-periphery distance calculated from all of the shapes in Fig. 14. In regard to equation 8, this is equivalent to stating that ΔE_s = 0.002 for the predicted ellipses in Fig. 15.) The predicted ellipses in Fig. 15 are good fits to the major and minor diameters (shown as "crosses") of MacAdam's obtained ellipses. (For ease of viewing, the data and the predicted ellipses are magnified by a factor of 10.0) An even better account of MacAdam's data can be produced if the model is modified to describe an imaginary observer (who could conceivably represent MacAdam's observer) whose D_1 system is increased by a factor of 1.18, and whose pupil diameter is somewhat larger than average, so that the appropriate retinal illumination is 700 td instead of 400 td. The predictions by the modified model are shown in Figs. 16 and 17.

It is a serious mistake to believe that MacAdam's ellipses represent color discriminations, in general, because the sizes of discrimination ellipses change drastically as luminance level changes, and their orientations change. Figure 18 shows four of the eight low-level discrimination ellipses that Brown,[13] who worked with MacAdam, obtained using himself as an observer. (These four ellipses were obtained at luminances ranging from about 3 to 10 td, and they are magnified by a factor of ten, which puts them on the same scale as those shown in Fig. 15.) Figure 19 illustrates that ATD95 predictions, again using $\Delta E_s = 0.002$, of relatively low-level (80 td) discrimination ellipses show the required very large increase in size. (The predicted low level ellipses are around a set of center points that are randomly chosen from those of MacAdam as shown in Fig. 15. In other words Figs IS and 19 shows model predictions of ellipses that MacAdam would have obtained if he had tested around some center points at both 400 and 80 td.)

Figure 20, which is also from Brown[13] illustrates how ellipses change in orientation as luminance decreases from about 175 to 8 td. For the indicated ellipses, and for ellipses in general, there is a tendency for their major axes to become oriented toward the violet corner of chromaticity space as luminance decreases. Predictions by ATD9S also show that tendency, as shown for one ellipse, at luminances ranging from 1000 to 100 td, in Fig. 21.

5. Chromatic Adaptation

The chromatic adaptation predictions made by ATD95 are very similar to those shown in a previous paper.[2] Accordingly, summaries of relevant experiments, and predictions of data from them, will not be given here.

6. Conclusions

It is thought that the ATD model for color perception and visual adaptation, which has evolved into the ATD95 model that is herein described, should now be seriously considered by the vision community as a replacement for all models that are currently used to make predictions (or to establish standards) that concern human color perception. The model makes superior predictions of not only the data that are shown in this paper, but also of many other visual responses such as have been considered in the initial[4] or more recent[2] version of the model.

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