Chromatic Aberration, Linear Models, and Matching Color Images

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Abstract

In an earlier paper,[7] we modeled the axial chromatic aberration of the human eye with an optical transfer function (OTF). The OTF quantifies the interaction between wavelength and spatial pattern in retinal image formation, and therefore has important consequences for color perception and image matching. However, using the OTF for practical calculations can be quite cumbersome. Here, we show how using finite dimensional linear models for surface and illuminant spectral functions greatly simplifies these calculations in certain cases. In our earlier wow, we applied our model to the problem of matching color images on missive displays; we show here how this application is a special case of this use of linear models.

Introduction

Because the human eye has an optical defect caned axial chromatic aberration, it can only be in focus at one wavelength at a time. Other wavelengths are out of focus and cause a blurred image to form on the retina The amount of blurring depends on both the wavelength composition of the light and its spatial pattern. This interaction between wavelength and spatial pattern has important consequences for color perception and image matching. In this paper, we show how the use of linear models greatly simplifies the calculations required to quantify these consequences, and how to apply them to the problem of matching color images.

We have modeled the transformation from the image at the eye, which we cad the *corneas* image, to the retinal image as an optical transfer function (OTF).[7] We based our calculations on an analysis of the chromatic aberration of a diffraction-limited optical system with a circular aperture described by Hopkins (1955).[5] We implemented Hopkins' calculation using the parameters of the human eye,[6,11,8,1,10,9,3] and we used the results of Williams *et al.*[14] to incorporate the wavelength-independent aberrations. The OTF takes the wavelength and spatial frequency distribution of the corneal image and produces the wavelength and spatial frequency distribution of the retinal image. Using the wavelength sensitivity of the retinal photoreceptors, we can then compute the spatial frequency distribution of photoreceptor responses.

Calculating the OTF enabled us to quantify a number of the interactions between wavelength and spatial frequency relevant for color perception and image matching. First, above 20 cycles per degree (cpd), only wavelengths near the accommodated wavelength can have detectable contrast in the retinal image, which implies that high spatial-frequency components play little role in color and contrast perception. Second, in the moderate spatial-frequency range, from 520 cpd, when the observer is accommodated to the yellow or green part of the spectrum, the visual system is dichromatic: there is no contrast in the short-wavelength receptor class.

Perhaps most important, the OTF we calculated suggests an improved procedure for matching color images. The conventional method of setting point-by-point matches between images fails to account for the fact that image points on different displays may not have same pointspread function on the retina Since the spatial patterns on the retina from individual points on the displays do not match, one cannot match the retinal images of two points simply by adjusting the intensities of the three display primaries. Instead, to equate photo-pigment absorptions between images on different displays, one must adjust the primary intensities in corresponding spatial-frequency bands. (We describe this procedure in more detail below.)

Because the OTF depends on the wavelength of the corneal image (as wed as its spatial frequency), using it to compute photoreceptor responses can be computationally quite expensive. When an image arises from a natural scene, representing the surface and illuminant spectral functions with finite-dimensional linear models greatly simplifies the computation. In that case, a simpler OTF can be computed that depends not on the wavelength of the corneal image but only on the weights of the basis functions that model the image. The number of weights win in most cases be much smaller than the number of wavelength samples, which is why the computation becomes so much less expensive.

This simpler OTF can also be used to predict matches of color images on emissive displays. This is because emissive displays can be represented with a three-di-mensional linear model. We presented our algorithm for color matching on emissive displays in an earlier paper.[7] Here we show that this use of the OTF is a special case of the OTF that arises from representing surface and illuminant functions with linear models.

The remainder of the paper is organized as follows. In the next section, we introduce notation for the OTF and show how to use it to predict photoreceptor responses. Next, we introduce linear models for surface and illuminant spectral functions and show how their use simplifies the prediction of photoreceptor responses. Then we show how our algorithm for matching color images across emissive displays is a special case of this simplification. Finally, we present a few conclusions.

The OTF and Retinal Images

To make these ideas more concrete, we introduce some notation and show how to use the OTF to compute retinal images. For simplicity, we deal only with one-dimensional images, but the extension to two dimensions is straightforward. Let the optical transfer function be $O(\nu, \lambda)$, where ν is spatial frequency in cycles per degree, and λ is wavelength in meters. Let the one-dimensional corneal image be $f(\chi, \lambda)$, where χ is spatial position in degrees of visual angle. At location $\chi 0$, for example, the SPD of the corneal image is $f(\chi 0, \lambda)$. We denote the corresponding retinal image as $g(\chi, \lambda)$. Finally, we denote the Fourier transform of these images with respect to the spatial variable, χ, using capital letters, $F(\nu, \lambda)$ and $G(\nu, \lambda)$.

The OTF O relates the Fourier transform of the corneal image, $F(\nu,\lambda)$, to the Fourier transform of the retinal image $G(\nu, \lambda)$, via

$$G(\nu, \lambda) = F(\nu, \lambda) O(\nu, \lambda)$$

To express the retinal image in terms of photoreceptor absorptions, we combine the OTF with the photopigment absorption curves $Ai(\lambda)$. When we compute the photoreceptor absorptions, Pi, from a uniform field with spectral power distribution $F(\lambda)$, we use the formula

$$P_i = \int_V F(\lambda) A_\iota(\lambda) d\lambda$$

where V is the range of visual wavelengths. But when we compute the Fourier transform of the spatial pattern of photoreceptor absorptions for the ith class of photoreceptors, $P_i(\nu)$, from an image with Fourier transform $F(\nu, \lambda)$, we must incorporate the OTF via the following equation:

$$P_i(v) = \int_v F(v,\lambda) O(\lambda, v) A_i(\lambda) d\lambda \qquad (1)$$

Linear Models for Surfaces and Illuminants

When a scene consists of surfaces and illuminants whose spectral functions can be described with finite-dimensional linear models, we can greatly simplify the prediction of retinal from corneal images. (For a introduction to the use of linear models for spectral functions, see Wandell[12]). Suppose for the illuminant SPD we have the linear model

$$E(\lambda) = \sum_{i=1}^{d_E} \omega_i^E E_i(\lambda),$$

where $E(\lambda)$ is the illuminant SPD, d_E is the dimension of the linear model, $E_i(\lambda)$ is the ith basis function, and ω_i^E the weight on the ith basis function. The spatial distribution of illuminant SPDs then becomes

$$e(\chi,\lambda) = \sum_{i=1}^{d_E} \omega_i^E(\chi) E_i(\lambda),$$

where the weights ω_i^E now depend on the spatial location χ. When we describe surface reflection functions with a linear model, the analogous equation for the spatial distribution of surface reflectances is

$$s(\chi,\lambda) = \sum_{j=1}^{d_s} \omega_i^s(\chi) S_j(\lambda).$$

The corneal image $f(\chi, \lambda)$ is the product of the illuminant and surface spatial distributions

$$f(\chi, \lambda) = e(\chi, \lambda)\, s(\chi, \lambda).$$

Expressing the illuminant and surface distributions in terms of their linear models, we have

$$\begin{aligned} f(\chi,\lambda) &= \left(\sum_{i=1}^{d_E} \omega_i^E(\chi) E_i(\lambda)\right) \left(\sum_{j=1}^{d_S} \omega_j^S(\chi) S_j(\lambda)\right) \\ &= \sum_{i=1}^{d_E} \sum_{j=1}^{d_s} \left(\omega_i^E(\chi)\, \omega_j^s(\chi)\right) \left(E_i(\lambda) S_j(\lambda)\right) \end{aligned} \qquad (2)$$

We now show that this defines a finite-dimensional model for the image $f(\chi, \lambda)$. Let $k = (i - 1)d_s + j$, so that k ranges from 1 to $d_F = d_E d_s$. Then define $F_k(\lambda) = E_i(\lambda) S_j(\lambda)$, and $\omega_k^F(\chi) = \omega_i^E(\chi)\, \omega_j^S(\chi)$. Substituting these expressions into Equation 2, we find that

$$f(\chi,\lambda) = \sum_{k=1}^{d_F} \omega_k^F(\chi) F_k(\lambda).$$

This implies a a d_F-dimensional linear model for $F(\lambda)$. The Fourier transform of $f(\chi, \lambda)$ is

$$F(v,\lambda) = \sum_{k=1}^{d_F} W_k^F(v) F_k(\lambda), \qquad (3)$$

where $W_K^F(v)$ is the Fourier transform of $\omega_k^F(\chi)$.

To compute the spatial pattern of photoreceptor absorptions, we again use Equation 1, but now we express the integral in a slightly different form by substituting the definition of F from Equation 3:

$$\begin{aligned} P_i(\nu) &= \int_V \left[\sum_{k=1}^{d_F} W_k^F(\nu) F_k(\lambda)\right] O(\lambda, \nu) A_i(\lambda) d\lambda \\ &= \sum_{k=1}^{d_F} W_k^F(\nu) \int_V F_k(\lambda) O(\lambda, \nu) A_i(\lambda) d\lambda \end{aligned}$$

By defining a new function,

$$C_{ik}(\nu) = \int_V F_k(\lambda) O(\lambda, \nu) A_i(\lambda) d\lambda$$

we can write the relationship between the spatial-frequency components of the d_F basis functions for F and the spatial frequency components of the spatial pattern of photoreceptors as

$$P_i(\nu) = \sum_{j=1}^{d_F} C_{ik}(\nu) F_k(\nu), \qquad (4)$$

which we can in turn write as a matrix multiplication computed separately for each spatial-frequency component:

$$\begin{pmatrix} P_1(v) \\ P_2(v) \\ P_3(v) \end{pmatrix} = \begin{pmatrix} C_{11}(v) & C_{12}(v) & C_{1d_F}(v) \\ C_{21}(v) & C_{22}(v) & C_{2d_F}(v) \\ C_{31}(v) & C_{32}(v) & C_{3d_F}(v) \end{pmatrix} \begin{pmatrix} F_1(v) \\ F_2(v) \\ \vdots \\ F_{d_F}(v) \end{pmatrix}. \quad (5)$$

We will express the spatial-frequency-dependent matrix multiplication using the matrix notation

$$P_v = C_v F_v, \quad (6)$$

where P_v is a three-dimensional column vector with entries $P_i(v)$, C_v is a three-by-three matrix with entries $C_{ik}(v)$, and F_v is a d_F-dimensional column vector with entries $F_k(v)$.

Color Matching on Emissive Displays

To model a emissive display in this framework, we observe that conventional displays have three primary lights, each with its own SPD. At each spatial location, the SPD of the display is a linear combination of the SPDs of the three primaries. Clearly this is the case of Equation 3, where the corneal image is expressed in terms of a finite-dimensional linear model. The dimension $d_F = 3$, and each $F_k(\lambda)$ is the SPD of one of the three primaries. In this case, the linear model is a consequence not of the linear models for the illuminant and surface spectral functions, but merely of the way that an emissive display forms images.

For an emissive display, Equations 4-6 generalize the matrix equation commonly used in color calibration from the special case of a uniform field. When we incorporate the OTF into the calculation, we can relate the primary intensities to the photoreceptor absorptions by expressing the images in the spatial-frequency domain. We must use a different matrix at each spatial frequency. The entries of the matrix are determined by the spectral power distributions of the display primaries, the photopigment absorption curves, and the optical transfer function of the eye. In the case of an emissive display, we call the collection of matrices C_v the *device-calibration matrices.*

Notice that the matrix C_0 defines the mapping from the display primary intensities of a uniform field (spatial frequency of zero) to the receptor responses. This three-by-three calibration matrix is widely used in conventional colorimetry.[2,4,13] The device-calibration matrices, which now depend on spatial frequency, generalize conventional colorimetric mapping from uniform fields to patterned images.

Using Equation 6, we can develop a method of equating the photoreceptor absorptions from images on display with different primaries. Suppose we have two displays with calibration matrices C_v and C'_v. Consider an image, F'_v. We can calculate the expected pattern of photoreceptor absorptions for the image on the first display from the matrix multiplication, $C_v F_v$. To equate photoreceptor absorptions from the two images requires that we find an image on the second display, defined by F_v such that at each spatial frequency,

$$C'_v F'_v = C_v F_v. \quad (7)$$

For each spatial frequency, v, we can solve for the image F'_v using

$$F'_v = (C'_v)^{-1} C_v F_v$$

In practice, we may be limited in how closely we can obtain the matches since the matrices C'_v may not be invertible, and the solutions may not lie within the color gamut of the second display.

Conclusions

Because of the eye's chromatic aberration, the wavelength and spatial frequency of corneal images interact in the formation of retinal images. In our earlier work we have modeled this interaction as an OTF, calculated the OTF based on optical and psychophysical measurements, and quantified some consequences of the OTF interaction for color perception and imaging matching. This led to a new algorithm for matching color images across emissive displays. Here, we showed that using linear models of surface and illuminant spectral functions greatly simplifies the use of the OTF for images arising from natural scenes, and that our algorithm for matching color images is a special case of this use of linear models.

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