Color Reproduction and Color Vision Modeling

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Abstract
The principles of spectral color reproduction, as exemplified by the Lippmann method, and of trichromatic color reproduction, as used by presentday systems, are reviewed. Colorimetry is also based on trichromatic principles, and provides the basis for the quantitative evaluation of color reproduction in television, photography, and printing. Being metameric, these reproductions are affected by changes in illuminant and observer. They are also sensitive to changes in their viewing conditions; these changes can be represented by color vision models, one of which is briefly described.

Spectral Color Reproduction
Colors are defined physically by their spectral power distributions. To reproduce colors with the same spectral power distributions as in the original scene is difficult. The most elegant method so far devised for doing this is the interference method invented by Lippmann. However, its low sensitivity and general inconvenience have made it obsolete for many years.

Trichromatic Color Reproduction
Because the eye does not respond to each wavelength of the spectrum, but only to the light in three broad bands, it is possible to reproduce colors trichromatically, by displaying them as mixtures of red, green, and blue light.

Colorimetry
Colorimetry also depends on the trichromatic principle. Colors of each wavelength of the spectrum were matched with mixtures of red, green, and blue light by groups of observers, and the results averaged to represent the response of Standard Colorimetric Observers. These data have led to the now universally-used CIE system of colorimetry, with its X,Y,Z tristimulus values, the x,y and u’,v’, chromaticity diagrams, and the CIELUV and CIELAB color spaces.

Television
If a television camera has the same spectral sensitivity functions as those of the three different types of cone in the eye, correct color reproduction does not occur because each display color stimulates more than one of the three different cone types. This is true whether the display device uses a cathode-ray tube, a triple projector, or a liquid crystal array. These unwanted stimulations result in the colors in the reproduction having reduced saturation and distorted hues. These errors can be corrected by passing the signals from the camera through a matrixing circuit which removes the effects of the unwanted stimulations; but this can only succeed within the gamut of reproducible colors. Colors outside this gamut require negative signals which cannot be displayed. The gamut limitation cannot be avoided, because it results from the overlapping nature of the cone spectral sensitivities.

Photography
In photography, the display normally consists of cyan, magenta, and yellow dye images, and the bands of wavelengths controlled by them also result in unwanted stimulations. But, unlike television, photography can provide no convenient way of introducing matrixing. What is usually done is to increase the wavelength separation of the three spectral sensitivities of the film as compared to those of the eye, and this improves the fidelity of most colors, although it introduces marked hue errors for a few colors such as certain types of blue flower. The dyes used in photography also absorb in parts of the spectrum where they should not do so, and these unwanted absorptions result in colors being too dark; however, the use of colored couplers and interimage effects are very effective in making the necessary corrections.

Half-tone and Digital Printing
In conventional printing, the modulation of the light in the display by the cyan, magenta, and yellow inks is achieved by varying the size of dots of ink that are small enough not to be resolved by the eye at normal viewing distances. In digital printing, it is necessary to construct these half-tone dots from patterns of micro-dots; to avoid spurious contouring effects, about ten times as many micro-dots are required per unit distance as compared to the number of half-tone dots. For high quality work this requires microdots of about 1/100 mm in diameter. However, by distributing the micro-dots in dithered patterns instead of in clusters, or by using error diffusion techniques (distribution over neighbouring pixels of the errors caused by using too few micro-dots) high quality results can be obtained with larger micro-dots.

Metamerism
Trichromatic reproductions inevitably result in almost all colors being reproduced with spectral power distributions that are markedly different from those in the original scene. This metamerism means that even if the reproduction matches the original for one illuminant and observer, it will not necessarily do so for others. Non-self-luminous displays, such as liquid crystal television devices, photographic transparencies and reflection prints, printed papers, and desktop publications, are therefore sensitive to changes in the spectral power distribution of their illuminant; and all...
displays, including self-luminous devices such as cathode-ray tubes, are sensitive to changes in observer, such as may arise because an individual observer differs significantly from a chosen CIE Standard Observer.

Evaluation of Color Differences

The colorimetric evaluation of color differences in color reproduction is usually carried out using the CIE 1931 Standard Colorimetric Observer; the data on which this observer is based were obtained with fields of 2° angular subtense, whereas those for the CIE 1964 Observer were obtained with fields of 10° subtense, which are much larger than typical uniform areas in pictures. The CIE color difference formulae associated with the CIEULV and CIELAB color spaces are widely used, the former usually for self-luminous displays, and the latter usually for non-self-luminous displays. These formulae are appropriate if the original and reproduction are viewed under the same conditions, and the white points are not too different from those that occur in typical daylight. For other illuminants, and for asymmetrical viewing situations, more elaborate procedures are necessary, and a model of color vision may be used.

A Model of Color Vision

The conditions under which colors are viewed affect their appearance. Because of this, it is not appropriate, for instance, to represent colors on reflection copy by displaying the same tristimulus values on a cathode-ray tube. It is necessary to determine the appearance of the colors in the reflection copy and then to display on the cathode-ray tube colors having those tristimulus values that result in the same appearance. A model of color vision can be used to determine these appearances. Details of the steps required to use one such model are given below. This model, referred to originally as the Hunt 91 model, has now been provided with improved predictors of chroma and colorfulness; these are incorporated in this paper, and this constitutes the version identified as the Hunt 94 model.

Main Features of the Model

For the spectral sensitivities of the cones of the retina, the model uses a linear combination of the CIE \( x, y, \) and \( z \) color matching functions; this means that the input to the model can be in the form of CIE tristimulus values, \( X, Y, \) and \( Z \). The way in which the cones respond to variations in stimulus intensity is represented by a function that has a gradual transition to a minimum at very low intensities, and a gradual transition to a maximum at very high intensities. Adaptation is allowed for by first dividing the cone responses by those for a suitably chosen reference white, and then reducing the effect of this division by means of a luminance-level adaptation factor, three chromatic adaptation factors, three cone bleach factors, and three Nelson-Judd effect factors. The resulting modified cone responses, \( \rho_a, \gamma_a, \) and \( \beta_a \), are used to form three color difference signals:

\[
C_1 = \rho_a - \gamma_a; \quad C_2 = \gamma_a - \beta_a; \quad C_3 = \beta_a \cdot \rho_a.
\]

An achromatic signal, \( A \), is also formed by combining the three responses, \( \rho_a, \gamma_a, \) and \( \beta_a \), in proportions that allow for the different abundances of the three types of cone, together with a response, \( A_S \), from the rods of the retina:

\[
A = [2 \rho_a + \gamma_a = (1/20) \beta_a + A_S + n] N_{bb}^* \\
A = [2 \rho_a + \gamma_a + (1/20) \beta_a + A_S + n] N_{bb}
\]

where \( n \) is a constant representing noise and \( N_{bb}^* \) is a parameter that allows for the effect of the luminance factor of the background. These four signals, \( A, C_1, C_2, \) and \( C_3 \), are then used to formulate predictors for hue, brightness, lightness, saturation, chroma, and colorfulness.

Achromatic colors are those that do not exhibit a hue (such as whites, greys, and blacks). The criterion adopted for achromacy is that \( \rho_a = \gamma_a = \beta_a \), and hence the three color difference signals, \( C_1, C_2, \) and \( C_3 \), are all zero. The chromatic response then increases as \( C_1, C_2, \) and \( C_3 \) become increasingly different from zero.

Constant hue corresponds to \( C_1, C_2, \) and \( C_3 \) being in constant ratios to each other. The unique hues are defined as corresponding to the following ratios:

- Unique red: \( C_1 = C_2 \)
- Unique green: \( C_1 = C_3 \)
- Unique yellow: \( C_1 = C_2/11 \)
- Unique blue: \( C_1 = C_2/4 \)

Brightness depends on the achromatic signal, \( A \) (together with a small contribution from the chromatic response of the cones), with due allowance being made for the effects of the surround. Lightness is a function of the brightness of a color relative to that of the reference white.

Saturation is represented by the chromatic response relative to the sum of the three cone responses, \( \rho_a + \gamma_a + \beta_a \); chroma by saturation with due allowance being made for the reduction in colorfulness that accompanies reductions in brightness relative to that of the reference white; and colorfulness by chroma with due allowance being made for the reduction in colorfulness that accompanies reductions in level of illumination.

Details of the input data required to use the model, and the steps for making the calculations, are given below.

Testing the Model Predictions

One way of recording observers’ perceptions of color appearance is to use magnitude estimation. In this technique observers use numbers to express the magnitudes of different perceptions, such as hue, lightness, and colorfulness. In the case of hue, the apparent percentage content of two of the appropriate neighboring unique hues, red, yellow, green, and blue are used; in the case of lightness, the apparent location on a scale extending from zero for a perfect imaginary black to 100 for the reference white is used; in the case of colorfulness, the apparent location on a scale from zero for whites, greys, and blacks, to larger numbers for colors that exhibit hue, is used.

* erratum shown in shaded box; 
revised errata is shown below original
In one very extensive investigation using this technique, hue, lightness, brightness, and colorfulness were scaled for related colors seen under a very wide variety of viewing conditions; these included: reflecting samples seen in an illuminated viewing booth; self-luminous samples displayed on a monitor; transmitting samples in large transparencies viewed on a light box; and transmitting samples in 35 mm slides projected in a dark room. The scalings obtained, referred to as the LUTCHI data base, have been used to determine the accuracy with which the Hunt 94 model predicts the scaled percepts, using percentage coefficients of variation (CVs). These CVs are calculated as 100 times the root-mean square of the differences between the predictions and the experimental results, divided by the average of the experimental results. The CV results are summarised in Table I, and are about 8 for hue, 12 for lightness, 11 for brightness, and 18 for colorfulness. These values are similar to those obtained when the difference between a single observer’s results are compared to those for the average of six observers. Observers’ consistency in scaling was thus best for hue, worst for colorfulness, and intermediate for lightness, and for brightness, and this was also true of the predictions given by the model.

Table I

<table>
<thead>
<tr>
<th>Conditions and Observer Consistency</th>
<th>Hue</th>
<th>Lightness</th>
<th>Brightness</th>
<th>Colorfulness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Booth &amp; Monitor</td>
<td>8</td>
<td>11</td>
<td>-</td>
<td>18</td>
</tr>
<tr>
<td>Booth at six levels</td>
<td>8</td>
<td>14</td>
<td>11</td>
<td>22</td>
</tr>
<tr>
<td>(omitting lowest level)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Large transparencies</td>
<td>7</td>
<td>10</td>
<td>-</td>
<td>18</td>
</tr>
<tr>
<td>Projected slides</td>
<td>7</td>
<td>12</td>
<td>-</td>
<td>18</td>
</tr>
<tr>
<td>One observer compared to the average for six observers</td>
<td>8</td>
<td>13</td>
<td>10</td>
<td>17</td>
</tr>
</tbody>
</table>

**Input Data Needed for the Model**

The following input data are needed:

**Chromaticity co-ordinates, x, y, and luminance factors, Y, in the illuminant considered:**

<table>
<thead>
<tr>
<th>Illuminant</th>
<th>xI, yI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adapting field</td>
<td>xA, yA</td>
</tr>
<tr>
<td>Background</td>
<td>xb, yb, Yb</td>
</tr>
<tr>
<td>Reference white</td>
<td>xW, yW, YW</td>
</tr>
<tr>
<td>Samples:</td>
<td>x, y, Y</td>
</tr>
</tbody>
</table>

By the adapting field is meant the total environment of the color element considered, extending to the limit of vision in all directions. By the background is meant the environment of the color element considered, extending typically for about 10° from the edge of the element in all, or most directions. If the adapting field is a typical average scene, or generally white, grey, or black, it is usually acceptable to take xA, yA as the same as xI, yI. If the background is a typical average scene, or generally white, grey, or black, xb, yb can be taken as the same as xI, yI, and a nonselective neutral can then be taken as the reference white and hence xW, yW are also the same as xI, yI. If this neutral is the perfect diffuser, then YW = 100. If the background is a typical average scene, Yb can be taken as 20; if the background is white, grey, or black, then Yb is the value of Y for the background considered.

**Luminances of Stimuli, and Scotopic Data:**

Photopic luminance of reference white in cd/m²: 
\[ L_W = \frac{L_p Y_W}{100} \]

where \( L_p \) is the luminance of the perfect diffuser in cd/m²; \( L_p = \frac{E}{\pi} \) where \( E \) is the illuminance in lux. (The photopic luminances, \( L \), of the samples, in cd/m², are given by \( L = \frac{L_p Y}{100} \).)

Photopic luminance of adapting field in cd/m²: 
\[ L_A \]

If the value, \( L_A \), is not available, \( L_A \) can be used as an approximation.

Scotopic luminance of adapting field in scotopic cd/m²: 
\[ L_{AS} \]

If the value, \( L_{AS} \), is not available, an approximation to it can be derived from \( L_A \) as:
\[ L_{AS} / 2.26 = L_A (T/4000) - 0.4 \]

where \( T \) is the correlated color temperature of the illuminant. \( L_{AS} / 2.26 \) is used throughout the model instead of \( L_{AS} \), because \( L_{AS} / 2.26 = L_A \) for the equi-energy stimulus \( S_E \). In the above formula this is achieved by putting \( T = 5600 \) for \( S_E \).

Scotopic luminances relative to reference white: \( S/S_W \)

If the scotopic values, \( S/S_W \), are not available, the equivalent photopic values, \( Y/Y_W \), can be used instead as an approximation.

**Values of the chromatic and brightness surround induction factors:**

Chromatic surround induction factor: \( N_c \)
Brightness surround induction factor: \( N_b \)

If optimised values, \( N_c \) and \( N_b \), are not available, the following values can be used:

Small areas in uniform light backgrounds and surrounds
Normal scenes
Television and VDU displays in dim surrounds
Large transparencies on light boxes
Projected photographs in dark surrounds

Values of the chromatic and brightness background induction factors:

Chromatic background induction factor: \( N_{cb} \)
Brightness background induction factor: \( N_{bb} \)

If optimised values, \( N_{cb} \) and \( N_{bb} \), are not available, the following values can be used:

\[ N_c = 0.725 \left( \frac{Y_W}{Y_b} \right)^{0.2} \]
\[ N_b = 0.725 \left( \frac{Y_W}{Y_b} \right)^{0.2} \]
**Steps for the Calculations**

**Step 1.** Calculate $X$, $Y$, $Z$ for the reference white, and for the samples.

$X = xY/y$ \quad $Y = Y$ \quad $Z = (1 - x - y) Y/y$

**Step 2.** Calculate $\rho$, $\gamma$, $\beta$ for the reference white, and for the samples.

$\rho = 0.38971X + 0.68898Y - 0.07868Z$

$\gamma = -0.22981X + 1.18340Y + 0.04641Z$

$\beta = 1.00000Z$

**Step 3.** Calculate $\rho/\rho_w$, $\gamma/\gamma_w$, $\beta/\beta_w$ for the samples.

$\rho_w$, $\gamma_w$, $\beta_w$ are the values of $\rho$, $\gamma$, $\beta$ for the reference white.

**Step 4.** Calculate $F_L$

$F_L = 0.2k^4(5L_A) + 0.1(1-k^2)^2(5L_A)^{1/3}$

where $k = 1/(5L_A + 1)$

**Step 5.** Calculate $F_\rho$, $F_\gamma$, $F_\beta$

$h_\rho = 3\rho W/ (\rho W + \gamma W + \beta W)$

$h_\gamma = 3\gamma W/ (\rho W + \gamma W + \beta W)$

$h_\beta = 3\beta W/ (\rho W + \gamma W + \beta W)$

$F_\rho = (1 + L_A^{1/3} + h_\rho) / (1 + L_A^{1/3} + 1/h_\rho)$

$F_\gamma = (1 + L_A^{1/3} + h_\gamma) / (1 + L_A^{1/3} + 1/h_\gamma)$

$F_\beta = (1 + L_A^{1/3} + h_\beta) / (1 + L_A^{1/3} + 1/h_\beta)$

If the color of the illuminant is discounted, $F_\rho$, $F_\gamma$, and $F_\beta$ are all set at unity.

**Step 6.** Calculate $\rho_D$, $\gamma_D$, $\beta_D$

$\rho_D = \varepsilon_a [ (Y_b / Y_W)F_L F_\gamma ] - \varepsilon_a [ (Y_b / Y_W)F_L F_\rho ]$

$\gamma_D = 0$

$\beta_D = \varepsilon_a [ (Y_b / Y_W)F_L F_\gamma ] - \varepsilon_a [ (Y_b / Y_W)F_L F_\beta ]$

where $\varepsilon_a[I] = 40 [I^{0.73} / (I^{0.73} + 2)]$

If there is no Helson-Judd effect, $\rho_D$, $\gamma_D$, and $\beta_D$ are all set at zero. (If $F_\rho$, $F_\gamma$, and $F_\beta$ are all set at unity, $\rho_D$, $\gamma_D$, and $\beta_D$ all become zero as a consequence).

**Step 7.** Calculate $\rho_a$, $\gamma_a$, $\beta_a$

$\rho_a = B_\rho [ \varepsilon_a (F_L F_\rho \rho/\rho_w) + \rho_D ] + 1$

$\gamma_a = B_\gamma [ \varepsilon_a (F_L F_\gamma \gamma/\gamma_w) + \gamma_D ] + 1$

$\beta_a = B_\beta [ \varepsilon_a (F_L F_\beta \beta/\beta_w) + \beta_D ] + 1$

where $B_\rho = 10^7 / [ 10^7 + 5L_A(\rho_w / 100) ]$

$B_\gamma = 10^7 / [ 10^7 + 5L_A(\gamma_w / 100) ]$

$B_\beta = 10^7 / [ 10^7 + 5L_A(\beta_w / 100) ]$

and

$\varepsilon_a[I] = 40 [I^{0.73} / (I^{0.73} + 2)]$

**Step 8.** Calculate $A_a$, $C_1$, $C_2$, $C_3$

$A_a = 2\rho_a + \gamma_a + (1/20)\beta_a - 3.05 + 1$

$C_1 = \rho_a - \gamma_a$

$C_2 = \gamma_a - \beta_a$

$C_3 = \beta_a - \rho_a$

**Step 9.** Calculate $h_t$

$h_t = \arctan ( [ 1/2 (C_2-C_3)/4.5 ] / [ C_1-(C_2/11) ] )$

**Step 10.** Calculate the hue quadrature $H$

$H = H_1 + \frac{100( (h_t - h_1) / e_1 )}{( (h_t - h_1) / e_1 + (h_2 - h_1) / e_2 )}$

where $H_1$ is either 0, 100, 200, or 300, according to whether red, yellow, green, or blue, respectively, is the hue having the nearest lower value of $h_t$. The values of $h_t$ and $e_1$, for the four unique hues are:

<table>
<thead>
<tr>
<th>Hue</th>
<th>$h_t$</th>
<th>$e_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>20.14</td>
<td>0.8</td>
</tr>
<tr>
<td>Yellow</td>
<td>90.00</td>
<td>0.7</td>
</tr>
<tr>
<td>Green</td>
<td>164.25</td>
<td>1.0</td>
</tr>
<tr>
<td>Blue</td>
<td>237.53</td>
<td>1.2</td>
</tr>
</tbody>
</table>

$e_1$ and $h_1$ are the values of $e_1$ and $h_1$, respectively, for the unique hue having the nearest lower value of $h_t$; and $e_2$ and $h_2$ are these values for the unique hue having the nearest higher value of $h_t$.

**Step 11.** Calculate the Hue Composition, $H_C$

Where $H_p$ is the part of $H$ after its hundreds digit, If

$H = H_p$, the Hue Composition is

$H_p$ Yellow, $100 - H_p$ Red

$H = 100 + H_p$, the Hue Composition is $H_p$ Green, $100 - H_p$ Yellow

$H = 200 + H_p$, the Hue Composition is $H_p$ Blue, $100 - H_p$ Green

$H = 300 + H_p$, the Hue Composition is $H_p$ Red, $100 - H_p$ Blue
Step 12. Calculate $e_s$

$$e_s = e_1 + (e_2 - e_1) \frac{(h_s - h_1)}{(h_2 - h_1)}$$

where $e_1$ and $h_1$ are the values of $e_s$ and $h_s$, respectively, for the unique hue having the nearest lower value of $h_s$; and $e_2$ and $h_2$ are these values for the unique hue having the nearest higher value of $h_s$.

Step 13. Calculate $F_t$

$$F_t = \frac{L_A}{(L_A + 0.1)}$$

Step 14. Calculate the yellowness-blueness, $M_{YB}$, the redness-greenness, $M_{RG}$, the chromatic response, $M$, the relative yellowness-blueness, $m_{YB}$, the relative redness-greenness, $m_{RG}$, and the saturation, $s$

$$M_{YB} = 100 \left[ \frac{1}{2} \left( C_2 - C_s \right) / 4.5 \right] \left[ e_s (10 / 13) N_{c_e} \right]$$

$$M_{RG} = 100 \left[ (C_1 - (C_2 / 11)) \right] \left[ e_s (10 / 13) N_{c_b} \right]$$

$$M = (M_{YB}^2 + M_{RG}^2)^{1/2}$$

$$m_{YB} = \frac{M_{YB}}{(\rho_a + \gamma_a + \beta_a)}$$

$$m_{RG} = \frac{M_{RG}}{(\rho_a + \gamma_a + \beta_a)}$$

$$s = \frac{50M}{(\rho_a + \gamma_a + \beta_a)}$$

Step 15. Calculate $F_{LS}$

$$F_{LS} = 3800 j^2(5L_{AS}/2.26) + 0.2 (1 - j^2)^4 (5L_{AS}/2.26)^{1/6}$$

where

$$j = 0.00001 / [(5L_{AS} / 2.26) + 0.00001]$$

Step 16. Calculate $A_S$

$$A_S = B_S \left( 3.05 \left[ e_n (F_{LS}/S/W) \right] + 0.3 \right)$$

where

$$B_S = 0.5 / \left[ 1 + 0.3 \left( 5L_{AS}/2.26(S/S_W) \right)^{0.3} \right] + 0.5 / \left[ 1 + 5 \left( 5L_{AS}/2.26 \right) \right]$$

and

$$e_n[I] = 40 / \left( P^{0.73}(P^{0.73} + 2) \right)$$

Step 17. Calculate $A$

$$A = N_{bb}A_w - 1 + A_r - 0.3 + (1^2 + 0.3^2)^{1/2}$$

Step 18. Calculate $A + (M/100)$

Step 19. Calculate $Q$ and $Q_W$

$$Q = \left\{ 7 \left( A + (M/100) \right) \right\}^{0.6} N_1 - N_2$$

where

$$N_1 = (7A_w)^{0.5} / (5.33 N_{1w}^{0.13})$$

$$N_2 = (7A_w N_{2w}^{0.362}) / 200$$

and $A_w$ is the value of $A$ for the reference white.

Step 20. Calculate $J$

$$J = 100(Q/Q_W)z$$

Step 21. Calculate the chroma, $C_{94}$, and the colorfulness, $M_{94}$

$$C_{94} = 2.44s^{0.69}(Q/Q_W)^{1.64 - 0.29(Y_b/Y_W)}$$

$$M_{94} = F_L^{0.15} C_{94}$$

Step 22. Tabulate the values of $H$, $H_C$, $M_{94}$, $s$, $Q$, $J$, and $C_{94}$.

References


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* erratum shown in shaded box; revised errata is shown below original